BME 5742
Bio-Systems Modeling and Control

Lecture 37

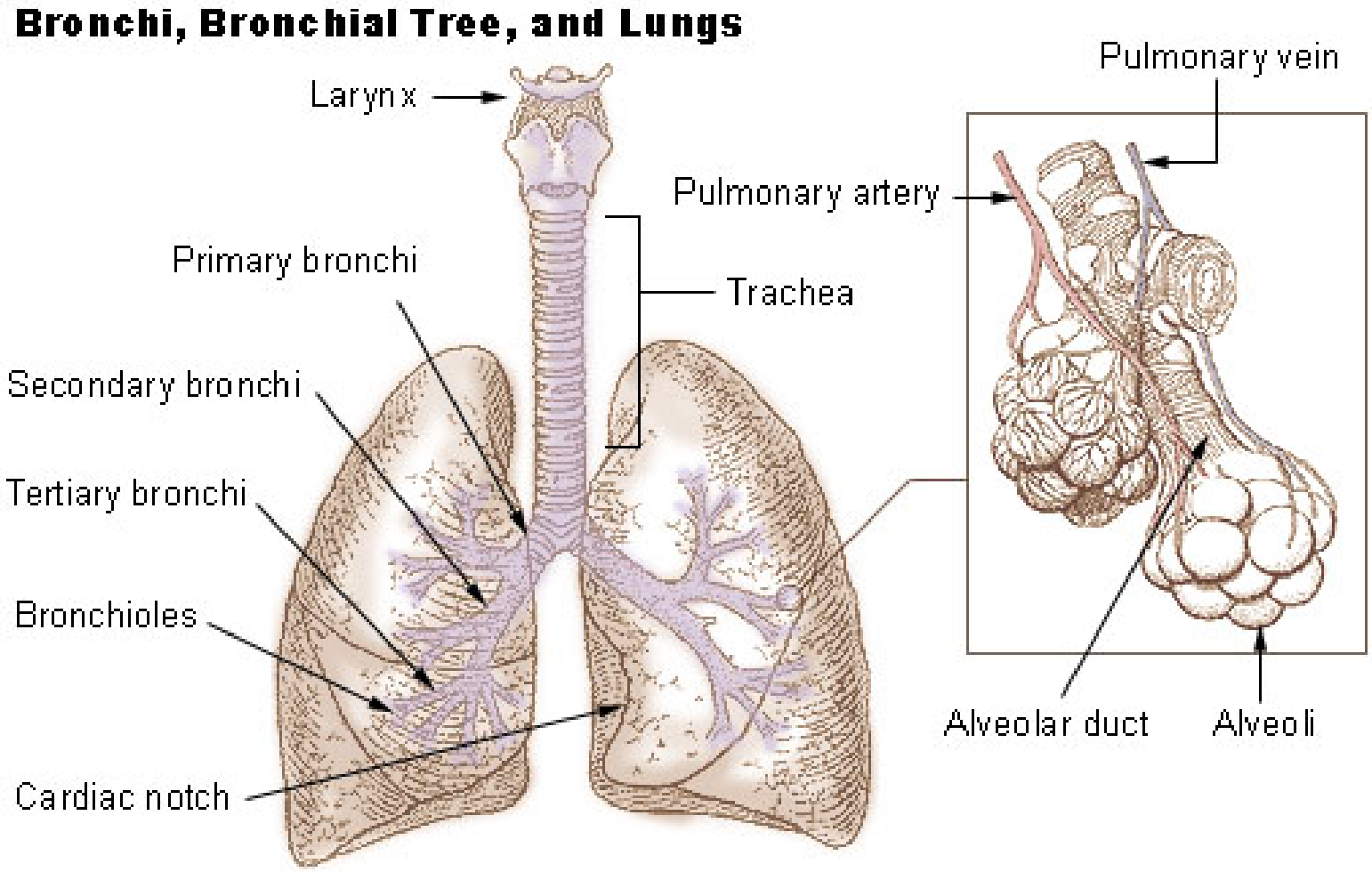
Gas Exchange in the Lungs

Single Alveolus Model

References

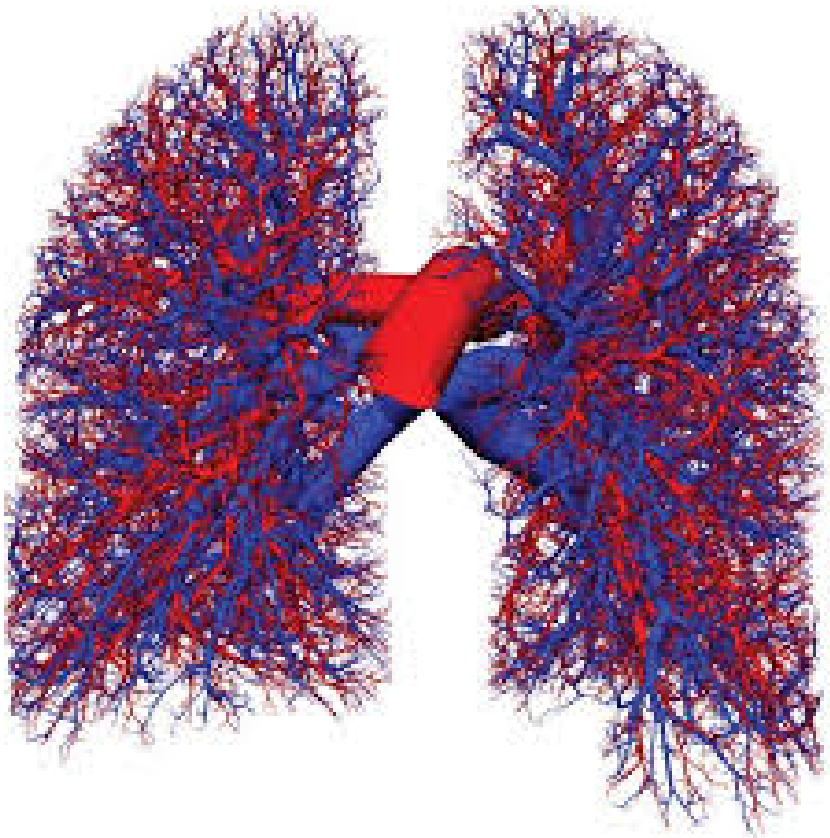
- Hoppensteadt & Peskin, “Modeling and Simulation in Medicine and the Life Sciences”, 2nd Ed., Springer 2002.
- Occasional images from Google Images

The Pulmonary Tissue consists of two separate Lungs



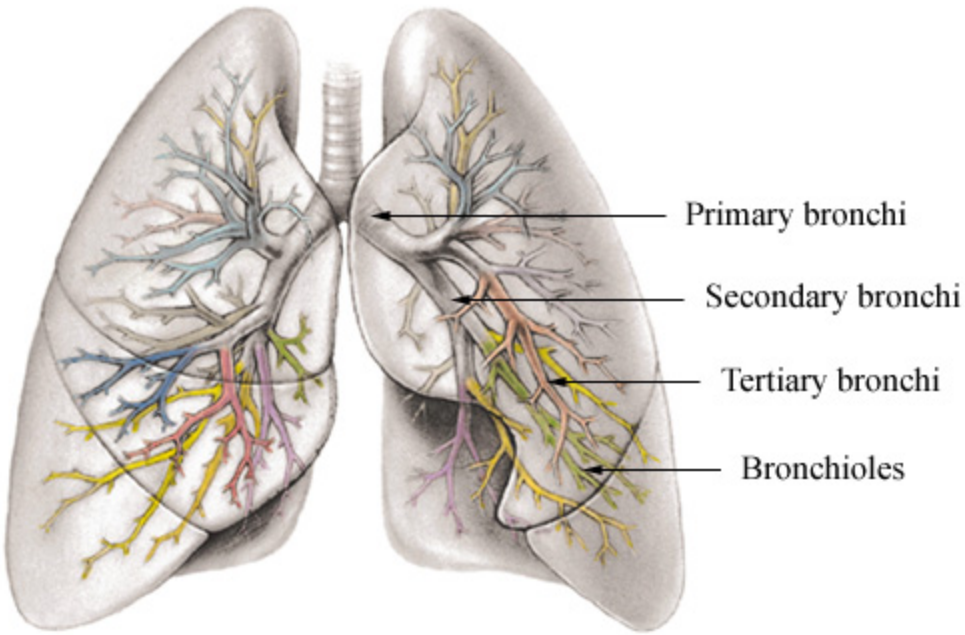
Pulmonary Circulation

- The blood leaving the right heart is subdivided many times by the branching pulmonary arterial tree before it reaches the alveoli.
- After passing through the alveoli, blood is collected by the pulmonary veins.
- Alveolar capillaries are all connected in parallel: the pulmonary blood flow is the sum of all alveoli flows.

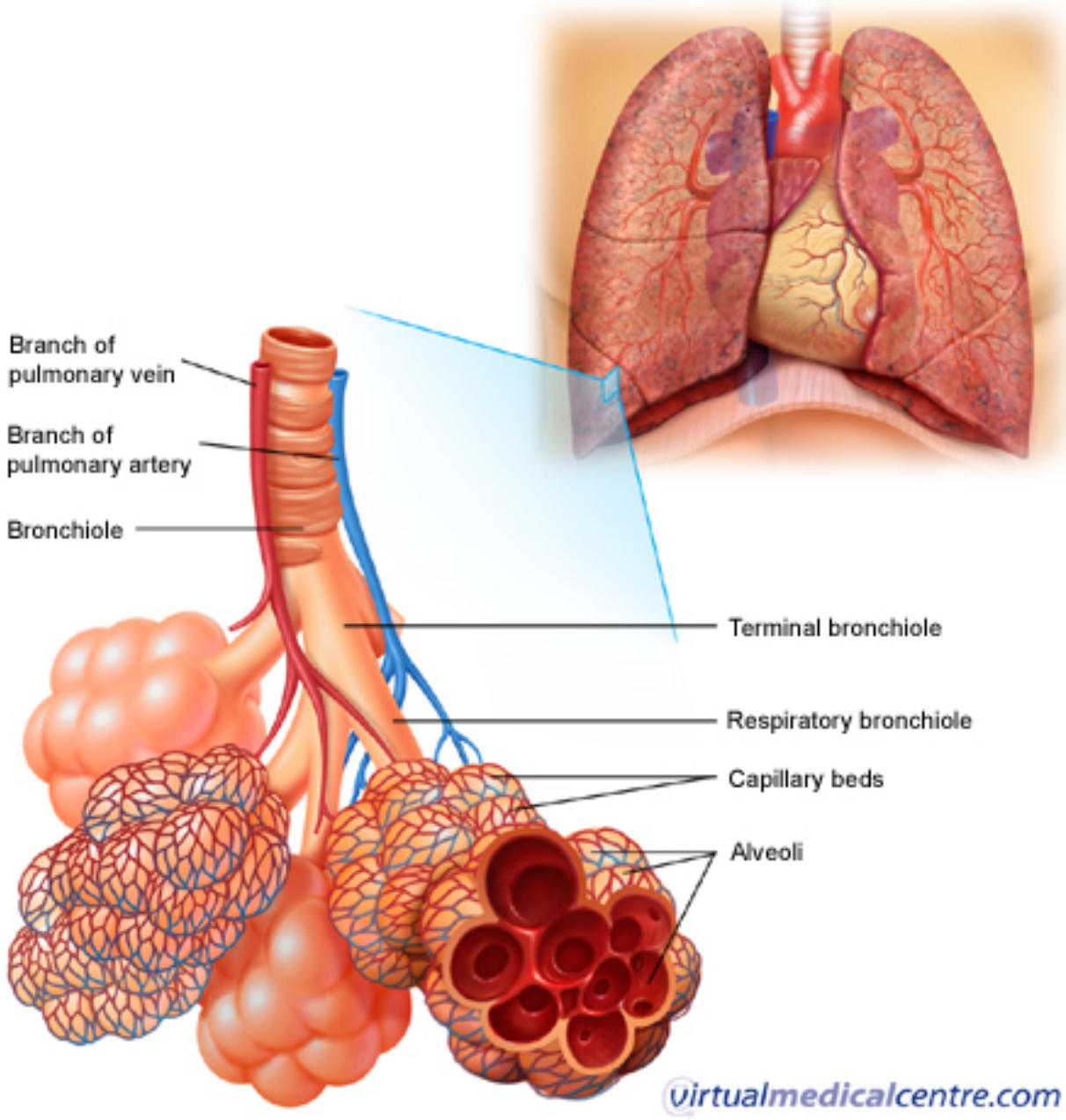


Lungs Facts - 4

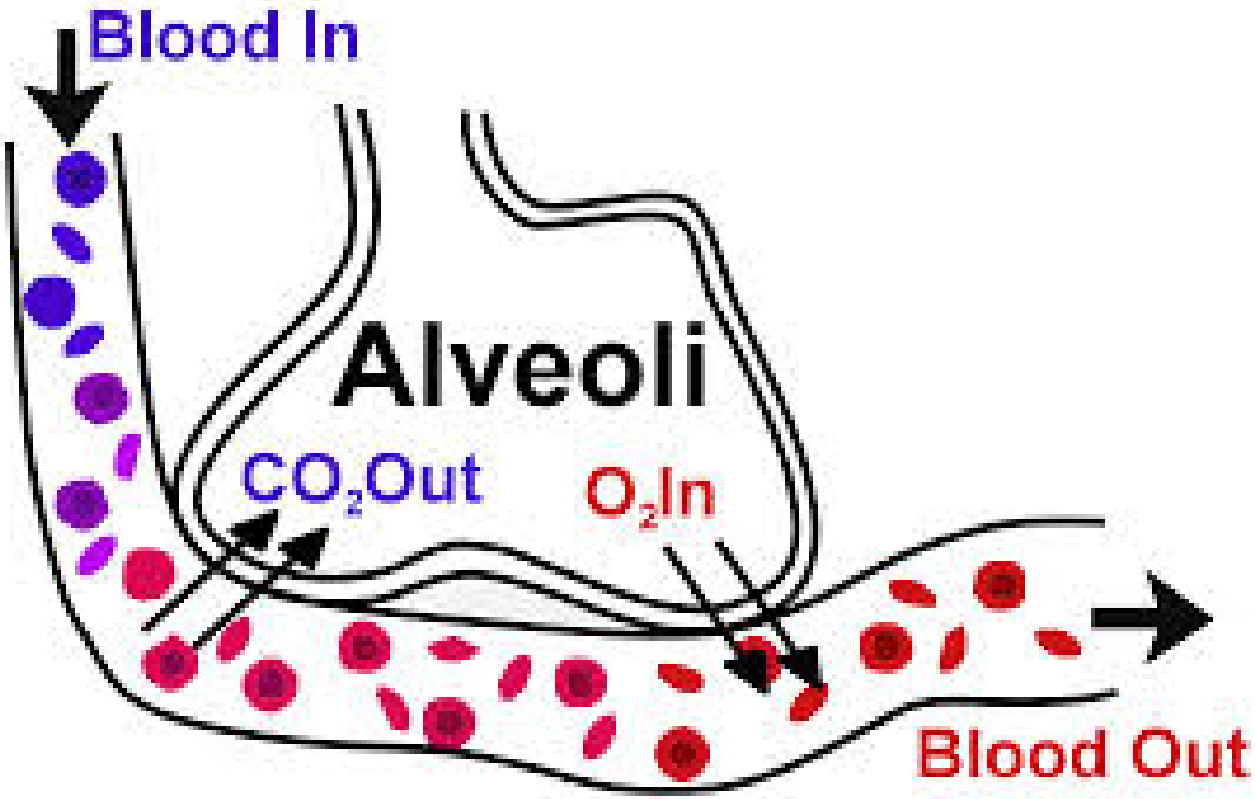
- Air entering the trachea is subdivided many times by the bronchial tree, before it reaches the alveoli.
- The total alveolar ventilation is the sum of the individual alveolar ventilations.



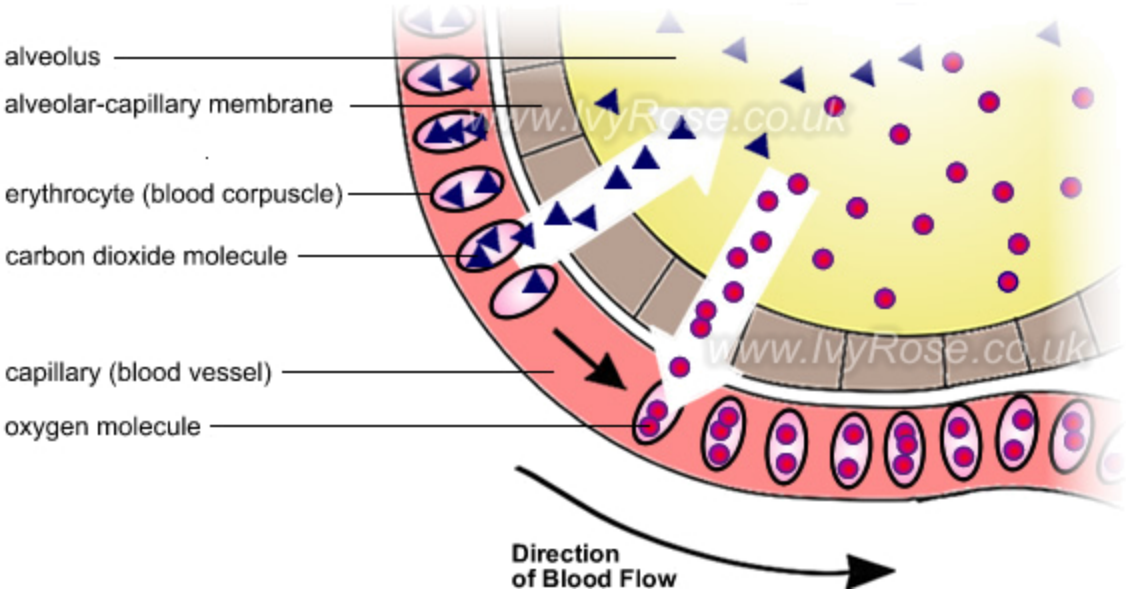
Unlike blood, the air leaves the alveoli, on expiration, by way of the same bronchial tree through which it entered on inspiration.

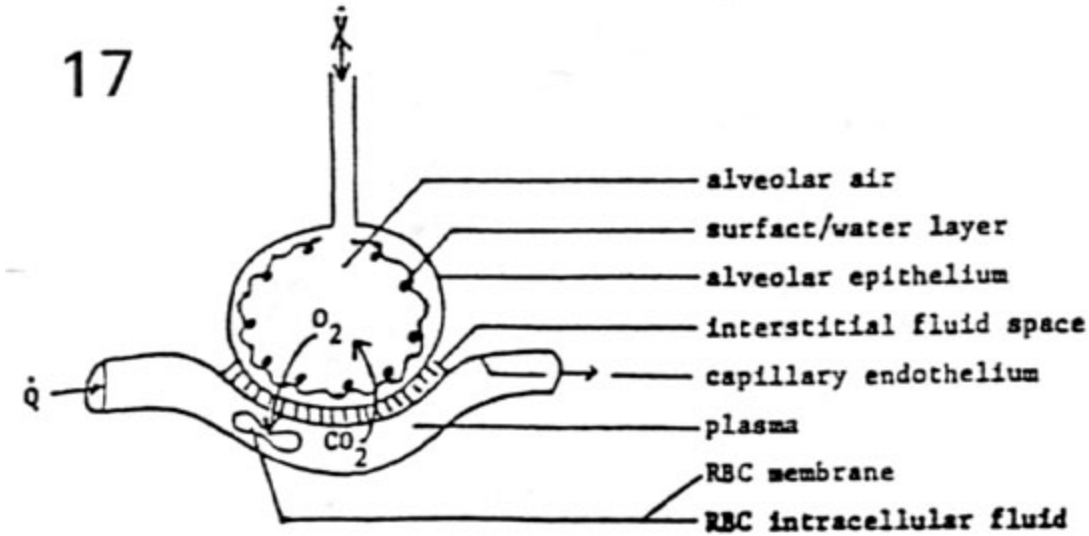


The two lungs contain about $3 \cdot 10^8$ alveoli (little sacs) in which air and blood are brought into close contact, so that gas exchange can take place.

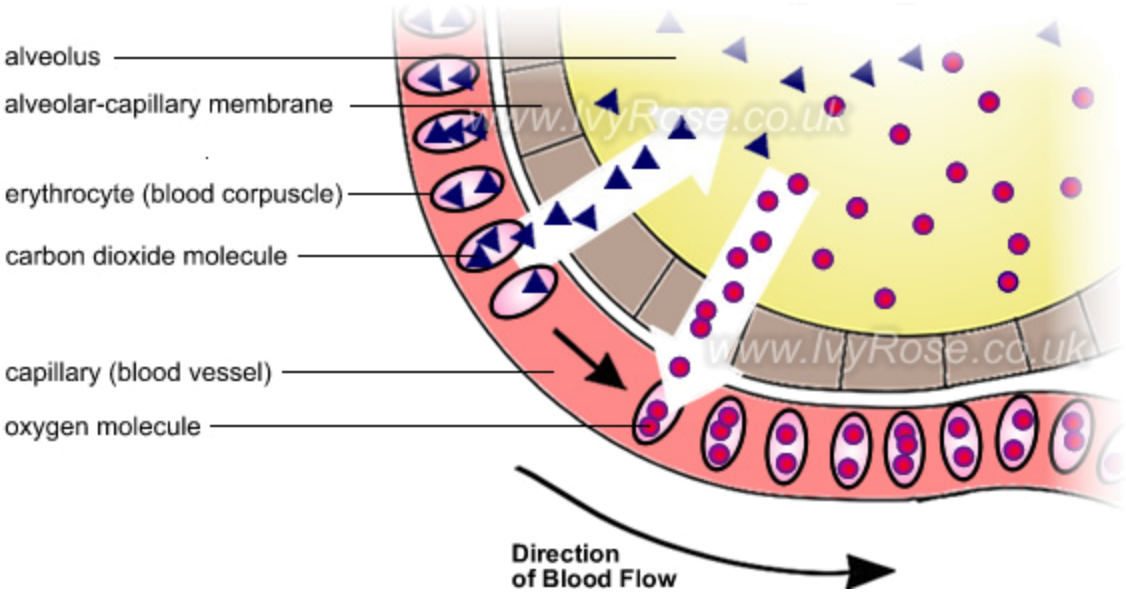


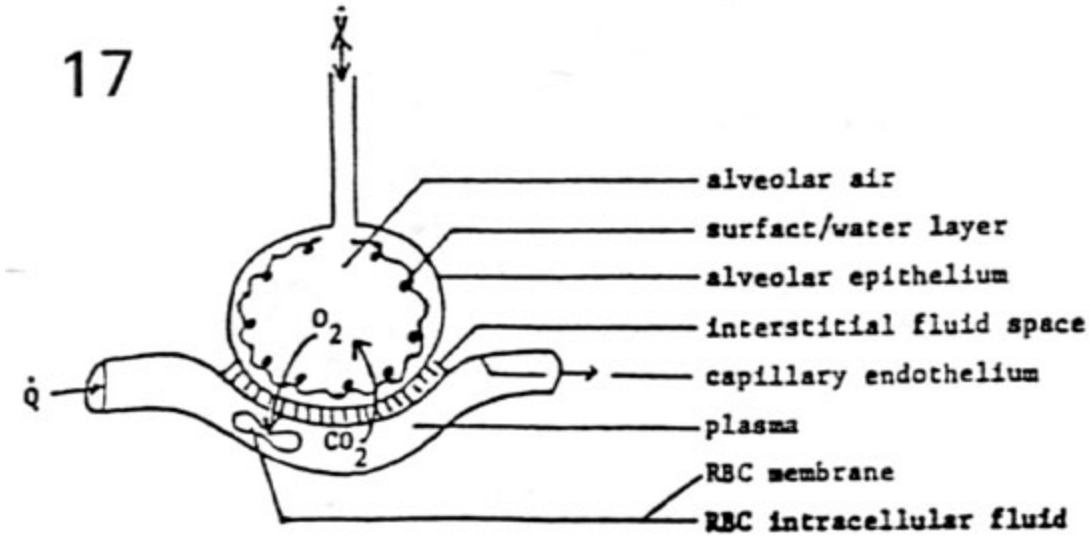
- The principal gases exchanged are O_2 , which is picked up by the blood, and CO_2, which leaves the blood and enters the air spaces of the lungs.



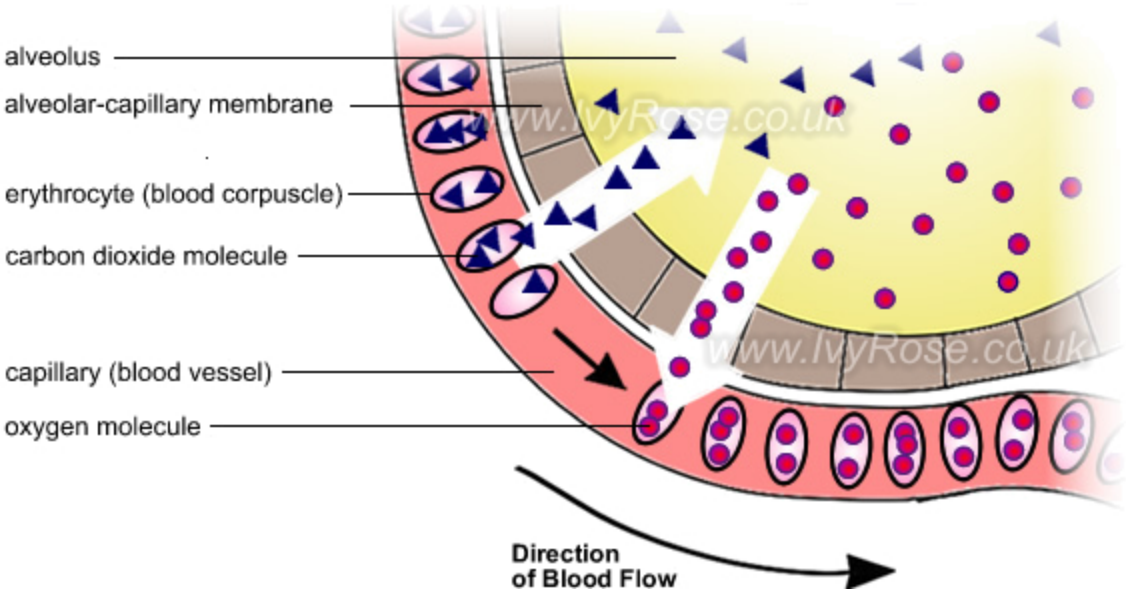


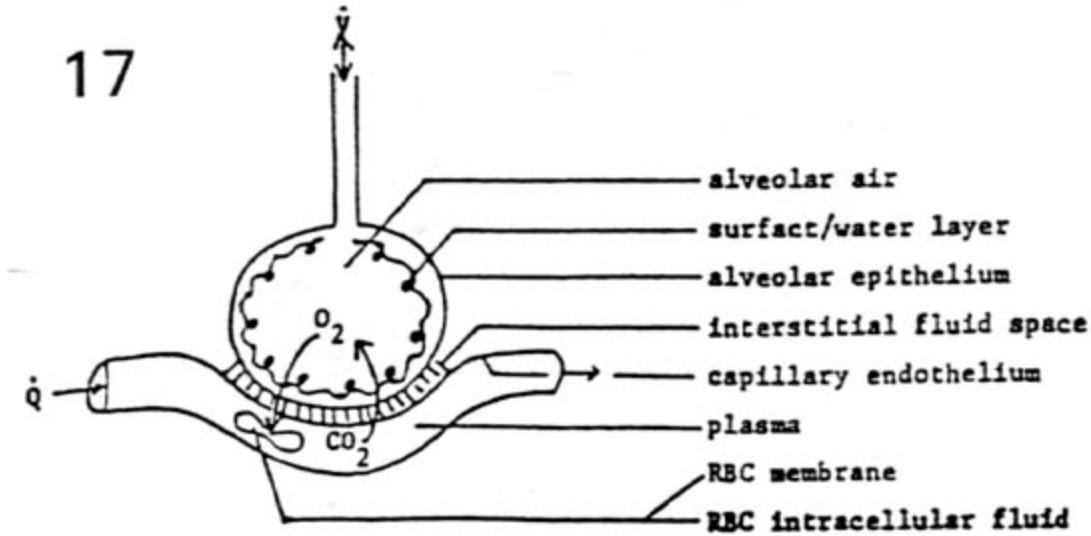
The gases need to cross the thin alveolar-capillary membrane. It is done by diffusion.





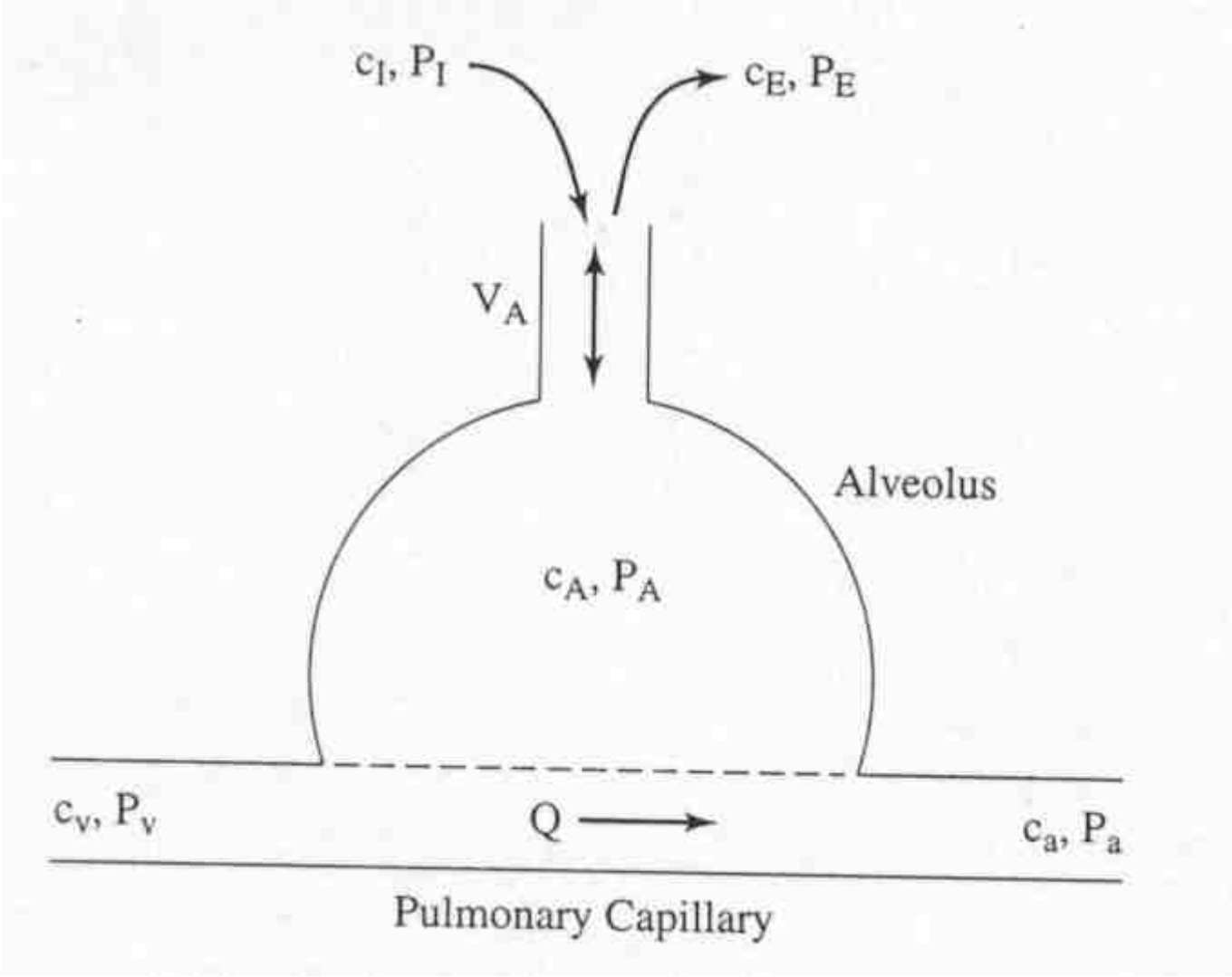
The blood in the capillaries of each alveolus quickly reaches an equilibrium with the alveolar air.



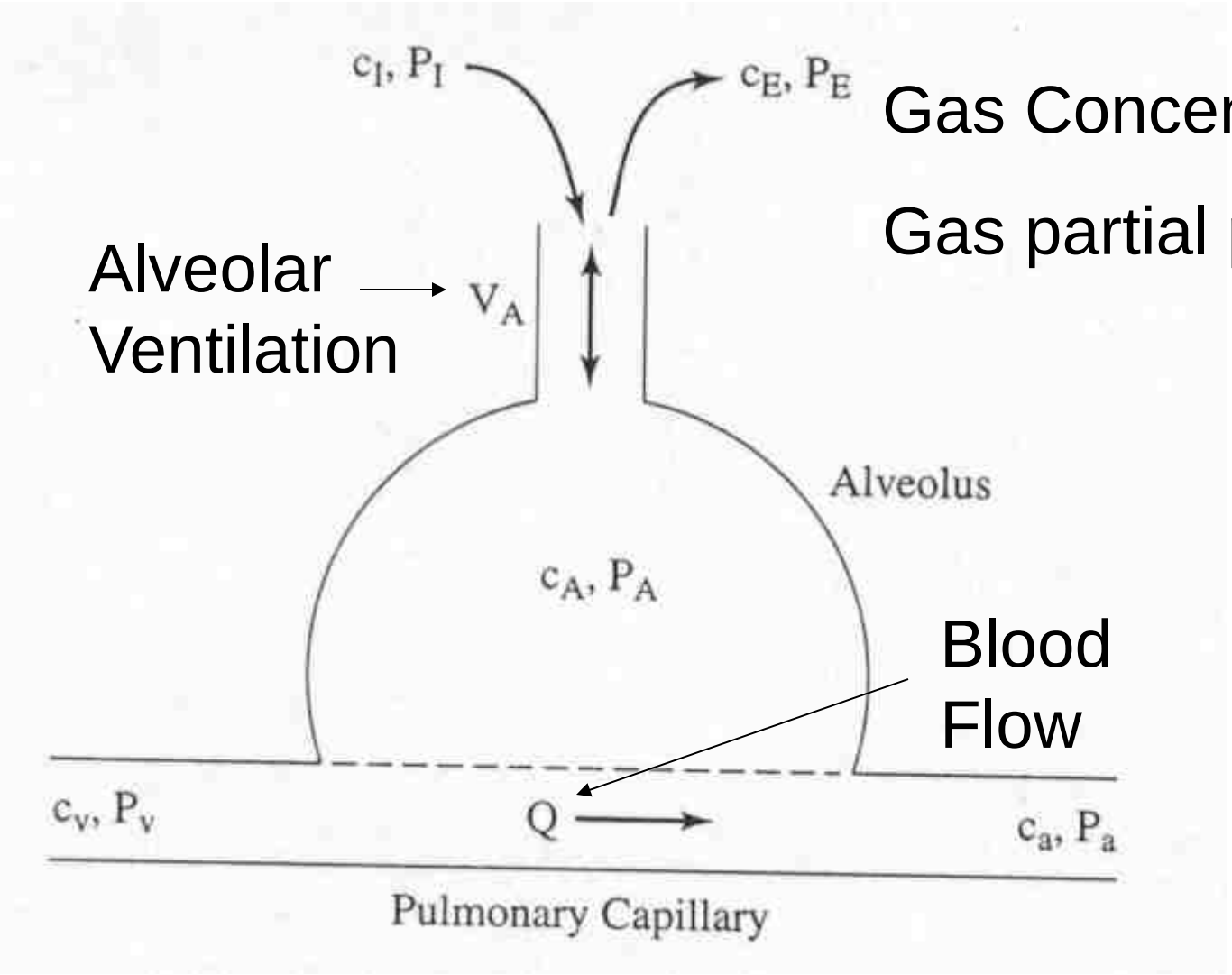


Typically, such equilibrium is established after the blood traversed about one third of a capillary length (from the pulmonary arteries side towards the pulmonary veins side). At that time, Hb molecules are all saturated.

Gas Transport in One Alveolus



Gas Transport in One Alveolus: Variables defined



Gas Concentration = c

Gas partial pressure = P

<u>Indices</u>:

I=inspired

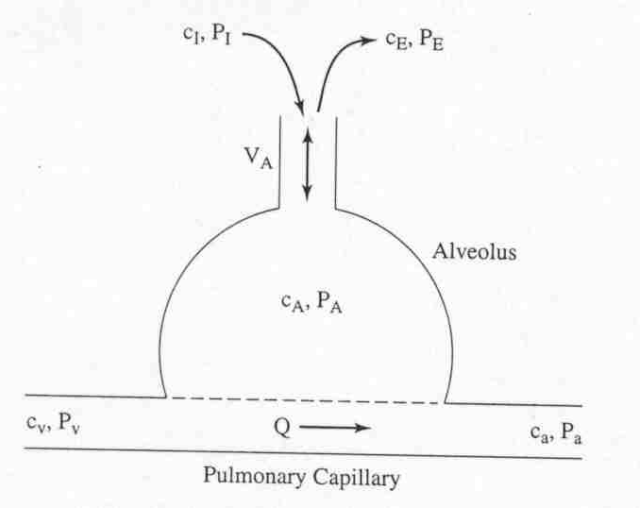
E=expired

A=alveolar

v=venous

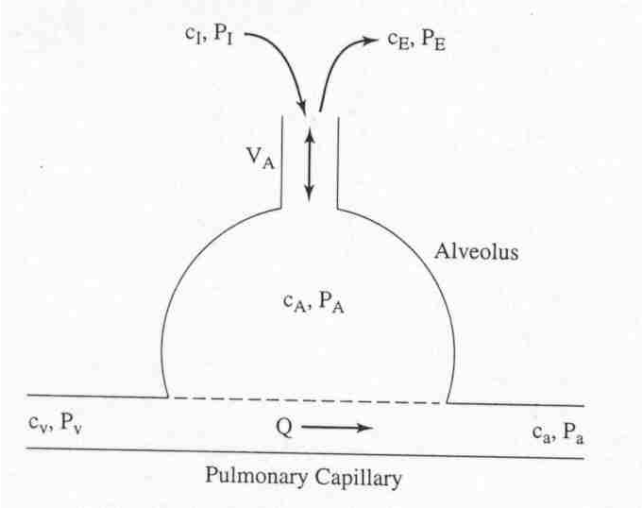
a=arterial

Alveolar Ventilation



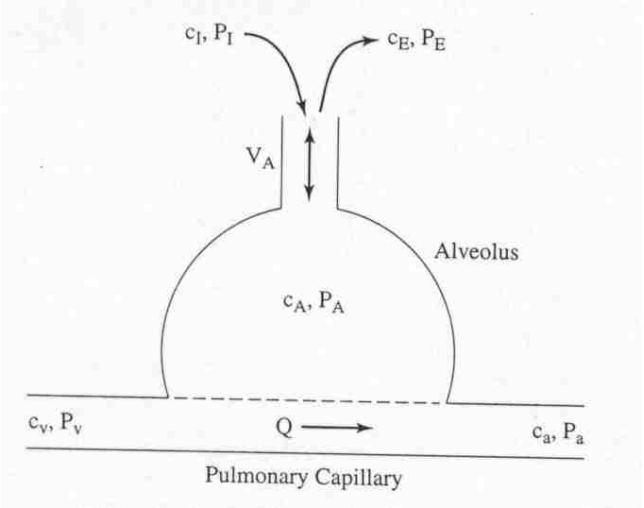
- Alveolar ventilation is the volume of fresh air delivered to the alveolus per unit time = (volume of air in one breath) x (number of breaths per unit time).
- The first installment of air reaching the alveolus in each breath is the air that was expelled on the previous breath and remained in the bronchial tree between breaths.

Alveolar Ventilation explained more



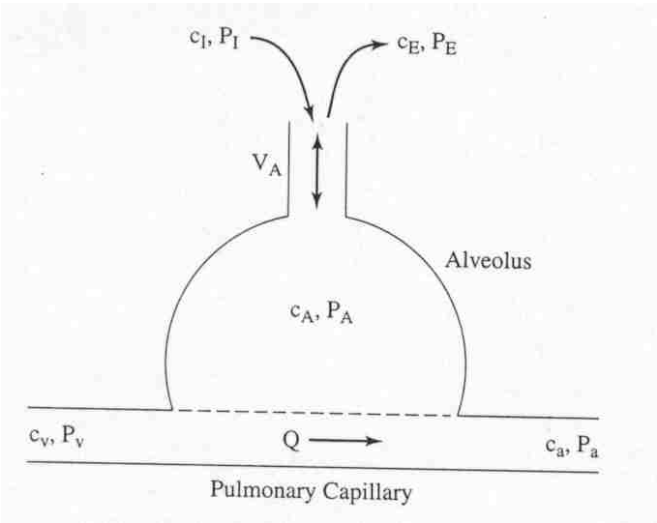
- Non-fresh air is not included in the alveolar ventilation.
- Alveolar ventilations of inspiration and expiration are not exactly equal, because O_2 consumption by the body is not exactly equal to the CO_2 production. We neglect this difference, and refer to both ventilations as V_A.

Blood is named according to its chemical composition



- We call the blood entering the alveolus <u>venous blood</u> , even though it flows in the pulmonary arterial tree. Likewise, <u>arterial blood</u> is the one leaving the lungs in the pulmonary veins.
- Venous blood is rich in CO_2 and its O_2 is relatively depleted. Arterial blood is rich in O_2 and is almost free of CO_2.

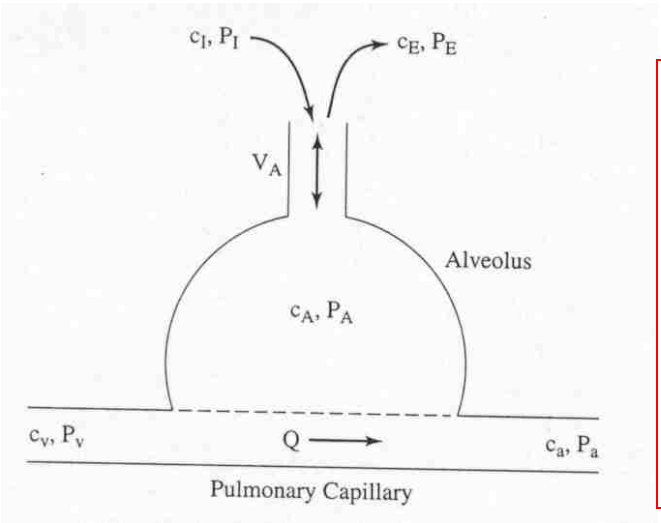
Alveolus Gas Transport Steady-State Assumption



$$V_A c_I + Q c_v = V_A c_E + Q c_a$$

- The number of molecules of a gas that enter the alveolus, per unit time, equals the number of molecules of that gas leaving per unit time.
- Molecules enter by air and blood, and leave by air and blood.

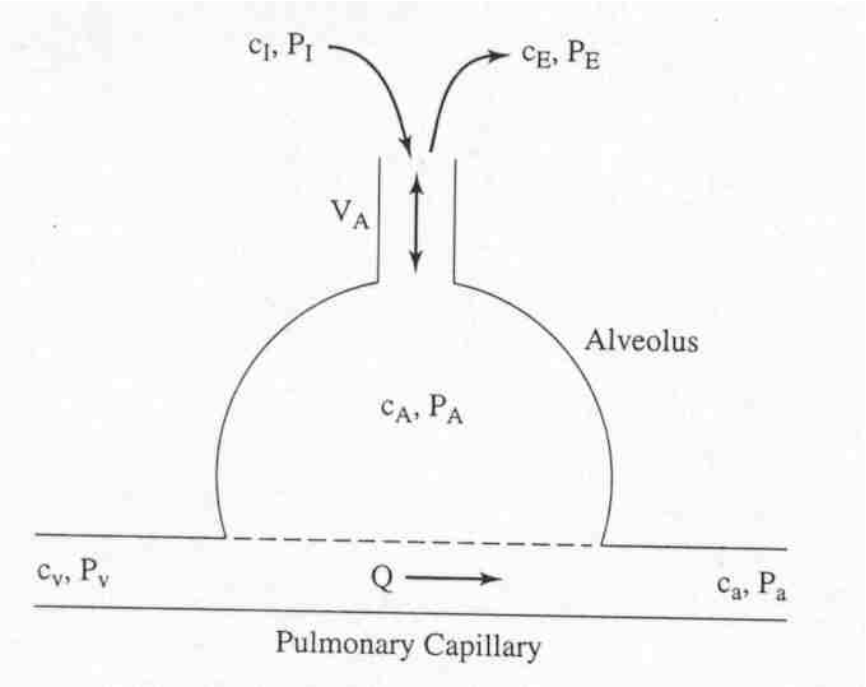
Assumption 1: Conservation of Number of Molecules



	Entering		Leaving	
	$V_A c_I$	$+ Q c_v$	$= V_A c_E$	$+ Q c_a$
By:	Air	Blood	Air	Blood

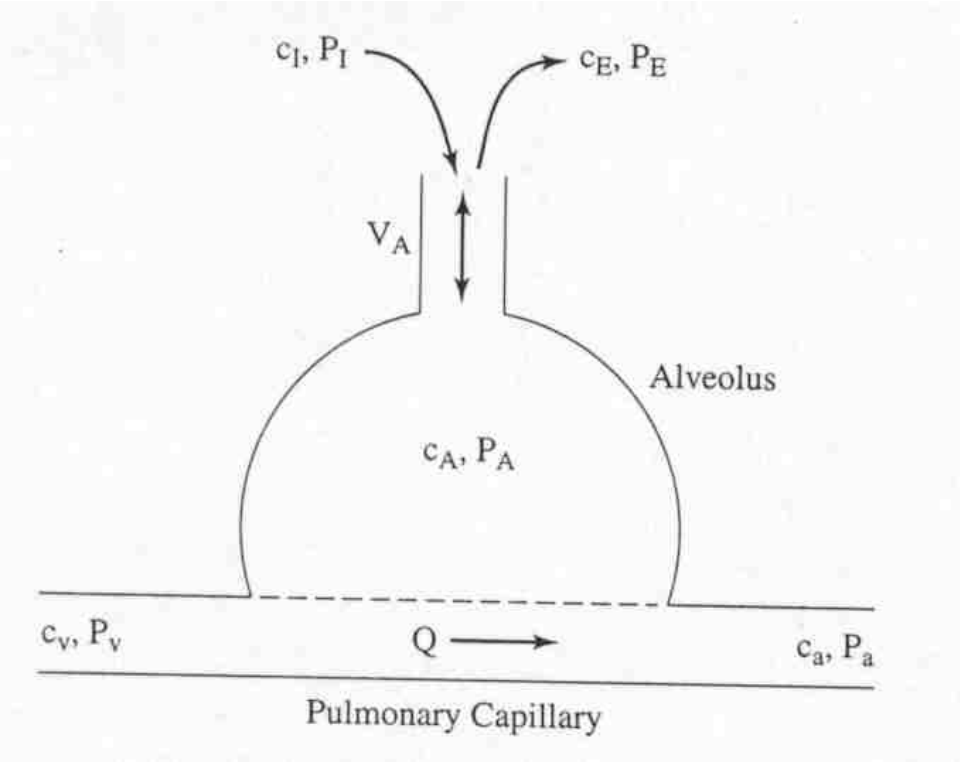
- The number of molecules of a gas that enter the alveolus, per unit time, equals the number of molecules of that gas leaving per unit time.
- Molecules enter by air and blood, and leave by air and blood.

Assumption 2: Expired air is a sample of alveolar air



$$c_E = c_A$$

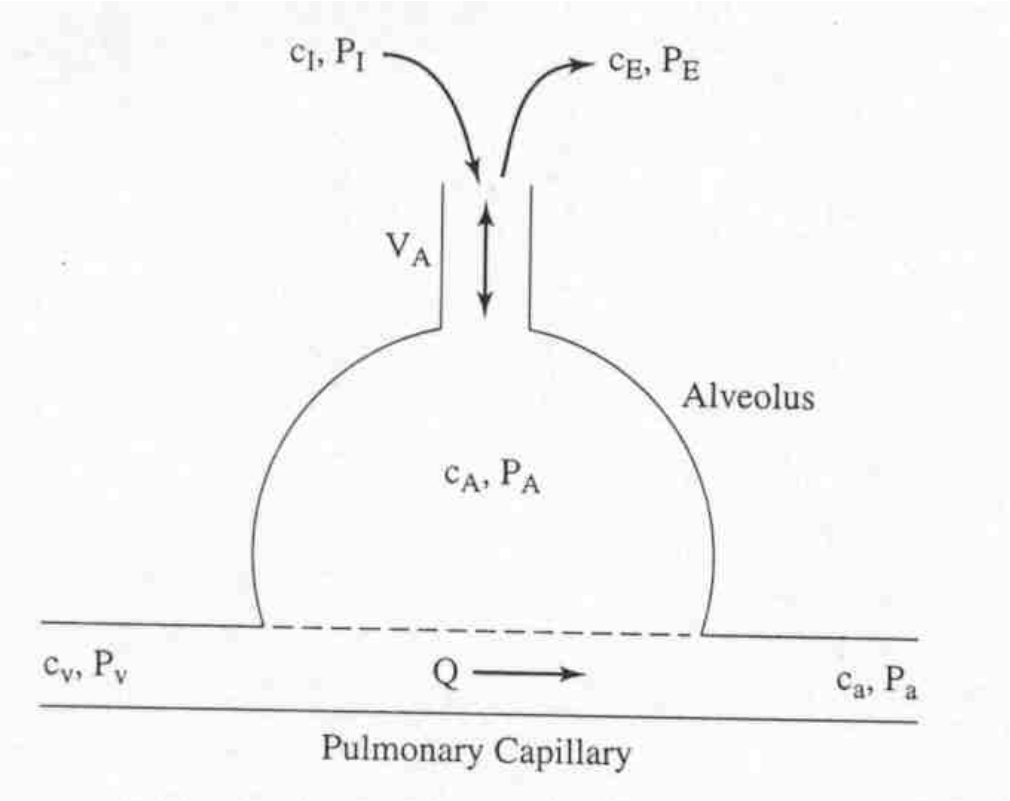
Assumption 3: Alveolar gases obey the Ideal Gas Law



$$P_A = KTc_A$$

K=Boltzmann's constant; T=Absolute temperature

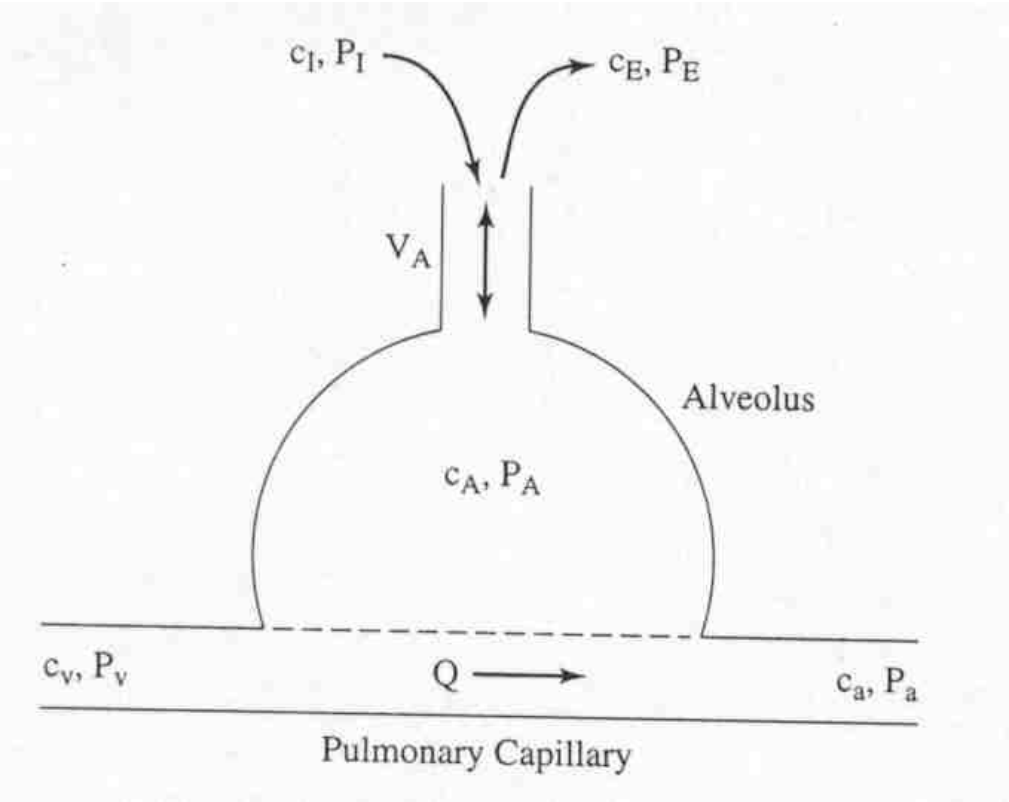
Assumption 4: Gas in alveolar arterial blood forms a simple solution



$$\sigma P_a = c_a$$

Solubility of gas = σ

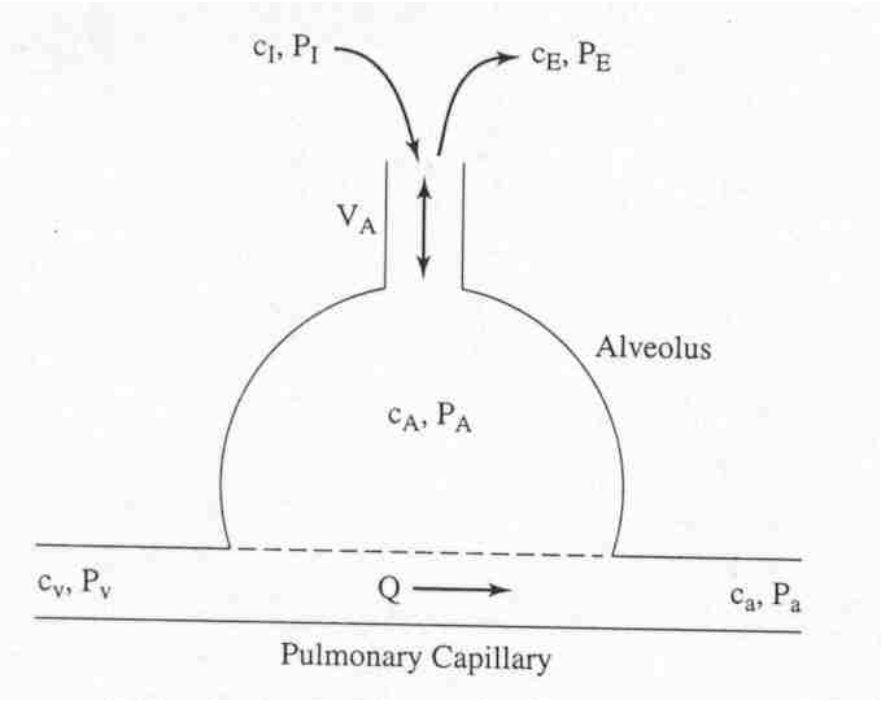
Assumption 5: Alveolar Blood-Air Equilibrium



$$P_a = P_A$$

Partial pressure of gas in blood leaving the alveolus = Partial pressure of gas in alveolar air

Gas molecules: From air to blood



$$V_A c_I + Q c_v = V_A c_E + Q c_a$$

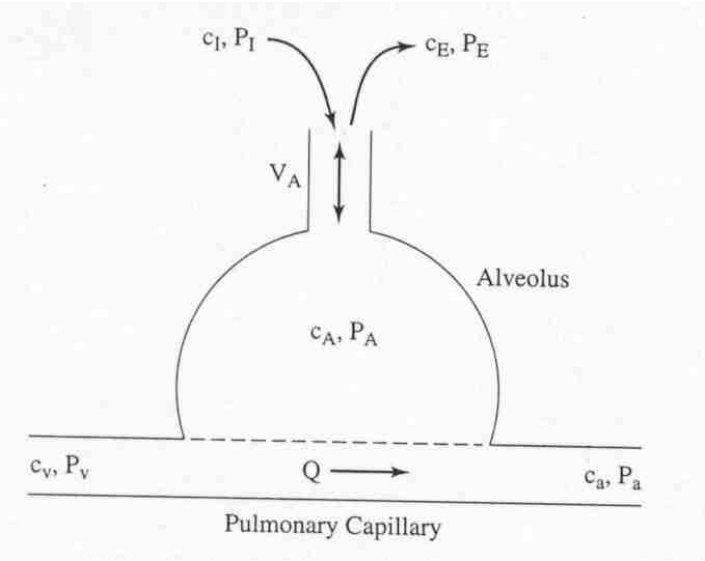
$$c_E = c_A$$

$$\Rightarrow \boxed{V_A (c_I - c_A) = Q (c_a - c_v)}$$

Net transport of a gas in alveolus

molecules given up by air are picked up by blood.

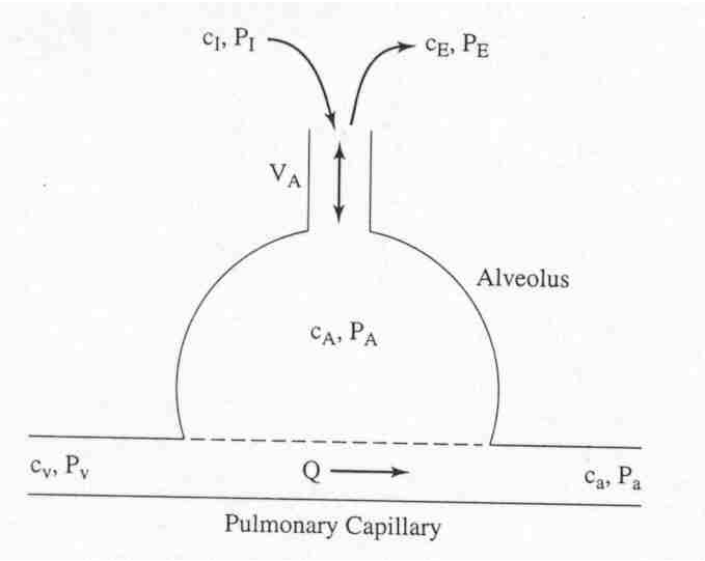
Net Transport of Gas in Alveolus -1



$$V_A(c_I - c_A) = Q(c_a - c_v)$$

- For a net transport of gas to occur, it is necessary that the composition of alveolar air and inspired air be different!
- Example: Partial pressure of O_2 in inspired air is around 160 mmHg, whereas in the alveoli it stands at around 100 mmHg.

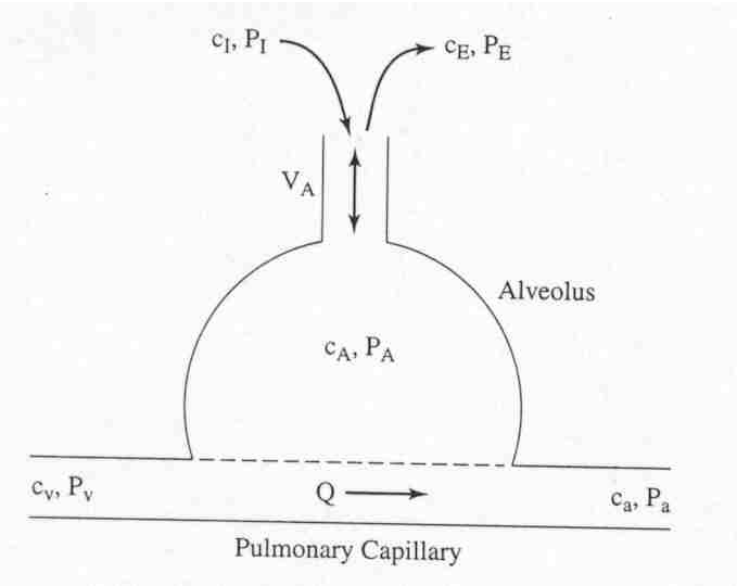
Net Transport of Gas in Alveolus -2



$$V_A(c_I - c_A) = Q(c_a - c_v)$$

- Example: Partial pressure of CO_2 in inspired air is very small (practically zero). Partial pressure of CO_2 in alveolus air is about 40 mmHg.
- The sign of c_I-c_A is positive for O_2 (direction of gas transport is inwards) and negative for CO_2 (outwards transport)

Equilibrium Solution for gas concentrations in alveolar air and arterial blood -1



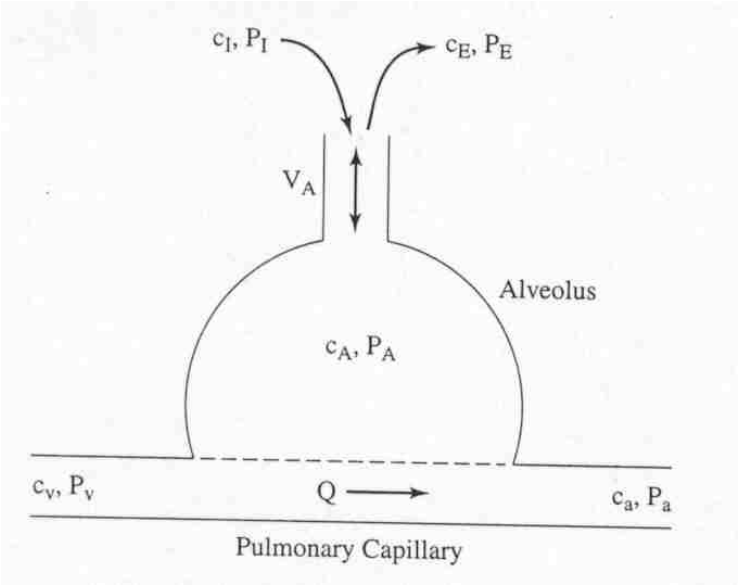
$$\sigma P_a = c_a$$

Combine: $$P_A = KTc_A$$

$$P_a = P_A$$

$$\Rightarrow c_a = \sigma KTc_A$$

Equilibrium Solution for gas concentrations in alveolar air and arterial blood - 2



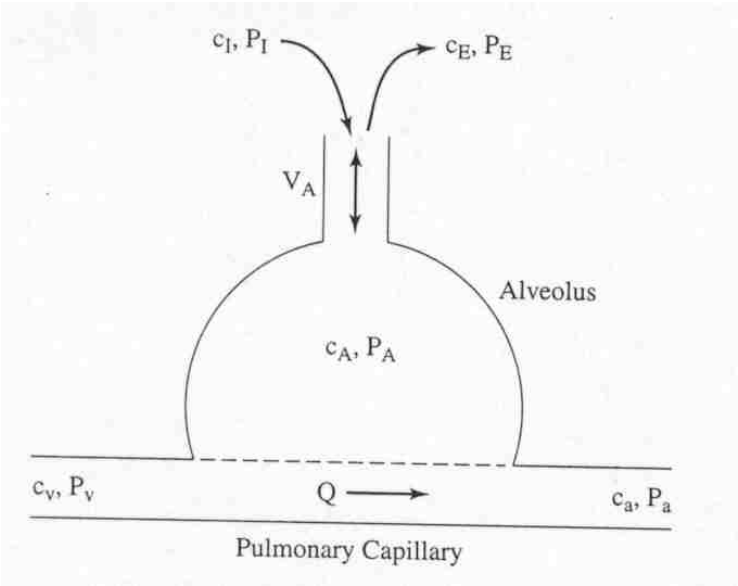
$$V_A(c_I - c_A) = Q(c_a - c_v)$$

$$c_a = \sigma K T c_A$$

Two equations with two unknowns c_a and c_A

Inputs are V_A, Q, c_I and c_v

Equilibrium Solution for gas concentrations in alveolar air and arterial blood - 3



$$V_A(c_I - c_A) = Q(c_a - c_v)$$

$$c_a = \sigma K T c_A$$

Solution for c_a and c_A:

$$\boxed{c_A} = \frac{V_A c_I + Q c_v}{V_A + Q\sigma K T} = \boxed{\frac{r c_I + c_v}{r + \sigma K T}}$$

$$\boxed{c_a = \sigma K T \frac{r c_I + c_v}{r + \sigma K T}} \qquad r = \frac{V_A}{Q}$$

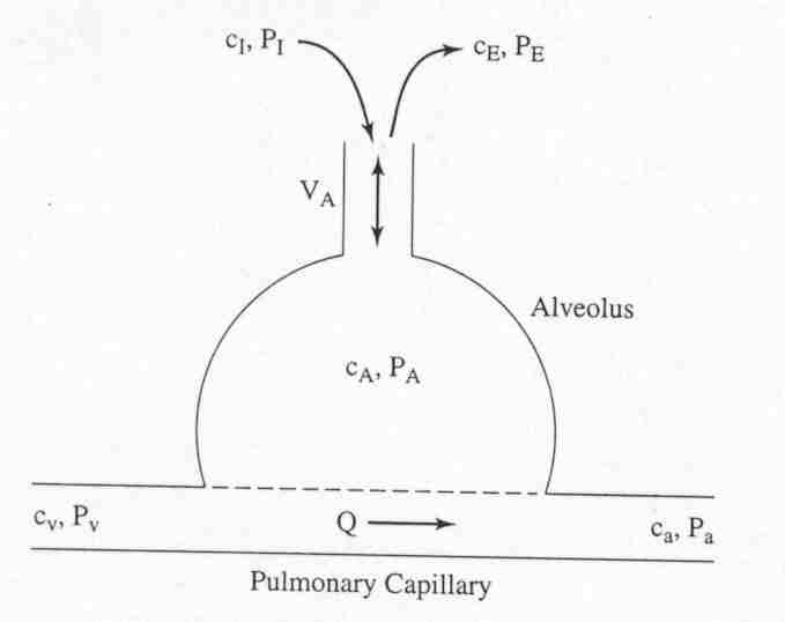
Alveolar gas concentrations depend only on r, the ratio of ventilation to blood flow

Ventilation-Perfusion Ratio r and its effects

$$c_A = \frac{rc_I + c_v}{r + \sigma KT}$$

$$c_a = \sigma KT \frac{rc_I + c_v}{r + \sigma KT}$$

$$r = \frac{V_A}{Q}$$



- If r becomes very large c_A approaches the value of c_I. Then c_a approaches σKTc_I.
- If r→0, c_a approaches c_v, and c_A approaches $c_v / \sigma KT$.
- Typical value of r is around 1.

Alveolar Net Gas Transport in terms of gas concentrations in inspired air and in venous blood

$$c_A = \frac{rc_I + c_v}{r + \sigma KT}$$

$$c_a = \sigma KT \frac{rc_I + c_v}{r + \sigma KT}$$

$$r = \frac{V_A}{Q}$$

$$f = V_A(c_I - c_A) = Q(c_a - c_v)$$

Substitute into either side to find f:

$$f = Qr \frac{\sigma KTc_I - c_v}{r + \sigma KT}$$

Alveolar Net Gas Transport in terms of gas partial pressures in inspired air and in venous blood

$$c_A = \frac{rc_I + c_v}{r + \sigma KT}$$

$$c_a = \sigma KT \frac{rc_I + c_v}{r + \sigma KT}$$

$$r = \frac{V_A}{Q}$$

$$f = V_A(c_I - c_A) = Q(c_a - c_v)$$

Substitute into either side to find f:

$$\boxed{f} = Qr\frac{\sigma KTc_I - c_v}{r + \sigma KT} = \boxed{Qr\sigma\frac{P_I - P_v}{r + \sigma KT}}$$

$$P_I = KTc_I \qquad \sigma P_v = c_v$$

Alveolar Net Gas Transport in terms of gas partial pressures in inspired air and in venous blood - Interpretation

$$f = Qr\sigma \frac{P_I - P_v}{r + \sigma KT} \qquad P_I = KTc_I \qquad \sigma P_v = c_v$$

- If venous blood were allowed to “equilibriate” directly with inspired air, the gas flux into the blood would be equal to $Q\sigma(P_I - P_v)$.
- The actual flux depends on r and is the product of the above with the ratio $r/(r+\sigma KT)$.