Low Power Hitch-hiking Broadcast in Ad Hoc Wireless Networks

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Abstract

Energy consumption is an important factor to be considered in designing algorithms for ad hoc wireless networks, where wireless nodes are assumed in general to be battery powered. The Hitch-hiking model introduced recently in [1] takes advantage of the physical layer design that facilitates combining of partial signals containing the same information in order to decode the complete message. Agarwal et al. also considered the Minimum energy Broadcast tree with Hitch-hiking (MBH) problem and proposed an algorithm WMH (Wireless Multicast with Hitch-hiking). The contribution of this paper is to prove that WMH has a constant performance ratio.

Key Words: broadcast, energy efficiency, ad hoc wireless networks, Hitch-hiking model.

1 Introduction and Related Work

Wireless networks provide rapid, untethered access to information and computing, eliminating the barriers of distance, time, and location for many applications in national security, civilian search and rescue operations, as well as in the area of personal communications. Ad hoc networks consist of wireless hosts that can communicate with each other in the absence of a fixed infrastructure. They are characterized by a dynamic topology, limited resources and limited security.

Wireless devices are battery powered and therefore have a limited operational time. Recently, the optimization of energy utilization of wireless terminals has received significant attention [6]. Different techniques for power management have been proposed at all layers of the network protocol stack. Power saving techniques can generally be classified in two categories: power savings by scheduling the wireless nodes to alternate between the active and sleep mode and power control by adjusting the wireless nodes transmission range. In this paper, we deal with the second power-control method.

The broadcasting task is an important communication mechanism, used for example in route discovery and in the dissemination of control packets in link state routing protocols. Due to the limitations of the energy resources in wireless networks, the design of efficient broadcasting is very important. This can be achieved by minimizing the number of forwarding nodes or by minimizing total transmission power by adjusting node transmission range.

Minimum energy broadcasting has been addressed previously in literature. An extensive discussion of energy-efficient broadcasting mechanisms in ad hoc wireless networks is presented by Ingelrest et al. in [5]. A few of these results are briefly reviewed next.

Cagalj et al.[2] prove that the minimum energy broadcast problem is NP-complete. Wieselthier et al. [11] have proposed three greedy centralized algorithms: BIP (Broadcast Incremental Power) which is a variant of the Prim's algorithm, BLU (Broadcast Least Unicast cost) that applies Dijkstra's algorithm between the source and each node, and BLiMST (Broadcast Link-based MST) when the broadcast tree is the MST. Wan et al. [10] proved analytical results for these three algorithms. The performance ratio of MST was shown to be between 6 and 12, for BIP between 13/3 and 12, and for SPT at least $n/2$, where n is the number of receiving nodes.

Li et al. [7] propose Broadcast with Local Minimum Spanning Tree (BLMST) where broadcast messages are simply relayed on a connected topology, which is previously constructed with Local minimum Spanning Tree (LMST) algorithm. The work in [2] proposes a centralized heuristic, Embedded Wireless Multicast Advantage (EWMA), that starts from the MST solution and improves energy consumption by increasing the energy of some nodes in order to change the state of some neighbors from forwarding node to leaf node. The paper also presents a distributed version of this algorithm.

Agarwal et al. [1] address broadcasting with Hitch-hiking model (see section 2). The authors proposed a centralized algorithm WMH (Wireless Multicast with Hitch-hiking) described in subsection 3.1 and its distributed version. This paper also proposed a protocol that reduces broadcast energy consumption by eliminating redundancy in the receive operation.

In [3], we have studied the effect of the Hitch-hiking model on the topology control problem. We defined the Topology Control with Hitch-hiking (TCH) problem, where the goal is to assign a power level to every node in the network such that the total energy consumption is minimized while the resulting Hitch-hiking based topology is strongly connected. We have proved that TCH is NP-complete, and have designed an algorithm, DTCH, which is distributed, localized and has constant performance ratio of $2/k$, where k is a characteristic of the physical medium.

The rest of the paper is organized as follows. Section 2 describes the Hitch-hiking and network models. We continue with a presentation of the MBH problem and WMH algorithm in subsection 3.1 and a proof of a constant performance ratio in subsection 3.2. The paper ends with conclusions in section 4.

2 Hitch-hiking and Network Models

Hitch-hiking model [1] applies to the physical layer and allows effectively combining partial signals in order to decode the full packet. The concept of combining partial signals using maximal ratio combiner [8] has been used traditionally in physical layer design of wireless systems to increase reliability. The Hitch-hiking model [1] introduces two thresholds related with SNR (signal to noise ratio): γ_p which is the threshold needs for successfully decoding the packet payload and γ_{acq} which is the threshold required for a successful time acquisition. The system is characterized by $\gamma_{acq} < \gamma_p$. We note with k the ratio of these two thresholds, $k = \gamma_{acq}/\gamma_p$. A packet received with a SNR γ is (1) fully received if $\gamma_{acq} < \gamma_p \leq \gamma$, (2) partially received if $\gamma_{acq} \leq \gamma < \gamma_p$, or (3) unsuccessfully received if $\gamma < \gamma_{acq}$.

The model also assumes omnidirectional antenna and employs a free space propagation energy model [9]. Consider that a wireless node x transmits a packet with power level $P_t = r^\alpha$, where α is a communication medium dependent parameter and r is the communication range of node x. A channel gain is often modeled as a power of the distance, resulting in $\gamma/\gamma_p = r^\alpha/d_{xy}^\alpha = (r/d_{xy})^\alpha$, where d_{xy} is the distance between nodes x and y. γ/γ_p is also referred as the coverage of node y by the node x. For example, consider $\gamma_{acq}/\gamma_p = 0.125$ and $\alpha = 2$. For a node y with $r/d_{xy} = 1/2$, the coverage is 0.25, whereas for the case $r/d_{xy} = 1/3$ the coverage is 0.

The basic idea in the Hitch-hiking model is that if the same packet is partially received n times from different neighbors with $\gamma_{acq} \leq \gamma_i < \gamma_p$ for $i = 1..n$ such that $\sum_{i=1}^{n} \gamma_i \geq \gamma_p$ then the packet can be successfully decoded. This is equivalent to saying that if the same packet is additively covered more than 1 by different senders, then the packet is considered to be received successfully.

The network model considered is a static ad-hoc wireless network with n nodes equipped with omnidirectional antennas and capable of receiving and combining partial received packets in accordance with the Hitch-hiking model. The network is represented by a directed graph $G = (V, E)$ where the vertices set V is the set of nodes corresponding to the wireless devices in the network and the set of edges E corresponds to the communication links between devices. Between any two nodes i and j there is an edge ij if the transmission from node i is received by node j with a SNR greater than γ_{acq}. Every node $i \in V$ has an associated transmission power level p_i. The coverage provided by an edge $ij \in E$ to the destination node is (1) 1 if $p_i/d_{ij}^\alpha \geq \gamma_p$ and (2) p_i/d_{ij}^α if $\gamma_{acq} \leq p_i/d_{ij}^\alpha < \gamma_p$, where d_{ij} is the Euclidean distance between nodes i and j. The case $p_i/d_{ij}^\alpha < \gamma_{acq}$ is not included since an edge will exist only when the SNR of the received signal is greater than γ_{acq}, that is $p_i/d_{ij}^\alpha \geq \gamma_{acq}$. Usually α equals 2 or 4.

In designing the solution $\gamma_p = 1$, which implies that if a SNR of a received signal is greater than or equal to 1, then the signal can be successfully decoded. Also, each node has assigned a power level between 0 and a maximum value P_{max} determined by the hardware constraints of the node.

3 Minimum-Energy Broadcast with Hitch-hiking (MBH) Problem

In subsection 3.1 we present the Minimum energy Broadcast with Hitch-hiking (MBH) problem and the Wireless Multicast with Hitch-hiking (WMH) algorithm as they were proposed in [1]. For more details the readers are referred to the paper [1]. In subsection 3.2 we prove that WMH algorithm has a constant performance ratio with the optimal solution of the MBH problem.

3.1 MBH Problem Definition and WMH Algorithm

The MBH problem can be defined as follows. Given a source node S and using the Hitch-hiking model, determine a set of forwarding nodes and their power level such that the message sent by S is received by all the nodes in the network and the total energy consumed for this task is minimized.

The MBH problem is NP-complete. This was proved in [1] by showing that the minimum energy broadcast problem without Hitch-hiking is a special case of MBH .

Next, we present the centralized wireless multicast with Hitch-hiking (WMH) algorithm [1] and show an example. In WMH, any node decides its final power based only on local information, of children and grandchildren nodes. Also, $\gamma_{acq} > 0$ and $\gamma_p = 1$.

WMH algorithm starts by constructing a Minimum Spanning Tree (MST). We assume that Prim's algorithm [4] is used to build a MST rooted at the source node S. The power level of each node is set to the power needed to reach the furthest children in the MST. The WMH algorithm decides the final power level of each node, one at a time, starting with the source node. Once a node decides its final power level, that value is not changed.

Let us introduce some notations. $pc(j)$ represents the coverage of node j in percentage and p_j is the power level assigned to node j. A node is fully covered if $pc(j) \geq 100$. Initially, the source node S has $pc(S) = 100$ and all others have pc set to 0. The goal is to have all nodes fully covered, that means all nodes will receive the message sent by the source S after a finite number of steps. A node j is a deciding node if it didn't decide its final power yet, $p_j > 0$ and $pc(j) = 100$. If more deciding nodes are available, the algorithm picks the one with smaller ID.

The approach taken by node i to decide its final power is as follows. The node i computes the gain for various power levels and will consider as final power the power level for which the gain is maximum. Let us introduce some notations: $PL(i)$ represents the set of power levels considered for node i; $CH(i)$ is the set of i's children nodes in the MST; $g_i(p)$ is the reduction in total power consumed when the power level of node i is set to p and $d_{i,j}$ is the distance between nodes i and j.

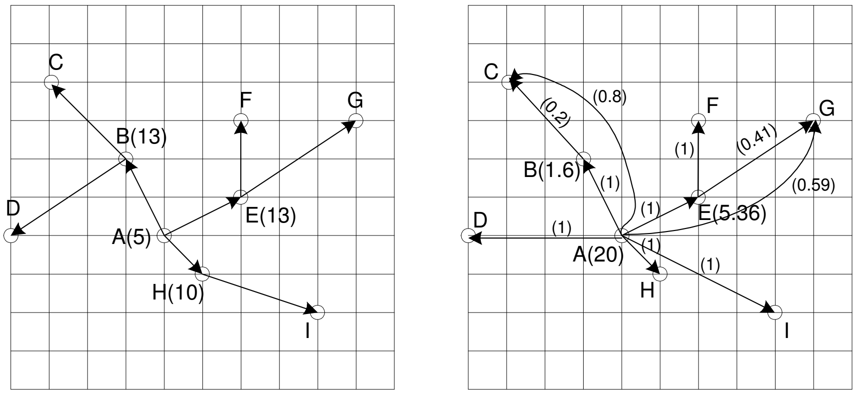
The power levels considered, $PL(i)$, are those power levels at which node i contributes to fully cover all the child nodes of node j for at least one $j \in CH(i)$. Basically, $PL(i)$ contains all power levels for which node i could reduce the power level of any child node j, by providing full coverage to all j's children.

The gain $g_i(p)$ is defined as the decrease in the total topology power, obtained by reducing the power level of some of the transmitting nodes, in exchange for the increase in node i's transmission power level to p. This is because when the power level of node i is increased, it provides partial or full coverage to more nodes in the network. For example, if k is a child of the node j, where $j \in CH(i)$, then an increase in the partial or full coverage of the node k will facilitate reduction of the power level of node j that has to provide less coverage to node k.

The process of computing the gain is performed for each power level $p \in PL(i)$. Once the gain for all power levels in $PL(i)$ is determined, final power level of node i is chosen as the one for which the gain is maximum. If no power level p achieves $g_i(p) > 0$, then p_i does not change. Once a node i has decided its final power, the coverage of the other nodes (children and grandchildren) is updated, based on the additional coverage provided by node i.

Let us now illustrate this mechanism on the example in Figure 1, with a topology consisting of nine nodes, when source node is A and $\alpha = 2$. The number on each node indicates the power level used by that node when forwarding the message. The number on each edge represents the coverage provided by that edge to the destination node. A value of 1 refers to full coverage while values less than 1 indicate the amount of partial coverage.

Figure 1 (a) represents an MST based broadcast tree, without Hitch-hiking. The power level assigned to each forwarding node is the power needed to reach the furthest children in the MST. In this case we obtain a total cost of 41.



a) MST without Hitch-hiking: cost 41

b) With Hitch-hiking, after applying WMH at node A: cost 26.96

Figure 1: *Example of energy-efficient broadcast trees*

In Figure 1 (b), we start from the MST topology and present the power assignment after node A executed the WMH algorithm. We assume $\gamma_{acq} = 0.1$. First, node A computes the set of transmission power levels $PL(A) = \{5, 20, 25, 34\}$. Then, by taking each power level in turn, the maximum gain is obtained for a power level of 20, as represented in Figure 1 (b). Then, the power levels of all A's neighbors are reduced due to the additional coverage provided by A to their children. For example, as A provides a coverage of (0.8) on C, the power level of B can be reduced to 1.6 such that to provide a coverage of (0.2) on C. These two partial coverages will suffice for a full coverage of node C. The total power cost obtained after running WMH on node A is 26.96.

The complexity of WMH algorithm, assuming that the level of transmission ranges is a constant, is $O(n^3)$, where n is the number of nodes in the network. Performance of the algorithm was analyzed through simulations. Simulation results show that power of the broadcast tree with WMH is up to 50% less than that of BIP [11] and EWMA [2].

3.2 WMH Has Constant Performance Ratio

In this section we prove that the WMH algorithm has a constant performance ratio of $12/k$, where $k = \gamma_{acq}/\gamma_p$ is a constant $k \in (0, 1]$, and represents a characteristic of the physical medium (see section 2). Let us denote with BMST the broadcasting with MST using Prim's algorithm [4]. In this approach, the power level of a node is set to power needed to reach the furthest children in the MST, where the root of the MST is the source node S.

Let us denote with MBG the minimum energy broadcast tree problem for the general case, that is, without Hitch-hiking. First we will show that the optimal solution of the MBG has a performance ratio of $1/k$ with the optimal solution of the MBH problem. Then we will prove that BMST is an approximation algorithm with constant performance ratio of $12/k$.

Theorem 1: *The performance ratio between the optimal solution of the MBG problem and the optimal solution of the MBH is upperbounded by* $1/k$.

Proof: Let us note an optimal solution of the MBG problem with OPT^{MBG} and an optimal solution of the MBH problem with OPT^{MBH}.

It is clear that $OPT^{MBH} \leq OPT^{MBG}$ since the solution set of the MBH problem includes that of the MBG problem. Next we show that $OPT^{MBG} \leq \frac{1}{k} \cdot OPT^{MBH}$.

Let us assume there are n nodes in the network, noted with $1, 2, ..., n$. Let us note node transmission ranges associated with OPT^{MBH} with $r_1, r_2, ..., r_n$. Then $OPT^{MBH} = r_1^\alpha + r_2^\alpha + ... + r_n^\alpha$. For a node i, we note with N_i^{MBH} the set of nodes partially or totally covered by i. Then $\forall j \in N_i^{MBH}$, $(\frac{r_i}{d_{ij}})^\alpha \geq k$ (see section 2), where d_{ij} is the distance between nodes i and j.

Let us now consider the case when each transmission range is increased by $k^{-\frac{1}{\alpha}}$. This corresponds to a solution SOL with node transmission ranges $r'_1, r'_2, ..., r'_n$:

$$SOL = \frac{1}{k} \cdot OPT^{MBH} = (r_1 \cdot k^{-\frac{1}{\alpha}})^\alpha + ... + (r_n \cdot k^{-\frac{1}{\alpha}})^\alpha = r_1^{\prime\alpha} + r_2^{\prime\alpha} + ... + r_n^{\prime\alpha}$$

For any node $i = 1..n$ and for any node $j \in N_i^{MBH}$, we have $(\frac{r'_i}{d_{ij}})^\alpha = (\frac{r_i \cdot k^{-\frac{1}{\alpha}}}{d_{ij}})^\alpha = \frac{1}{k} \cdot (\frac{r_i}{d_{ij}})^\alpha \geq 1$. Therefore all nodes that were previously partially covered in the MBH solution, are now fully covered. Also, since the power level of the nodes are increased, the property that all nodes will receive the message from the source S is preserved. Therefore, SOL is also a solution of the MBG problem, with $OPT^{MBG} \leq SOL$. This results in $OPT^{MBG} \leq \frac{1}{k} \cdot OPT^{MBH}$.

To summarize, we have proved that $OPT^{MBH} \leq OPT^{MBG} \leq \frac{1}{k} \cdot OPT^{MBH}$, therefore $\frac{OPT^{MBG}}{OPT^{MBH}} \leq 1/k$.□

Theorem 2: *BMST is an approximation algorithm with a constant performance ratio of* $12/k$, *where* $k = \frac{\gamma_{acq}}{\gamma_p}$ *is a constant* $k \in (0, 1]$ *and represents a characteristic of the wireless communication medium.*

Proof: Let us note an optimal solution of the MBG problem with OPT^{MBG}, and an optimal solution of the MBH problem with OPT^{MBH}.

$BMST$ is a solution of the MBG problem. Since MBG is a particular case of MBH, then $BMST$ is also a solution of MBH.

It is proved in [10] that $BMST$ as a solution for MBG has a performance ratio between 6 and 12, therefore $BMST \leq 12 \cdot OPT^{MBG}$. In Theorem 1 we proved that $OPT^{MBG} \leq \frac{1}{k} \cdot OPT^{MBH}$, therefore $OPT^{MBH} < BMST \leq \frac{12}{k} \cdot OPT^{MBH}$.

This concludes our proof that the $BMST$ is an approximation algorithm for the MBH problem and has a constant performance ratio of $12/k$.□

WMH is a localized algorithm for the MBH problem (see subsection 3.1) that starts from the $BMST$ solution and improves it, using the Hitch-hiking advantage that allows combining partial messages in order to fully decode the message. Therefore $WMH \leq BMST$, which implies that WMH has also a constant performance ratio of $12/k$ with an optimal solution of the MBH problem.

4 Conclusion

In this paper, we have addressed the minimum energy broadcasting operation in static ad hoc wireless networks. This problem has been largely considered in literature in various settings. One approach is to use the Hitch-hiking model in order to reduce power consumption. This model applies to the physical layer and allows to effectively combine partial signals in order to decode the full packet. In this way, a packet can be delivered with less transmission power. We described MBH problem and WMH algorithm proposed in [1]. We also proved that WMH algorithm has a constant performance ratio.

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