BME 5742 Bio-Systems Modeling and Control

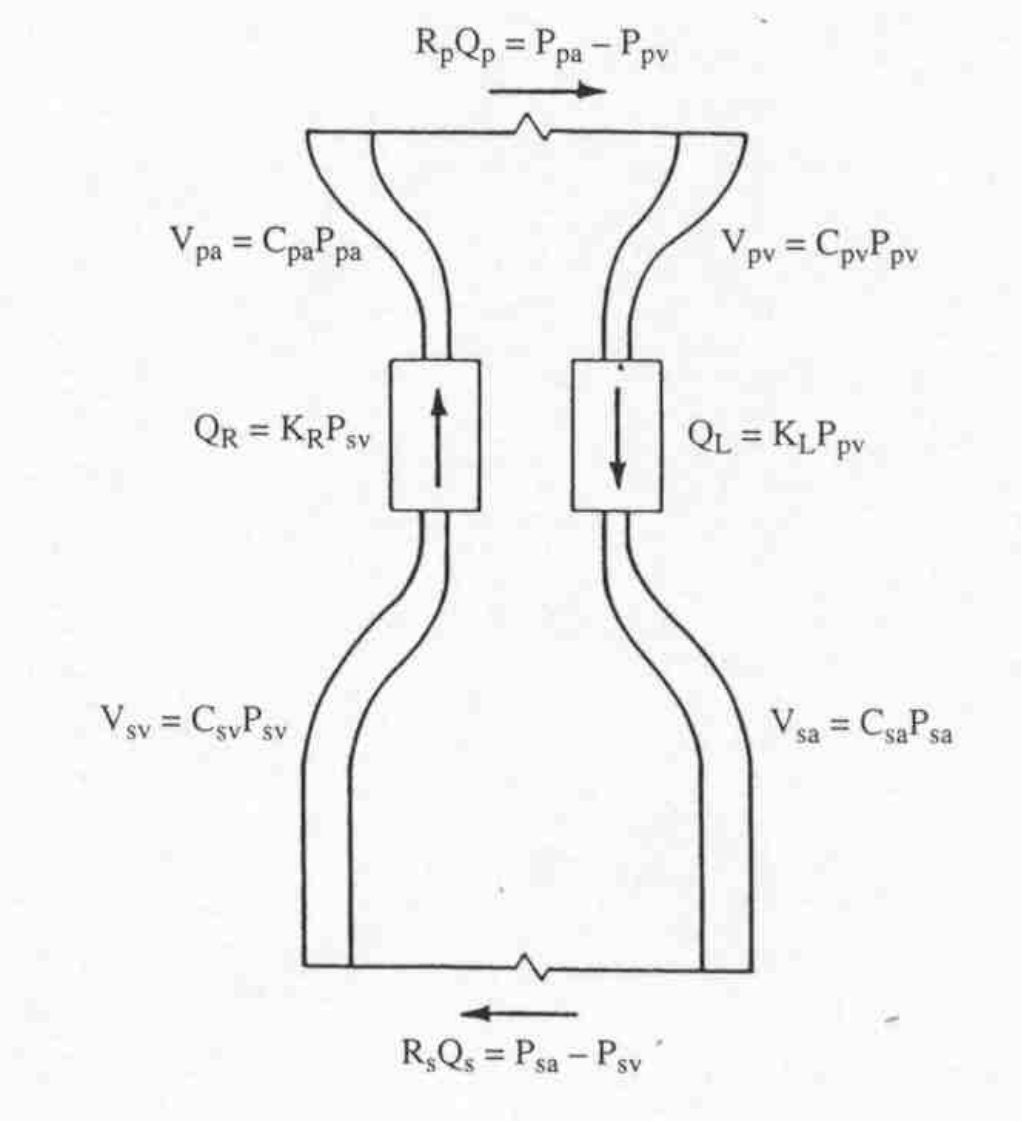
Lecture 42
Heart & Blood Circulation
Steady-State Flow-Pressure-Volume

Model of the Uncontrolled Circulation

- We like to explore the various flow relationships, and constraints.
- The model (even though being highly incomplete, in the sense that no control mechanisms are shown) is useful in understanding <u>self-regulating</u> properties of blood circulation.
- Need to understand why additional (external to the circulation system) controls are needed.

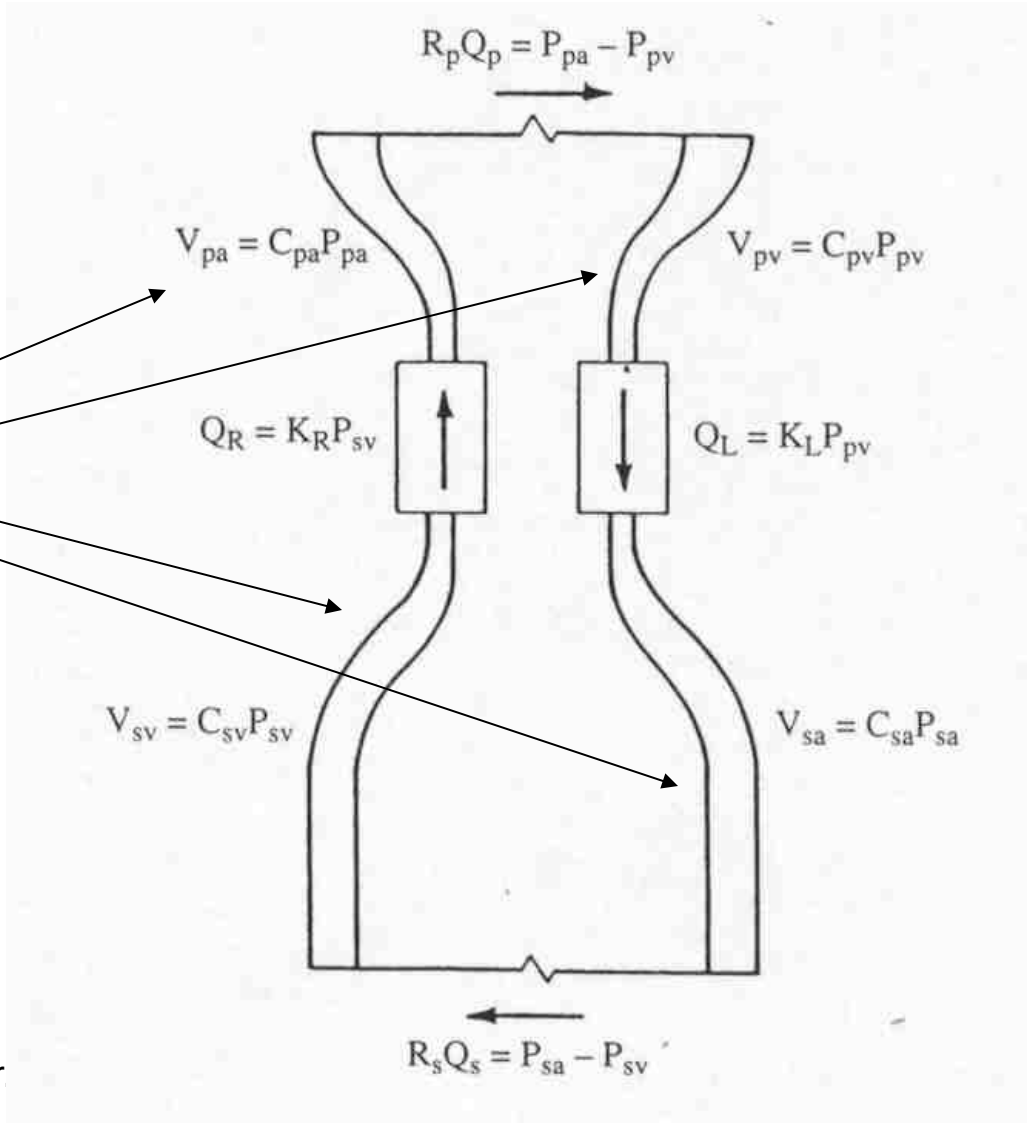
Simplified Circulation Model

- This minimal aggregated model features two pumps, four compliance vessels and two resistance vessels.
- Math law is given for each component



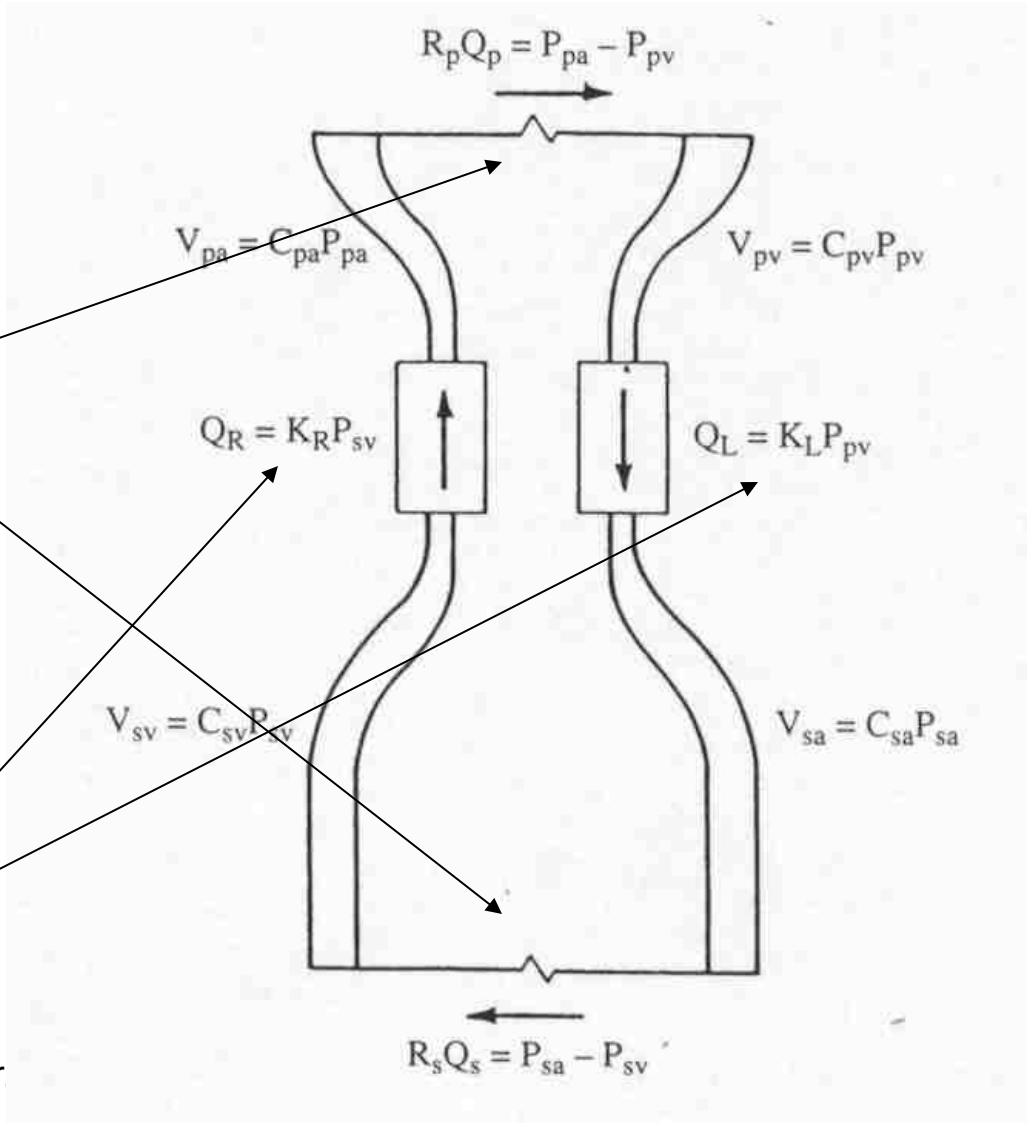
Model Compliance Vessels Assumptions

- Systemic (s) and pulmonary (p) arteries (a) and veins (v) are compliance vessels.
- We may (as a first approximation) neglect the dead volumes V_d.



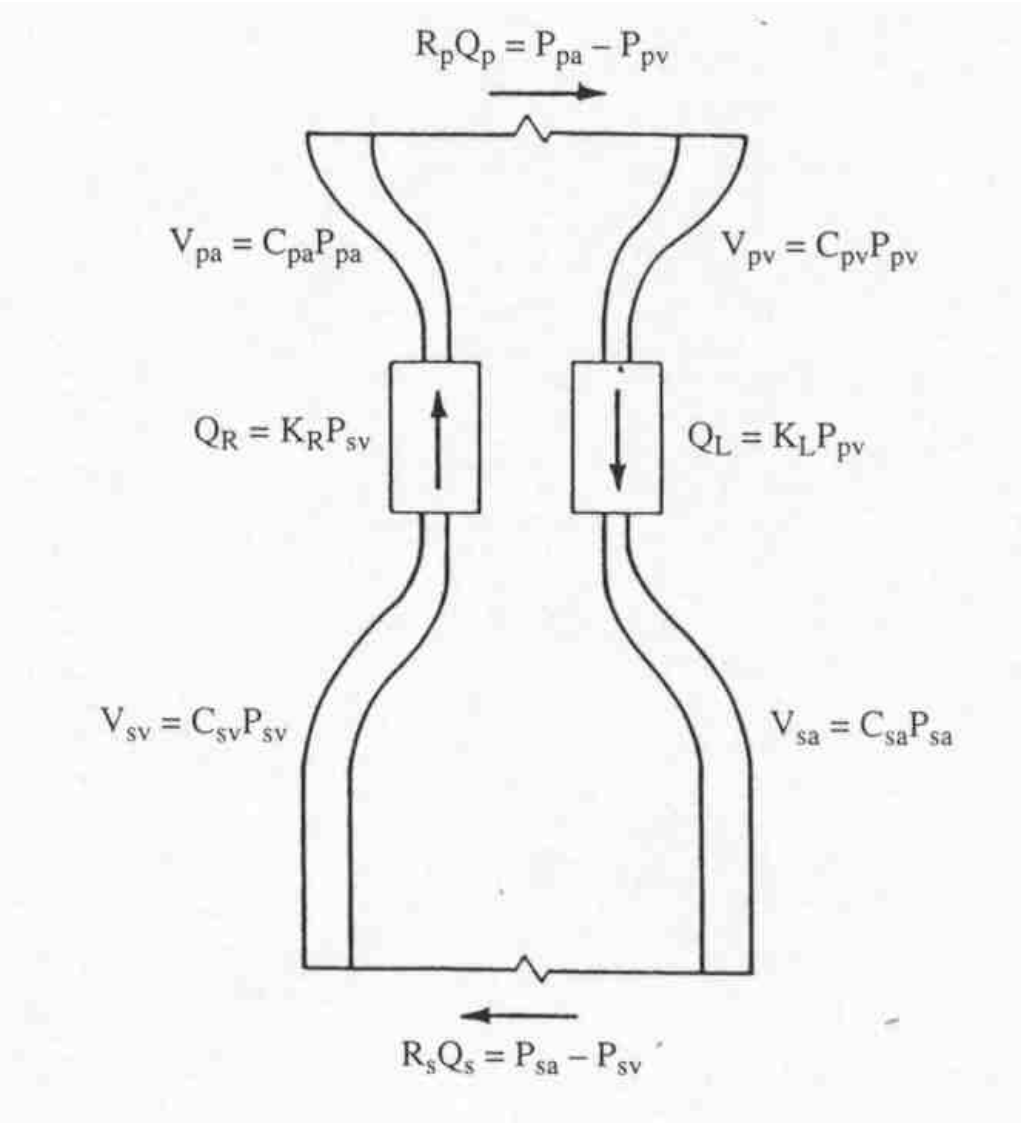
Model Resistance Vessels and Pumps Assumptions

- Systemic (s) and pulmonary (p) tissues act like resistance vessels.
- We neglect the systole compliance of the heart chambers.



Model Equations

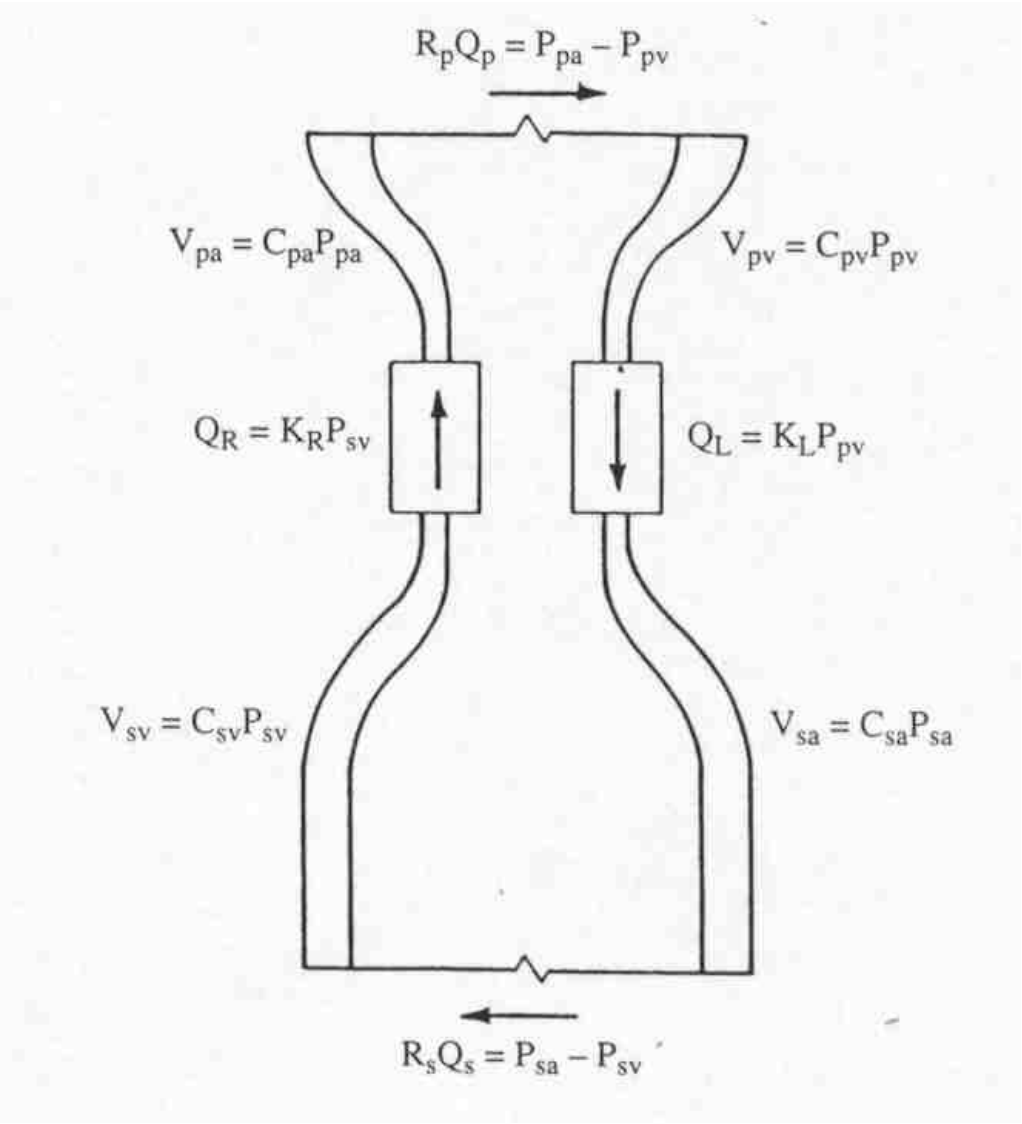
- In this eight components model, each component is characterized by one simplified equation (see figure).
- All flows, pressures and volumes are unknown.



Model Unknowns

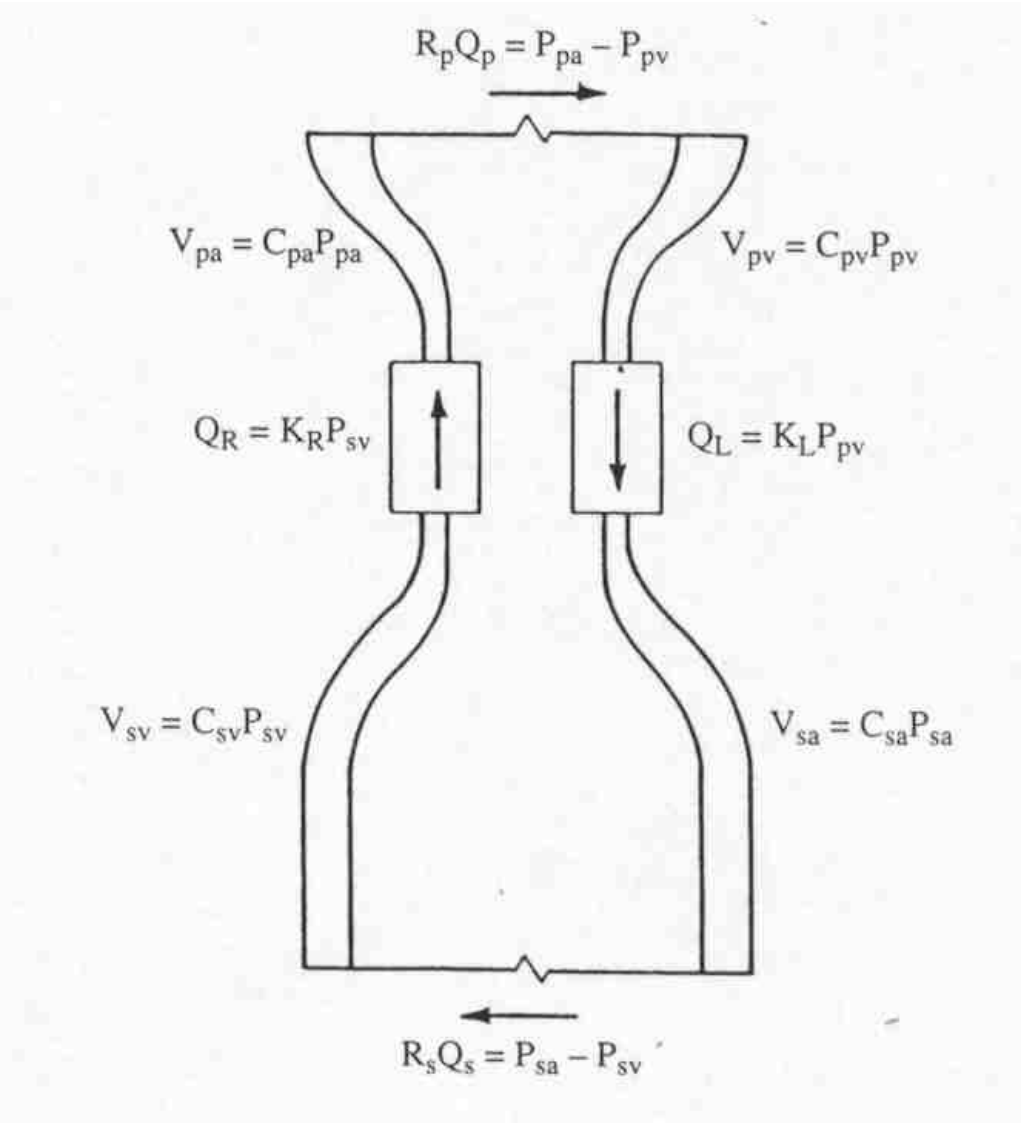
All flows, pressures and volumes are unknown: Q_R ,Q_L ,Q_s,Q_p, P_{sa}, P_{sv}, P_{pa}, P_{pv}, V_{sa}, V_{sv}, V_{pa}, V_{pv} → 12 unknowns, and only 8 equations.

- How do we resolve the discrepancy?



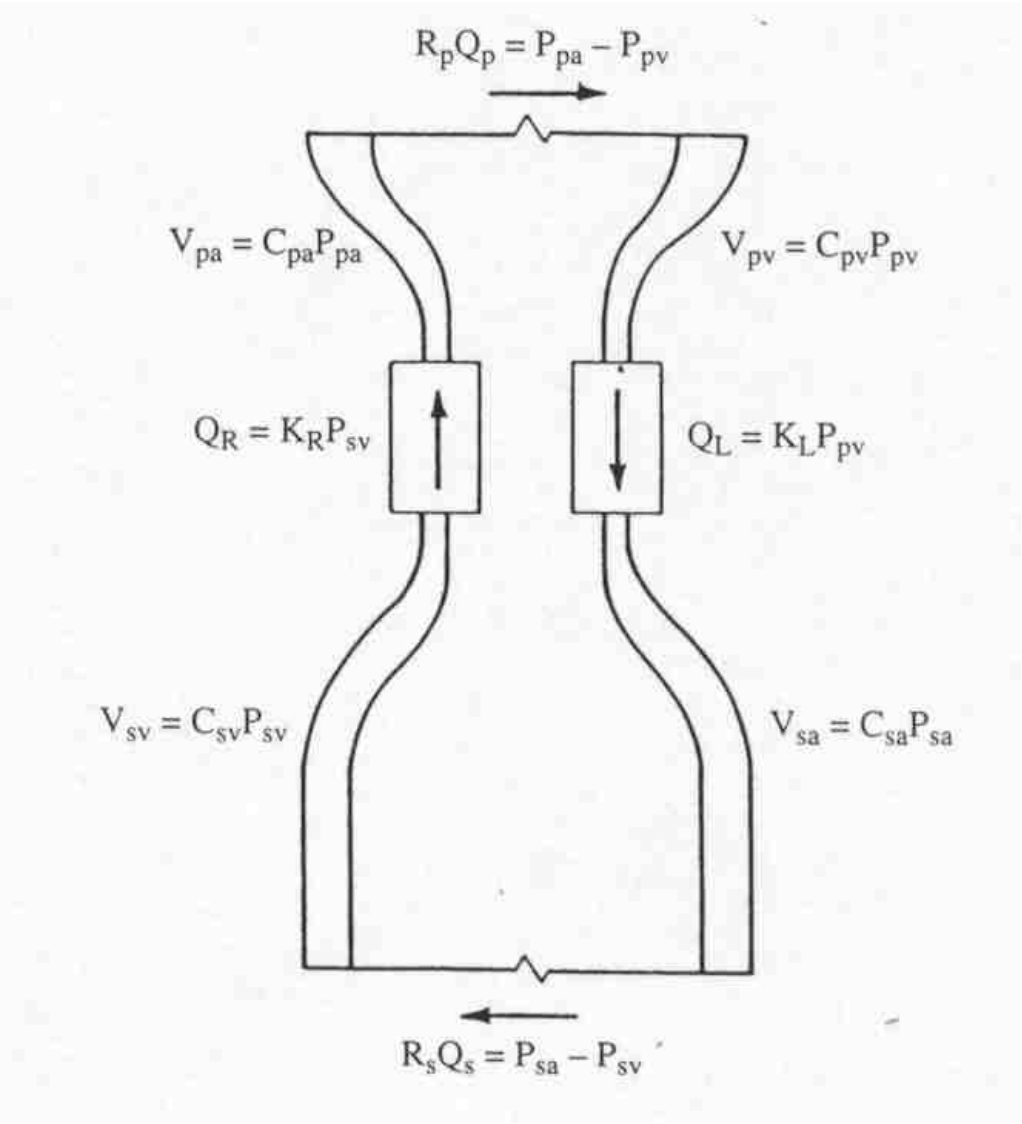
Model Unknowns at Steady-State :

- At steady-state all flow rates are equal: $Q_R = Q_L = Q_s = Q_p = Q$
- Q, P_{sa}, P_{sv}, P_{pa}, P_{pv}, V_{sa}, V_{sv}, V_{pa}, V_{pv} → 9 unknowns, and only 8 equations.
- Still a discrepancy



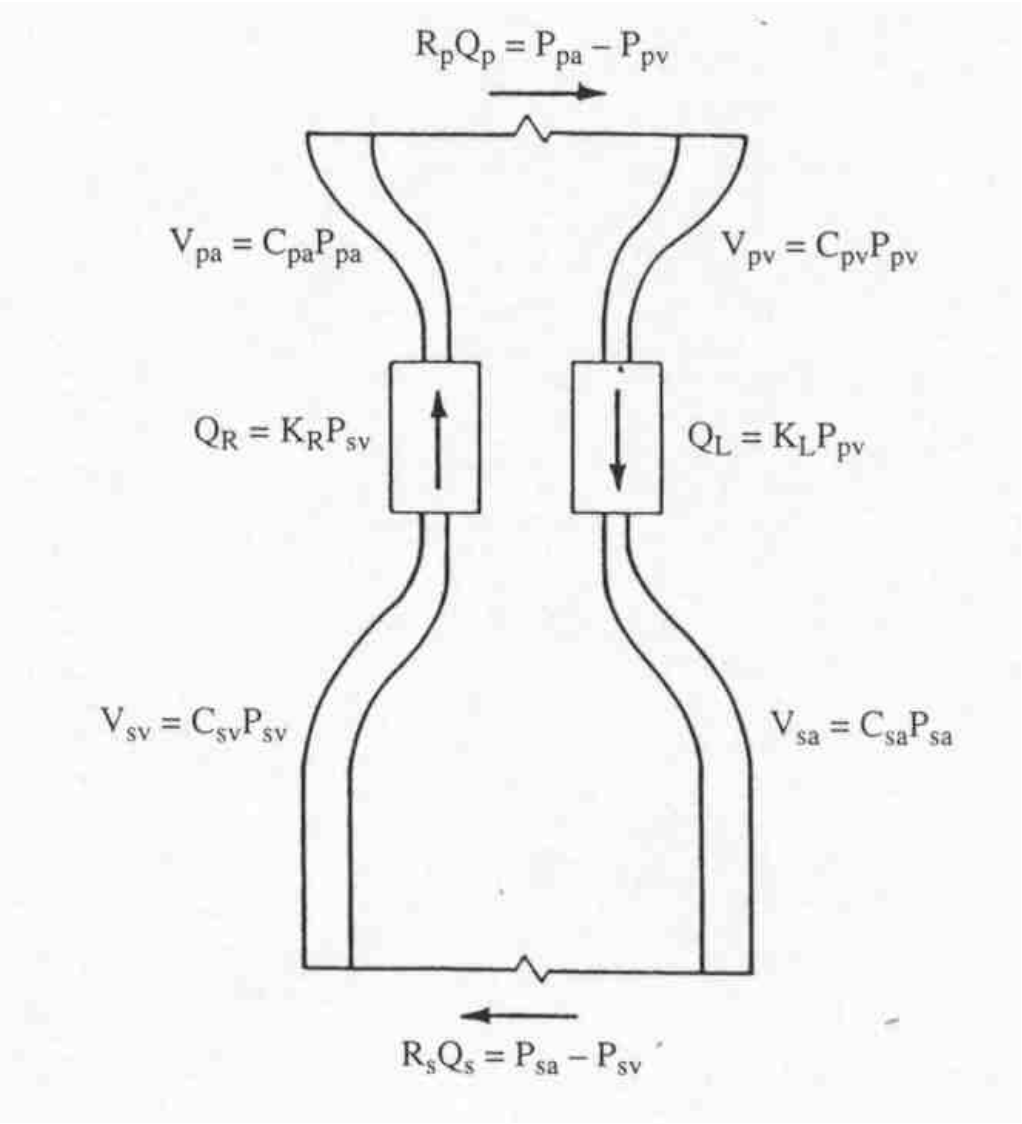
Model Unknowns at Steady-State taking into account Constraints:

- All volumes add up to the total blood volume V_0. If V_0 is known, we have an equality constraint: $V_{sa}+V_{sv}+V_{pa}+V_{pv}=V_0$
- Now we have 8 equations with 8 unknowns



Equations Solution Strategy:

- Step 1: Express heart pressures as functions of Q.
- Step 2: Find the resistance vessels pressures as functions of Q
- Step 3: Find all volumes as functions of Q.
- Step 4: Find Q from volume equality constraint.



Dr

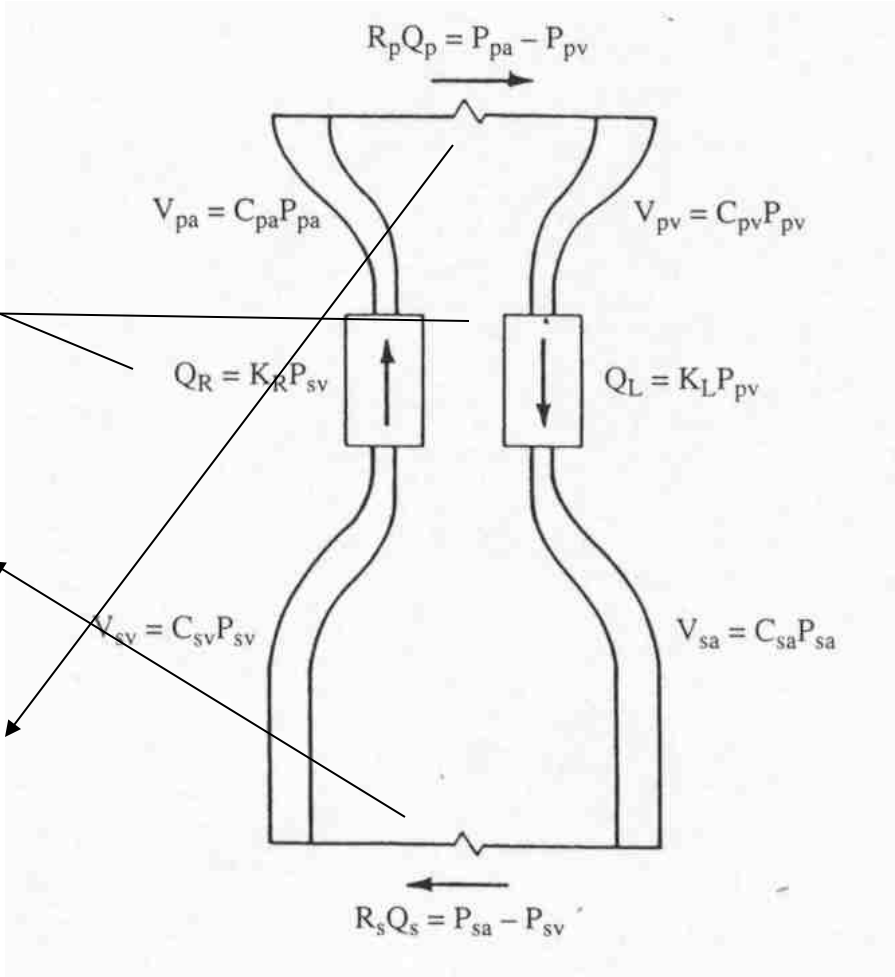
Pressures as Functions of Flow

$$P_{sv} \approx \frac{Q}{K_R}$$

$$P_{pv} \approx \frac{Q}{K_L}$$

$$P_{sa} = QR_s + P_{sv} \approx QR_s + \frac{Q}{K_R}$$

$$P_{pa} = QR_p + P_{pv} \approx QR_p + \frac{Q}{K_L}$$



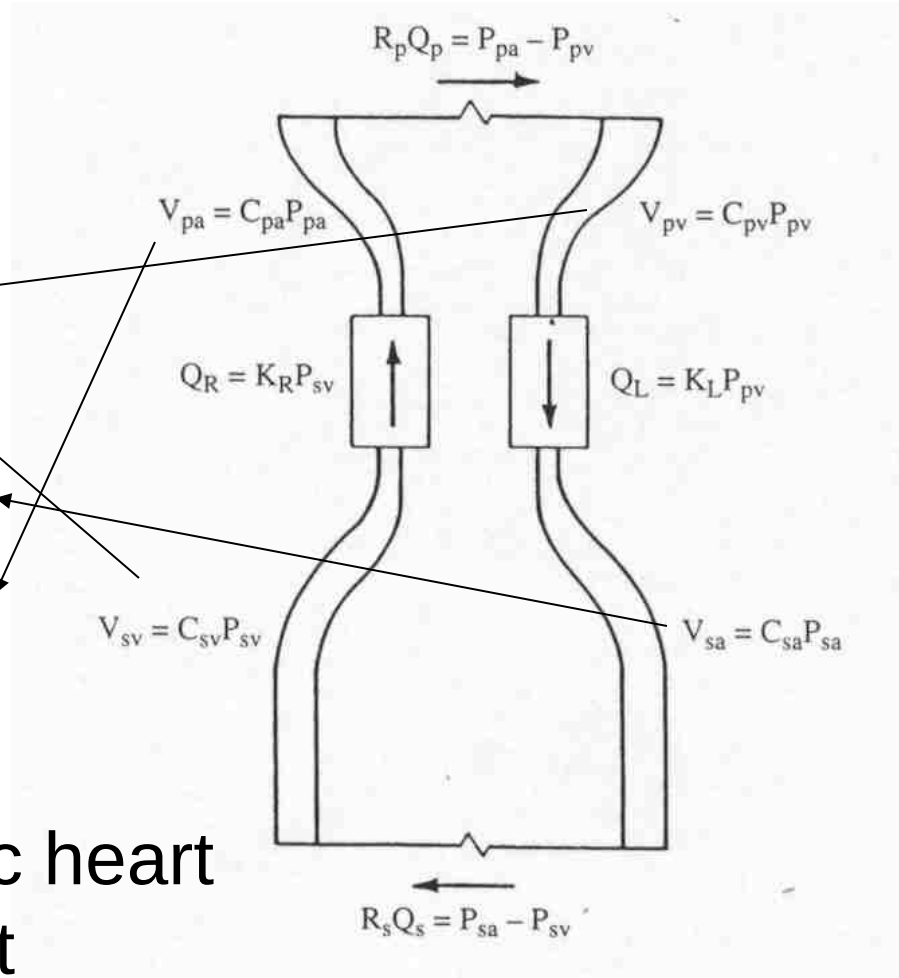
Volumes as Functions of Flow

$$P_{sv} = \frac{Q}{K_R} \Rightarrow V_{sv} = C_{sv}\frac{Q}{K_R}$$

$$P_{pv} = \frac{Q}{K_L} \Rightarrow V_{pv} = C_{pv}\frac{Q}{K_L}$$

$$P_{sa} = QR_s + \frac{Q}{K_R} \Rightarrow V_{sa} = (C_{sa}R_s + \frac{C_{sa}}{K_R})Q$$

$$P_{pa} = QR_p + \frac{Q}{K_L} \Rightarrow V_{pa} = (C_{pa}R_p + \frac{C_{pa}}{K_L})Q$$



Recall: we neglect systolic heart compliance and compliant vessels' dead volumes

Note about Units of all Coefficients:

- If $V=aQ$, where $[V]=$liter and $[Q]=$liter/min, then $[a]=$min.
- A coefficient C_1/K where $K=C_2F$ means that $[C/K]=[F^{-1}]=$min.
- As we know $[CR]=$min.
- So we see that all coefficients in the V vs. Q equations have time units.

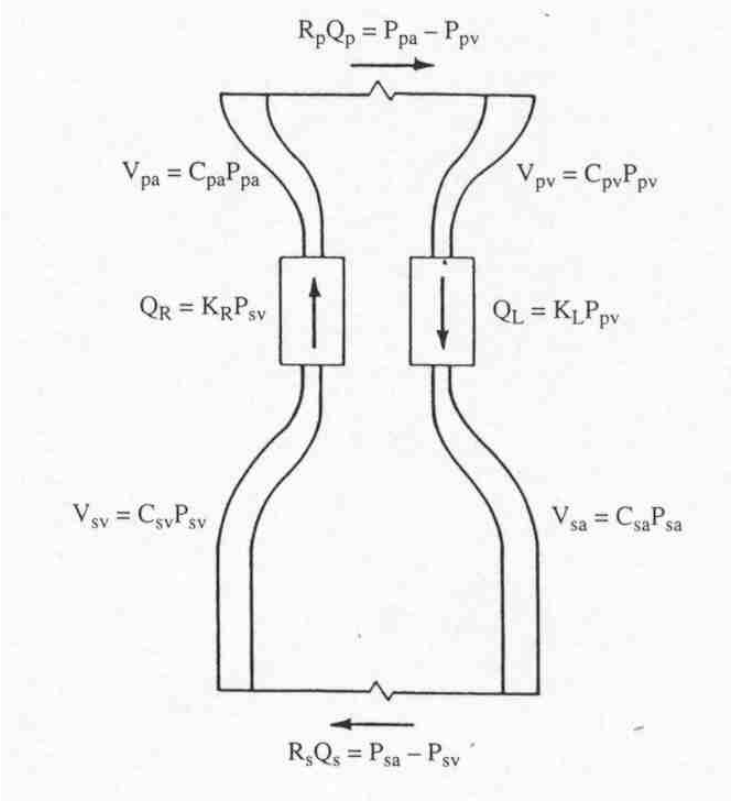
Defining Time Constants:

$$V_{sv} = C_{sv}\frac{Q}{K_R} = T_{sv}Q$$

$$V_{pv} = C_{pv}\frac{Q}{K_L} = T_{pv}Q$$

$$V_{sa} = (C_{sa}R_s + \frac{C_{sa}}{K_R})Q = T_{sa}Q$$

$$V_{pa} = (C_{pa}R_p + \frac{C_{pa}}{K_L})Q = T_{pa}Q$$



$$V_i = T_iQ \qquad i = sv, pv, sa, pa$$

Solving for Q from the Volume Constraint Equation:

$$V_{sv} = T_{sv} Q$$

$$V_{pv} = T_{pv} Q$$

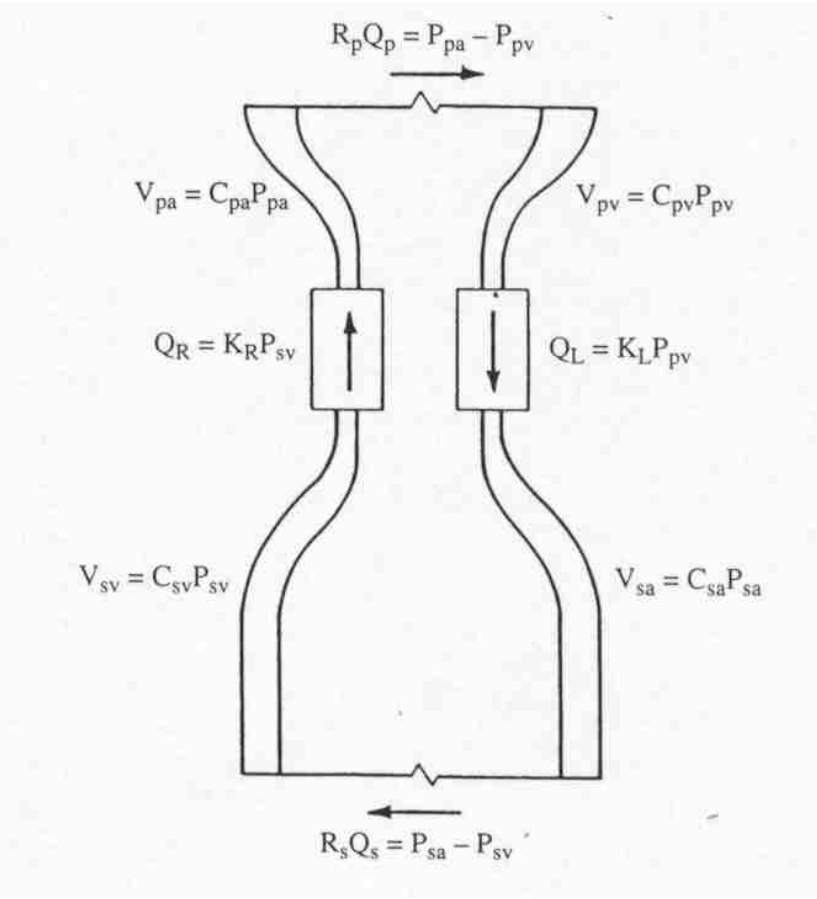
$$V_{sa} = T_{sa} Q$$

$$V_{pa} = T_{pa} Q$$

$$V_0 = V_{sv} + V_{pv} + V_{sa} + V_{pa} \Rightarrow$$

$$\Rightarrow V_0 = (T_{sv} + T_{pv} + T_{sa} + T_{pa}) Q$$

$$\boxed{Q = \frac{V_0}{T_{sv} + T_{pv} + T_{sa} + T_{pa}}}$$



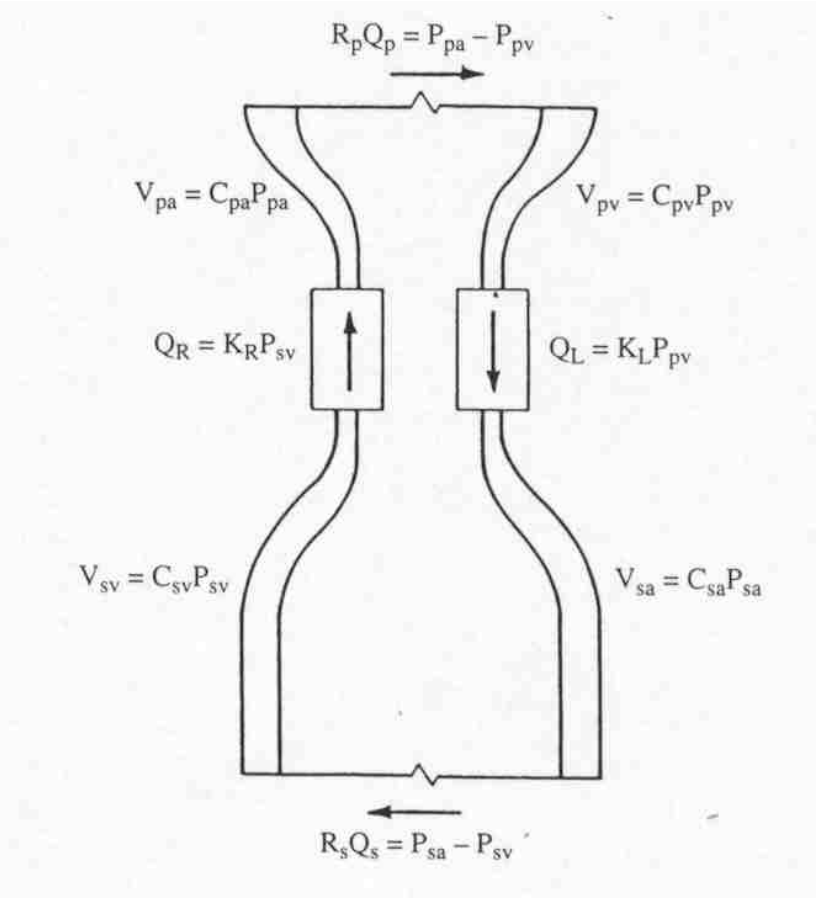
Solving for Volumes and Pressures from the Q Expression

$$Q = \frac{V_0}{T_{sv} + T_{pv} + T_{sa} + T_{pa}}$$

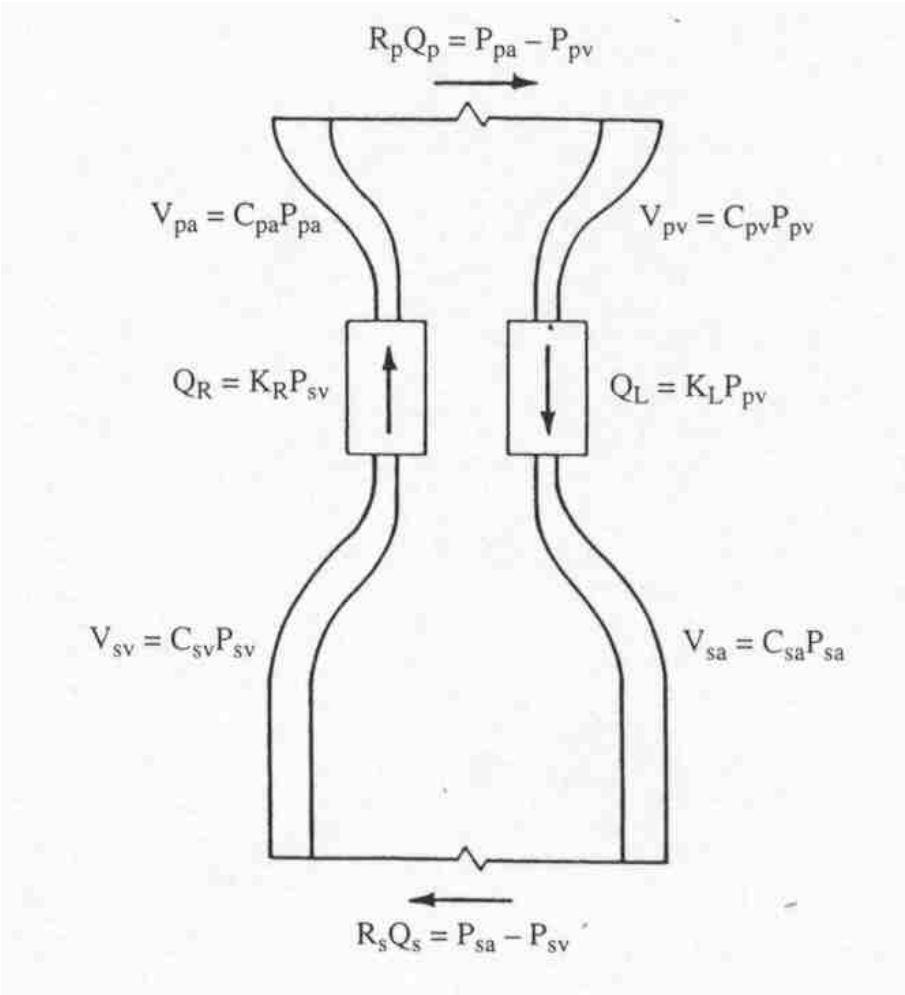
$$V_i = T_i Q \qquad i = sv, pv, sa, pa$$

$$P_i = \frac{V_i}{C_i} \qquad i = sv, pv, sa, pa$$

$$\boxed{\begin{aligned} V_i &= \frac{T_i V_0}{T_{sv} + T_{pv} + T_{sa} + T_{pa}} \\ P_i &= \frac{T_i V_0}{C_i (T_{sv} + T_{pv} + T_{sa} + T_{pa})} \end{aligned}}$$



Finding the Vessels' Compliances from the Resting Pressures and Volumes:

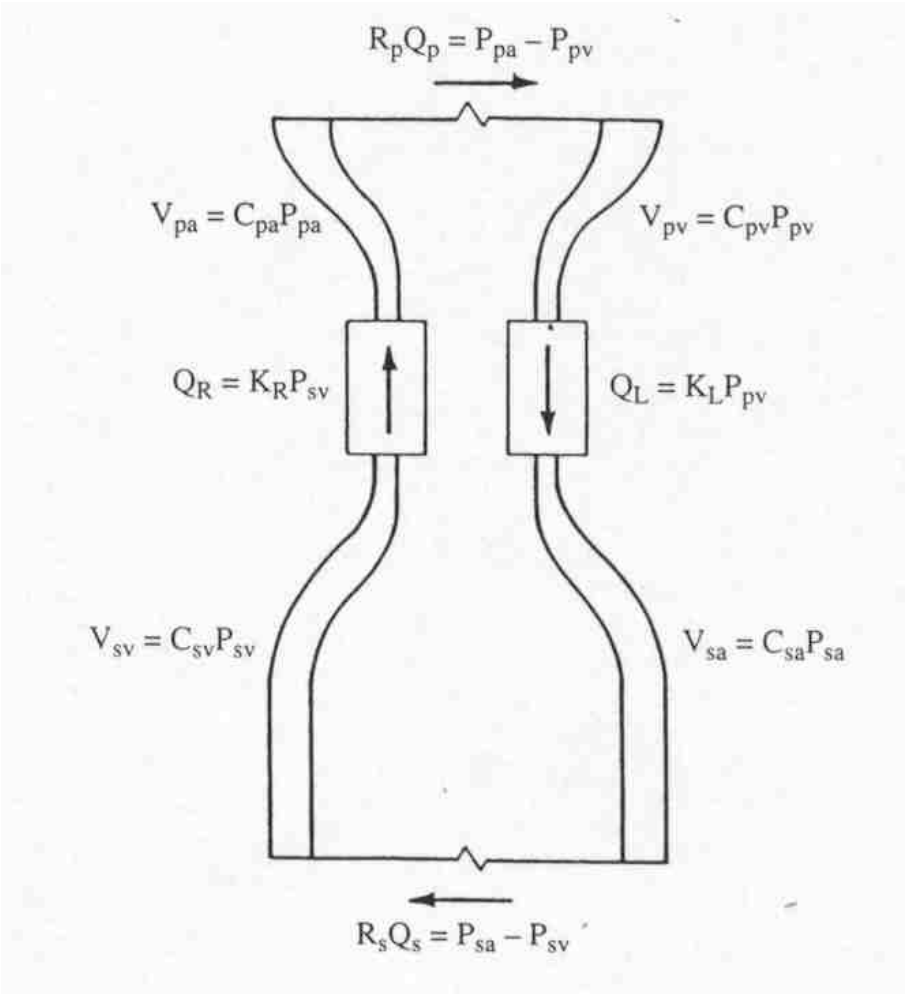


	P [mmHg]	V [liters]
sa	100	1.0
sv	2	3.5
pa	15	0.1
pv	5	0.4

	C_i [liters/mmHg]
sa	0.01
sv	1.75
pa	0.00667
pv	0.08

Finding the Systemic and Pulmonary Resistance

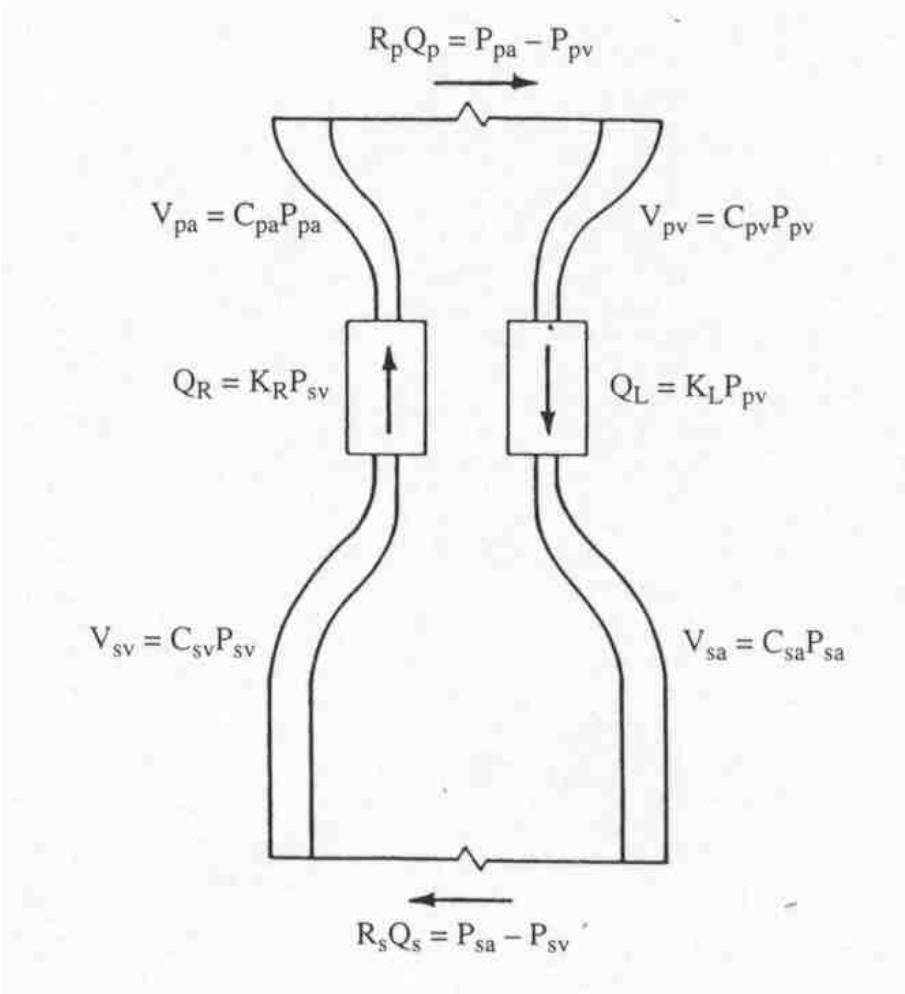
(Recall: For pulse rate of 80 [bpm] and V_0=5[liters] → Q=5.6[liters/minute])



	P [mmHg]	V [liters]
sa	100	1.0
sv	2	3.5
pa	15	0.1
pv	5	0.4

R_s	17.5 [mmHg/(liters/minute)]
R_p	1.79 [mmHg/(liters/minute)]

Finding the Pumps Coefficients (Recall: For pulse rate of 80 [bpm] & V_0=5[liters] → Q=5.6[liters/minute])

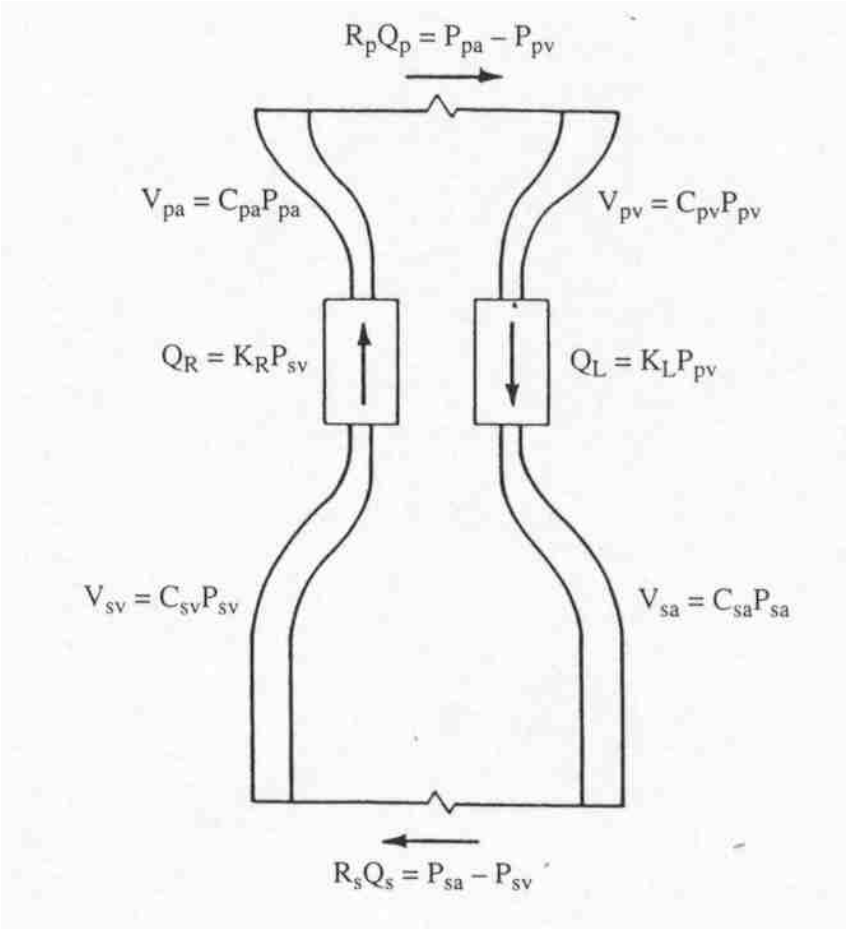


	P [mmHg]	V [liters]
sa	100	1.0
sv	2	3.5
pa	15	0.1
pv	5	0.4

K_L	1.12 [(liters/minute) / mmHg]
K_R	2.8 [(liters/minute) / mmHg]

Finding the four Time Constants (typical values)

i	T_i [seconds]
sv	37.5
pv	4.3
sa	10.7
pa	1.1



Need to Coordinate the Two Heart Sides

- What keeps the cardiac outputs of the right and left heart equal?
- What mechanisms control the partition of blood volume between the systemic and pulmonary circulations?
- Fact: If the left cardiac output exceeds the right cardiac output by only 10% for one minute, the entire pulmonary circulation system will be emptied.

Transient Flow and Built-in Negative Feedback

- The right and left cardiac output are equal at steady-state, but may differ during transient.
- A sudden change in any coefficient (resistance, compliance or pump coefficient) may cause a temporary deviation between Q_R and Q_L. Consequently, pressures and volumes change, and eventually restore flow equality between the two sides.
- The steady-state flow-pressure-volume relationships contain within them some “negative feedback” restoration mechanisms!

Example to Self-Restoration: Assume that K_R is Suddenly Reduced

$$V_{sv} = C_{sv}\frac{Q}{K_R} = T_{sv}Q$$

$$V_{pv} = C_{pv}\frac{Q}{K_L} = T_{pv}Q$$

$$V_{sa} = (C_{sa}R_s + \frac{C_{sa}}{K_R})Q = T_{sa}Q$$

$$V_{pa} = (C_{pa}R_p + \frac{C_{pa}}{K_L})Q = T_{pa}Q$$

- Time constants T_{sv} and T_{sa} increase, whereas T_{pv} and T_{pa} remain unchanged.
- A sudden drop of K_R causes a sudden drop of Q_R (recall: $Q_R=K_RP_{sv}$) compared to Q_L.

If $Q_R < Q_L$ there will be a net transfer of blood volume away from the pulmonary circulation and into the systemic circulation

$$Q = \frac{V_0}{T_{sv} + T_{pv} + T_{sa} + T_{pa}}$$

$$V_i = T_i Q \qquad i = sv, pv, sa, pa$$

$$P_i = \frac{V_i}{C_i} \qquad i = sv, pv, sa, pa$$

$$V_i = \frac{T_i V_0}{T_{sv} + T_{pv} + T_{sa} + T_{pa}}$$

$$P_i = \frac{T_i V_0}{C_i (T_{sv} + T_{pv} + T_{sa} + T_{pa})}$$

- P_{sv} increases and P_{pv} decreases, which increases Q_R and decreases Q_L.
- This restores the two flow rates Q_R and Q_L into a new equality.
- Systems settles into a new equilibrium, with modified pulmonary and systemic volumes.

Steady-State Volume Ratios as Function of the System Parameters

$$\frac{V_p}{V_s} = \frac{V_{pa} + V_{pv}}{V_{sa} + V_{sv}} = \frac{T_{pa} + T_{pv}}{T_{sa} + T_{sv}} = \frac{\dfrac{C_{pa} + C_{pv}}{K_L} + C_{pa} R_p}{\dfrac{C_{sa} + C_{sv}}{K_R} + C_{sa} R_s}$$

- V_p is the total pulmonary blood volume, and V_s is the total systemic blood volume.
- Temporary flow disparity cause volume and pressures changes that act to restore flow parity.
- Key mechanism: Q dependence on P_v (cardiac output depends on the venous pressure)

Stroke Volume and Cardiac Output

- Maximum volume: $V_{ED}=V_d+C_{diastole}P_v$
- Minimum volume: $V_{ES}=V_d+C_{systole}P_a$
- Stroke Volume: $V_{stroke} = V_{ED} - V_{ES} =$
  $= C_{diastole}P_v - C_{systole}P_a \approx C_{diastole}P_v$
  assuming that $C_{systole}$ is very small.
- Let F be the heart rate [beats/minute], then:
- Cardiac Output = $Q = FV_{stroke} \approx FC_{diastole}P_v$

Why is it that Q Dependence on P_V is so Critical?

- Imagine that the values of $C_{systole}$, $C_{diastole}$, P_a and P_v are such that:
- $Q=FV_{stroke}\approx FC_{systole}P_a$
- Need to show (as an exercise) that there is no longer built-in negative feedback in the flow-pressure-volume system.

The Flow-Pressure-Volume auto-regulation is however insufficient!

There is a need for more feedback mechanisms. “Why” and “How” will be explored in the next part.