

# Camera-Aided Robot Calibration

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## PREFACE

Many of today's industrial robots are still programmed by a teach pendant: The robot is guided by a human operator to the desired application locations. These motions are recorded and are later edited, within the robotic language residing in the robot controller, and played back, for the robot to be able to repetitively perform its task. Examples of typical robotic applications, for which such a strategy is more than sufficient, include pick-and-place of objects and automobile painting. For a successful run of such applications it is required that the robot be repeatable and that its work environment be unchanged, i.e., all parts and tools must be in well-defined fixed locations. Slight inaccuracies, while being tolerable in applications such as spray-gun painting, are not allowed for precision assembly, such as placement of surface mount electronic components on a circuit board.

Modern automation trends, on the other hand, have placed an increasing emphasis on sensor-guided robots and off-line programming. In the first, sensors such as vision are often employed to detect differences between actual and desired part locations. These offsets are then communicated to the robot controllers, so that the robot can correct its preprogrammed path. The robot motion commands are generated off-line from a CAD system. The programming task, in such a case, can be greatly simplified with the aid of interactive computer graphics, to simulate the effects of planned motions, without actually running the robot. Such software programs utilize data on the work piece and the robot that already exist in CAD databases. For a successful accomplishment of these more advanced tasks the robots need to be not only repeatable but also accurate. One of the leading sources of robot imperfect accuracy are differences between the geometric model of the robot as exists in its "blueprint drawings" and its actual geometry, as results from its construction tolerances. Robot calibration is a process by which the accuracy of a robot manipulator is enhanced, sometimes by orders of magnitude, through modification of the robot control software. To be able to better tune the robot geometric model through calibration, a sufficient amount of precision measurement data must be collected. Such data include the measured internal robot joint positions, and the coordinates of one or more points on the robot with respect to a designated reference frame.

These and many more fundamental ideas on robot accuracy and calibration are nicely laid out in several earlier references, most noteworthy is the book *Fundamentals of Manipulator Calibration*, by Mooring, Roth, and Driels (1991). Other books include Stone (1987) and Bernhardt and Albright (1993). Robot calibration, being somewhat a "tough sell" to industry users and

robotics researchers of the 1980's, has evolved in the 1990's into an active mainstream robotics research area, as evidenced by numerous publications on the subject and many significant recent contributions.

Our book may, on one hand, be viewed as a compilation of our own research results, developed mostly in the early 1990's. In that sense, the book complements earlier books that focused on developments during the 1980's. This book is not intended, on the other hand, as a comprehensive review of all new results in the area of robot calibration. It is aimed at robotics researchers and practitioners as a handy and self-contained reference in the, narrower yet critically important, area of "robot calibration using computer vision." The key issue is no longer - "should we calibrate?", but rather "How to do it fast and cheap?". Shifting the burden of calibration from the robot manufacturer to the robot user raises issues of measurement rate, total calibration time, automated operation, user-friendliness, non-invasiveness and total cost.

Robot calibration consists of four steps: selection of a suitable robot kinematic model, measurement of robot end-effector's pose (i.e. position and orientation) in the world coordinate system, estimation of the robot model parameters, and compensation of robot pose errors. The measurement phase is unquestionably the most critical step towards a successful robot calibration.

Many robot pose measurement techniques discussed in the Robot Calibration literature are still far from being attractive tools for robot users who need to calibrate their robots on the manufacturing floor. Calibration measurement instruments such as theodolites, laser tracking systems, and coordinate measuring machines are either too slow or overly expensive or both.

Cameras and vision systems have become standard automation components. A typical robotic cell may feature an integrated multiple-camera system for product inspection, part presentation, and real-time monitoring of assembly accuracy. Some of these cameras may be fixed in the robot cell area, whereas others may be permanently attached to the moving robot arm to assist in on-line component alignment. Implementation of a robot calibration system using cameras may require little additional hardware, and only a modest amount of additional software.

Calibration by a camera system is potentially fast, automated, non-invasive and user-friendly. Cameras can also provide full pose measuring capability. There are two typical setups for vision-based robot pose measurement. The first is to fix the cameras in the robot environment so that while the robot changes its configuration the cameras can "view" a calibration fixture mounted on the robot end-effector. The second setup is to mount a camera or a pair of cameras on the end-effector (hand) of the manipulator.

If the cameras in the system are calibrated in advance, the locations of the calibration fixture in world coordinates for various robot measurement configurations can be computed by the vision system. The stationary-camera setup is non-invasive, as the cameras are often placed outside the robot

workspace, and need not be removed after robot calibration. The major problem existing in all stationary camera setups is that in order to have a large field-of-view for the cameras, one has to sacrifice measurement accuracy. By using higher resolution cameras, the cost of the system and in particular its image processing part may increase dramatically.

The moving camera approach can resolve the conflict between high accuracy and large field-of-view of the cameras. The cameras need only perform local measurements, whereas the global information on the robot end-effector pose is provided by a stationary calibration fixture.

Methods for robot calibration using hand-mounted cameras can be further classified into "two-stage" and "single-stage" methods. In a two-stage approach, the cameras are calibrated in advance. The calibrated cameras are then used to perform robot pose measurements. In a single-stage approach, the parameters of the manipulator and those of the camera are jointly and simultaneously estimated. Depending upon the number of cameras mounted on the robot hand, these methods can be further divided into stereo-camera and monocular-camera methods. In the stereo-camera case, two cameras that have the same nominal optical characteristics are mounted on the robot hand. In the monocular case, only one camera is used.

This book, being the first on the topic of robot calibration using computer vision technology, covers the entire process of vision-based robot calibration, including kinematic modeling, pose measurement, error parameter identification, and compensation. It also addresses the issue of hand/eye calibration.

It is assumed that the reader is familiar with the basic theory of and practical approach to cameras, lenses, and image processing algorithms such as image preprocessing and segmentation. Moreover, even though most basic definitions are provided, it is assumed that this book is not the reader's first exposure to robotics. Sufficient familiarity with robots at the practical level (i.e. programming) and an introductory course on Robotics would be very helpful.

The book starts with an overview that emphasizes the authors' personal perspective on the history of robot calibration and of available techniques with focus on vision-based methods. It addresses some standing issues related to kinematic modeling, pose measuring, kinematic identification, camera calibration and autonomous calibration.

Chapter 2 covers the review of camera calibration techniques that are relevant to the robot calibration problem. It starts with the distortion-free pin-hole camera model to introduce the concept of camera calibration. By using a lens distortion model, a number of camera calibration techniques which are suitable for camera-aided robot calibration are presented. The chapter also addresses relevant issues such as the estimation of the image center and the compensation for perspective projection distortion. Finally, camera calibration simulation and experimental results are given that

demonstrate the effectiveness of the camera calibration techniques outlined in this chapter.

Chapter 3 studies the properties of kinematic modeling techniques that are suitable for robot calibration. It summarizes the well-known Denavit-Hartenberg (D-H) modeling convention and points out the well-known drawbacks of the D-H model for robot calibration. After the presentation of a modified D-H model, the chapter then develops the Complete and Parametrically Continuous (CPC) model and the modified CPC model, both designed to overcome the D-H model singularities.

Pose measurement is a key element in a successful robot calibration task. If cameras are mounted on the robot hand, poses of the robot end-effector can be measured by a single camera or by a pair of stereo cameras. On the other hand, if stationary cameras are used, at least two cameras must be used. Chapter 4 discusses in great detail various vision-based pose measurement techniques. Methods for the identification of the relationship between the robot tool coordinate frame and the camera coordinate frame are also discussed.

Kinematic identification is a central element in a robot calibration task. Chapters 5 to 10 address this issue from different perspectives. Chapter 5 concentrates on error-model-based kinematic identification, while Chapter 6 presents linear solution approaches under the assumption that the robot measurement configurations follow a certain pattern.

Autonomous calibration of robot and camera systems is important in certain applications. In Chapter 7, a procedure for simultaneous calibration of a robot-camera system is developed.

Although hand/eye calibration is a highly practical problem, this issue is not addressed in existing books. Chapter 8 is devoted to this particular area. The chapter starts with a review of quaternion algebra, a key mathematical tool. Linear solution approaches for estimating the unknown rotation matrix are then covered. These methods are fast, but less accurate, comparing nonlinear approaches, which are also discussed in this chapter. This chapter demonstrates the pros and cons of the various approaches.

The geometric relationships in a robot system that most frequently need to be calibrated are the base and tool transformations. Chapters 9 and 10 deal with this aspect of calibration. Whenever the entire pose of the robot can be measured, the calibration of the base transformation becomes very simple. However when only point measurements are available, the task is more complex. Chapter 9 presents a linear approach that solves for the base transformation using point measurements only.

The final stage of a robot calibration task is accuracy compensation, using the identified kinematic error parameters. Chapter 11 presents a number of accuracy compensation algorithms, including the intuitive task-point redefinition algorithm and the linear quadratic regulator algorithm. The first is accurate and fast, provided that the robot is not in its singularity points. On the other hand, the latter is more robust. A simple bilinear interpolation

method suitable for 2D compensation is also given in this chapter.

Off-line optimal selection of measurement configurations can significantly improve the accuracy of kinematic identification. In Chapter 12, a number of procedures that are designed for robot measurement configuration selection are outlined.

In Chapter 13, we present experimental results which were obtained by calibrating two industrial robots. Practical considerations important for conducting robot calibration experiments are also given in this chapter.

A brief appendix to the book provides readers additional mathematical background.

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## Chapter 1

### OVERVIEW OF ROBOT CALIBRATION

#### I. THE MOTIVATION

*Robot Calibration* is the process of enhancing the accuracy of a robot manipulator through modification of the robot control software. Humble as the name "calibration" may sound, it encompasses four distinct actions, none of which is trivial:

- Step 1: Determination of a mathematical model that represents the robot geometry and its motion (*Kinematic modeling*).
- Step 2: Measurement of the position and orientation of the robot end-effector in world coordinates (*Pose Measurement*).
- Step 3: Identification of the relationship between joint angles and end-point positions (*Kinematic Identification*).
- Step 4: Modification of control commands to allow a successful completion of a programmed task (*Kinematic Compensation*).

The need for robot calibration arises in many applications that necessitate off-line programming and situations that require multiple robots to share the same application software. Examples of the first are assembly operations, in which costly hard-automation (such as the use of accurate  $x$ - $y$  positioners for the assembly part) to compensate for robot inaccuracies may be avoided through the use of calibration, as shown in Figure 1.1.1. An example of the latter is robot replacement, where calibration is an alternative to robot reprogramming, as shown in Figure 1.1.2.

Without calibration, robots which share application programs may experience significant accuracy degradation. The need to reprogram the machine upon replacement (or upon other maintenance actions that may cause permanent changes in the machine geometry) may result in a significant process down-time. Robots should be calibrated in a time period which is a fraction of the reprogramming time for calibration to be economically justifiable.

The growing importance of robot calibration as a research area has been evidenced by a large number of publications in recent years, including books and survey papers. Readers interested in surveys of robot calibration and detailed reference lists are referred to the book by Mooring, Roth, and Driels (1991) and a survey paper by Hollerbach (1988).



This book is not intended to be one more comprehensive survey of calibration. It focuses on camera-based techniques for robot calibration utilizing a unified modeling formalism developed and refined by us over recent years. We naturally chose to put main emphasis on our own research results; but, of course, these results were not developed in "empty space", as will be shown in the next section.

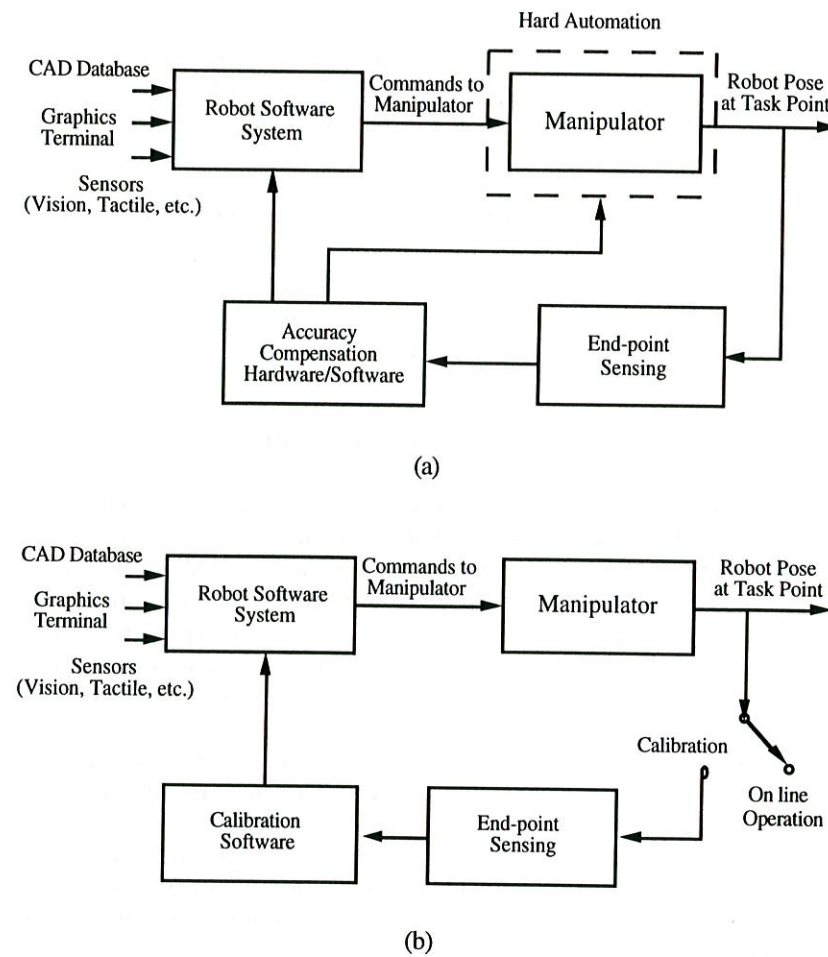


Figure 1.1.1. Assembly Operations: (a) Without robot calibration  
(b) With robot calibration

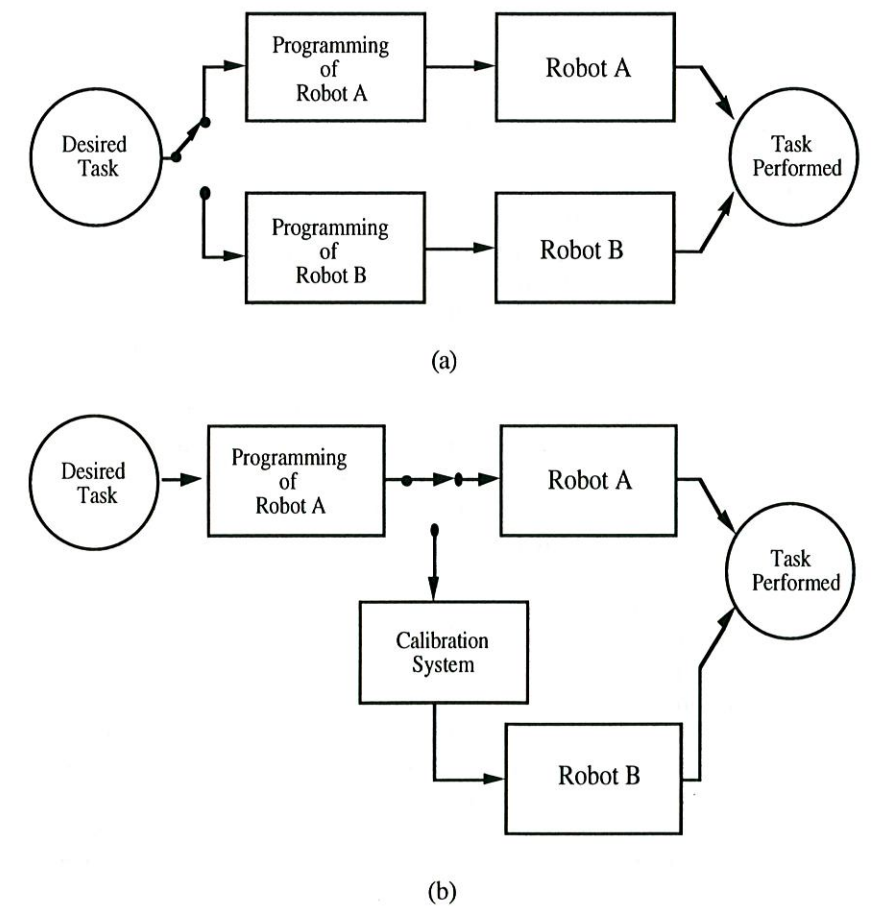


Figure 1.1.2. Robot replacement: (a) Individual programming of each robot  
(b) With calibration

## II. HISTORICAL PERSPECTIVE

The following brief historical review of Robot Calibration research and practice portrays our own subjective view of key references and achievements that were most related to our own work and have had the biggest influence on us.

The booming growth of Robotics research in the late 1970's and early 1980's was a direct result of successful application of robot manipulators to automated manufacturing, particularly in the automotive industry and parallel to that the rapid growth in the computer industry. The predominant method of robot programming, suitable for the applications at that time, was "Teaching by Doing"; that is, physically moving the manipulator to each task

point, recording and later replaying the joint-space description at these joints. Manipulators were designed to be highly repeatable and most applications involved a relatively low number of task points with minimal interaction between the robot and external sensors.

Richard Paul's book (1981) has been a major influence on all robotics researchers of the 1980's. His systematic use of homogeneous transformations and the Denavit-Hartenberg (D-H) mechanism kinematic modeling formulation for robot path planning and control quickly became universal standards. Many manipulator controllers were subsequently built featuring D-H forward and inverse models. The theoretical tractability of Task-Space robot path planning and the advent of sophisticated sensors such as solid state cameras, force sensors, and various types of proximity sensors and the personal computer revolution all fueled the high expectations that robot manipulators would be soon used to implement fully automated "factories of the future", and be key elements in many sophisticated multi-step applications involving task-space description and on-line interaction with large magnitudes of sensory data.

Such applications require, in principle, repeated use of the robot inverse kinematic model. In addition there is a need to program the robots off-line to move to task points never visited before by the robot. Such off-line programmed robots must be designed not only to be repeatable but, more importantly, to be accurate. The accuracy of a manipulator depends strongly on the accuracy of the robot geometric model implanted within its controller software. Robotics researchers and practitioners from academia and industry began to study the effects of joint offsets, joint axis misalignment and other accuracy error sources on the manipulator end-effector position and orientation errors.

The first major discovery, found independently by Mooring (1983) and Hayati (1983), which in retrospect established Robot Calibration as a new research area, was that the D-H model is singular for robots that possess parallel joint axes. More specifically, the common normal and offset distance parameters may undergo large changes when consecutive joint axes change from parallel to almost-parallel. Both researchers offered alternative robot kinematic models: Hayati introduced a modification to the D-H model which gained popularity and was subsequently adopted by many other researchers. Mooring advocated a model introduced earlier in the mechanism kinematics literature (for instance, refer to Suh and Radcliffe (1978)) based on the classical Rodrigues equation. Since most industrial manipulators are designed to be "simple", that is, to have parallel or perpendicular consecutive joint axes, this singularity problem is a major issue from a practical view point.

During the 1980's many robot calibration researchers came with their own model versions. In fact, the number of models almost equaled the number of researchers. An excellent survey of this flood of models, categorized into 4-, 5- and 6-link parameters models is Hollerbach's paper (1988). The survey

concluded with the following comment:

*One issue that should be settled in the future is the choice of coordinate system representation. One strong alternative seems to be the Hayati modification of the Denavit-Hartenberg representation. It is not clear at this point what advantage the six-parameter representations would have for modeling lower-order kinematic pairs, while they have the disadvantage of redundancy.*

To be able to improve the accuracy of the robot one needs to be able to measure the world coordinates of the robot end-effector at different robot joint-space configurations and record the joint positions at such configurations. If end-effector position and orientation, as predicted by the robot nominal forward kinematic model by plugging in the joint readings at the selected measurement configurations, differ from the actual end-effector pose measurement, the robot model needs to be suitably adjusted. That is what Robot Calibration is all about.

From a data collection point of view Robot Calibration is not different from Robot Performance Evaluation (in particular for assessing repeatability and accuracy). Much work on evaluation of machine tools and robot manipulators was performed during the 1980's at the National Bureau of Standards and one excellent review compiling many such testing techniques is the book chapter by Lau, Dagalakis and Myers (1988). Another beautiful survey of robot end-joint sensing techniques is the paper by Jiang, Black, and Duraisamy (1988). One of the first major reports of actual robot calibration experiments was the paper by Whitney, Lozinsky and Rourke (1986). Data collection was performed by Whitney and his co-investigators using theodolites.

Many Calibration or Robot Testing studies during the 1980's were done using a variety of measurement techniques ranging from expensive Coordinate Measuring Machines (CMM) and Tracking Laser Interferometer Systems to ones that employed inexpensive customized fixtures. The "heart of the matter" is the measurement in "world coordinates" of one point on the robot end-effector. World coordinates are often defined by the calibration measurement equipment itself. The measured point represents the end-effector position. The measuring of the coordinates of three or more non-colinear end-effector points provides the full pose (position and orientation) of the end-effector. Some measurement devices are capable of measuring the full 6-dimensional pose, some can measure only the 3D position and others, such as single theodolite, measure even less than that.

A major contribution to the Kinematic Identification phase of Robot Calibration was the paper by Wu (1984) in which the Identification Jacobian, a matrix relating end-effector pose errors to robot kinematic parameters errors, is systematically derived. This mathematical tool is very useful for both machine accuracy analysis and machine calibration. Another contribution by Wu and his co-authors was the paper (Veitschegger and Wu (1988)) that introduced two techniques for Accuracy Compensation.

Casting the full robot calibration problem as a four-step problem – modeling, measurement, identification and compensation, was featured in the survey paper by Roth, Mooring and Ravani (1987) and expanded into a full scope book (Mooring, Roth and Driels (1991)). That book was indeed a comprehensive survey of all phases of manipulator calibration as evidenced from research done mostly during the 1980's. The book is a simply written tutorial to many of the fundamental concepts and methods and is highly recommended for first-time robot calibration practitioners.

In addition to Hollerbach's "standing question" regarding calibration models, many other open research issues have lingered, such as:

1. What is the relative importance of robot geometric errors compared to non-geometric errors?
2. How is the calibration quality related to the resolution and accuracy of the calibration instrumentation and the method of calibration?
3. How should robot measurement configurations be optimally chosen?
4. How is observability of robot kinematic error parameters related to the selection of calibration configurations and method?

Some of these problems, even today, are not yet fully answered. Practical implementation questions were even more acute:

1. How can robot calibration be done "fast" and "cheap"?
2. Should calibration be done primarily by the robot manufacturer or can the calibration load be shifted to the robot user?
3. Should robots be designed differently from a hardware and software point of view to accommodate on-line calibration capability?
4. What current technology will make robot calibration, performed "on the manufacturing floor", economically feasible?

Starting with some of the research issues, we believe that our Complete and Parametrically Continuous (CPC) type models, as introduced in the thesis by Zhuang (1989), the paper by Zhuang, Roth and Hamano (1992) and explained in detail in this book, are a step forward toward answering Hollerbach's question. The CPC model was inspired by a paper by Roberts (1988) in the Computer Vision literature, which discussed a very useful line representation with respect to a local coordinate frame using the directional cosines of the line. In the case of robot modeling a joint axis directional vector is represented in terms of a coordinate frame located on the previous joint axis. The CPC model is a natural evolution of Hayati's model (1983), Mooring and Tang's model (1984) and Sheth and Vicker's model (1972). Hayati's model utilizes a plane perpendicular to one of the joint axes. Mooring's model also represented joint axes, however with respect to the world frame. Sheth and Vicker introduced the concepts of Motion and Shape

Matrices. Readers are also referred to Broderick and Cipra (1988).

Important observability issues of kinematic error parameters can be addressed through a generic link-by-link error model, originally introduced by Everett and Suryohadiprojo (1988). These are explained in detail in this book.

The CPC models and error-models apply uniformly to manipulator internal links as well as the *BASE* and *TOOL* transformation. This makes the model highly convenient to robot partial calibration. Ideas of progressive calibration – starting from *BASE* only through *BASE* and joint offsets only, to full scale calibration, were first pursued by Mooring and Padavala (1989).

It is important to fully recognize that the kinematic identification and accuracy compensation processes are merely least squares fittings of a suitable number of design parameters to improve on the overall accuracy. Calibration can be done with any number of available design parameters depending on the actual physical set up. Some robot software systems do not allow the user access to all the coefficients of the robot kinematic model. For instance, in some commercial SCARA arms a user is allowed to modify only the joint variable offsets and the link length parameters. In this light the question about relative importance of non-geometric errors may not be fully meaningful as the least squares fitting is done based on noisy data affected by both geometric and nongeometric sources.

Breaking robot calibration into different levels and focusing on specific partial calibration problems such as Hand-Eye Coordination and Robot Localization is one of the central themes of this book.

With regard to the implementation issues, the "fast and cheap" guideline automatically rules out expensive instrumentation such as Laser Tracking Systems or methods that are highly invasive such as placing contract calibration fixtures within cluttered and application dependent robot work environments. From a calibration cost viewpoint the use of cameras and vision systems is extremely beneficial as these already exist as integral components of most industrial robotic cells. For instance, electronic assembly operations often require the use of a multiple camera setup, one that is attached to the robot end-effector which monitors "fiducial" points on the circuit boards and transmits data which are used to finely adjust the end-effector location and another that may be located within the conveyor system monitoring from below the relative alignment between the robot and the assembly area. Hence, calibration implementation may involve, at most, additional camera calibration boards and specialized calibration software at a cost which is a tiny fraction of the total robotic cell cost. Data acquisition systems using cameras are non-invasive, very fast (potentially) and, in principle there is no increase in the level of difficulty in monitoring more than one point on the robot. In other words, full pose measuring ability is easily feasible.

The major stumbling block that prevented widespread use of cameras for machine tool and robot metrology has been camera resolution, most importantly image sampling due to nonzero pixel size. It can be shown, for

instance, that with typical off-the-shelf stereo cameras arranged in stationary locations, the coordinates of a target point located on the moving robot and viewed from a few feet away may have a diameter of uncertainty of nearly 1 mm. Taking directly end-effector position measurements using common stereo cameras is of course totally unacceptable for most robotic applications that typically may require repeatability in the order of magnitude of 0.001-0.1 mm. Sklar (1988) experimented with robot calibration using 1024x1024 pixel camera and customized vision software. With today's technology such a solution increases dramatically the overall cost of the calibration system.

The question "can low resolution sensors be used to provide (indirectly) accurate end-effector position readings?" received an interesting answer in three different independent studies done in the 1980's. Stone (1987) used triangulation of very noisy acoustic sensors tracing a spark-generating target. These noisy points were obtained through moving each robot joint one at a time. For a revolute joint the collection of measured points is used to fit circles in 3D space. These circles define accurately the plane of rotation and center of rotation for the respective joint. For a prismatic joint the measured points are used to fit 3D lines which describe the respective joint axis direction. Similar studies using stereo cameras were performed by Barker (1983) and Sklar (1988). Another good reference is a thesis by Chen (1987). The reader should keep in mind that any robot kinematic modeling starts by specifying the manipulator's joint axes, in an arbitrary configuration, followed by set-up of link coordinate frames and construction of the link transformations using any convention for selection of link parameters. Joint axis identification provides a solution to robot calibration that is radically different from the method of identification of error parameters using linearized accuracy error models. This solution, which is potentially "cheap", is most certainly not "fast". Accurate joint axis identification requires the measurement of a large number of target points along each joint travel. When using stationary stereo cameras to track a light source mounted on the robot, the issue of target visibility arises, complicating further the calibration process.

Resolving the difficult tradeoff between camera position measurement resolution and the camera field of view, necessitates that the camera(s) always remain close to the moving robot target. In other words, it is necessary that the cameras, used to calibrate the robot, move together with the robot. This idea may appear somewhat counterintuitive to metrology practitioners. After all, position measurement using stationary stereo cameras first requires careful calibration of the cameras. These calibrated cameras subsequently define the robot "world coordinate frame". Presumably any intentional or accidental displacement of these cameras may take the entire measurement system out of calibration.

Reported experimental results by Puskorius and Feldkamp (1987) and by Zhuang, Roth and K. Wang (1994) showed that robot calibration using moving cameras is feasible. The point is that cameras attached to the robot

hand continue to remain calibrated with respect to a fictitious camera calibration fixture that moves together with the moving robot.

The field of Camera Calibration has been a rich area of research. Readers are referred to Tsai (1987) and Weng, Cohen and Herniou (1992) and others for references. Camera models could range from the simplest distortion-free "pin-hole" model that leads to the well-known perspective transformation to more involved models that take into account lens distortion effects. Of particular interest to robot calibration are camera models that contain an explicit description of the camera pose (position and orientation) with respect to the viewed object.

Key references are Tsai's paper (1987), proposing a method for calibration of the camera pose and relevant camera internal parameters using a single flat calibration board, and the paper by Lenz and Tsai (1989) describing the following interesting idea for robot calibration: Since the camera model includes the pose of the camera and since the camera is rigidly attached to the robot arm, the pose of the camera represents the pose of the manipulator. In other words, the robot calibration measurement phase is done by recalibrating the camera at each robot joint space measurement configuration.

The above references opened up the entire research area of Camera-Aided Robot Calibration in which methods based on moving cameras play a central role. Scanning this issue is what this book is all about.

In recent years Robot Calibration became a very popular research area as evidenced by many recent publications and special sessions in Robotics Conferences. Camera-Aided Robot Calibration is only a minor representative of the current research directions in this area.

The inherent challenge and the critical importance of robot pose measuring in the world coordinate frame prompted several researchers to strongly consider the idea of "Autonomous Calibration". In autonomous calibration, kinematic identification is to be performed based only on data obtained internally within the robot. Most prominent of this new wave is Hollerbach and his co-researchers (Bennett and Hollerbach (1989, 1991)). The idea in its basic form was to create a closed kinematic chain by addition of several links that connect the robot end-effector to the ground. This type of kinematic redundancy created more kinematic unknowns offset by a larger number of equations, which all-in-all enabled the identification of the robot kinematics as well as the additional parameters of the calibration system.

A natural development of this idea was that rather than closing the loop mechanically, one can close the loop optically. Bennett, Geiger and Hollerbach (1991) reported autonomous calibration using servo actuated stereo cameras attached to the robot hand. The camera readings were conveniently considered "internal sensory data".

Recently, Zhuang, Wang and Roth (1995) explored simultaneous calibration of a robot and a single camera attached rigidly to the robot hand. This idea is described in detail later in this book.

Much attention has been given recently to the calibration of parallel

manipulators and other multiple degrees of freedom systems such as machine tools and laser tracking systems (Zhuang, Li, Roth, and Xie (1992)). In all the concept of autonomous calibration is central. The level of accuracy required in machine tools and laser tracking systems prohibits the use of off-the-shelf cameras. Cameras can play an important role in the calibration of large parallel robots such as Stewart platforms, but this has not yet been fully explored experimentally and we chose not to include this topic in the book. Most of the techniques that are presented in this book have been verified experimentally.

It is important to stress that although much work has been done in recent years, Camera Aided Robot Calibration as a research area is far from being closed. One of the book's purposes is to encourage robotics researchers to find solutions to many of the numerous open problems listed throughout the book. The goal of "cheap and fast" calibration, to allow off-line accurate, yet user-friendly, robot programming right on the manufacturing floor remains one of the key goals in manufacturing and automation. Successful attainment of it carries a potential enormous economic pay-back. We hope that this book contributes in documenting the current knowledge.

## Chapter 2

### CAMERA CALIBRATION TECHNIQUES

#### I. INTRODUCTION

It is appropriate to discuss first camera models and camera calibration. The main intent is not to provide an exhaustive and comprehensive review of this rather rich area, but to focus on those techniques that have practical relevance to the subsequent problem of robot calibration.

This chapter is organized as follows. Relevant camera models as well as the so-called perspective transformation matrix (PTM) method are overviewed in Section II. Tsai's radial alignment constraint (RAC) method for camera calibration is described in Section III. A simplified RAC-based algorithm is given in Section IV. A procedure that handles a near singular case is presented in Section V. Weng's two-phase nonlinear optimization approach is outlined in Section VI. Methods for determining the ratio of scale factors and estimating the image center are derived in Sections VII and VIII. An analysis of distortion of the centroid of a circular point due to perspective projection is brought in Section IX. Calibration simulation and experimental results are presented in Section X. The chapter concludes with references and discussion.

#### II. CAMERA MODELS

##### A. A DISTORTION-FREE CAMERA MODEL

The purpose of the model is to relate the image coordinates of an object point visible by the camera, to the coordinates of this point in a reference coordinate system. Let  $\{x_w, y_w, z_w\}$  denote the world coordinate system;  $\{x, y, z\}$  denote the camera coordinate system, whose origin is at the optical center point  $O$ , and whose  $z$  axis coincides with the optical axis; and  $\{X, Y\}$  denote the image coordinate system centered at  $O_I$  (the intersection of the optical axis  $z$  and the image plane) (refer to Figure 2.2.1).  $\{X, Y\}$  lies on a plane parallel to the  $x$  and  $y$  axes.

The transformation from the world coordinates  $\{x_w, y_w, z_w\}$  to the camera coordinates  $(x, y, z)$  is