BME 5742 Biosystems Modeling and Control

Lecture 45

The Hoppensteadt-Peskin Heart and Blood Circulation Simulator: Systemic Arterial Model and the Heart Valves

Simulator's m. Files

- Simulator is implemented in MATLAB.
- Go to Dr. Charles Peskin's web site (below) and download the files related to Chapter 1.
- http://www.math.nyu.edu/faculty/peskin
- Put all files in your regular Matlab Work directory. The files are tied to each other. When you run one, it may refer to some of the others where certain functions are implemented.

About the Simulator - Cons

- You may opt to create your own Simulink simulator (using the resistors, capacitors etc in the model). By doing so you will surely benefit from available more sophisticated numerical integration methods.
- Hoppensteadt-Peskin's simulator uses a basic Euler Integration (this is done by replacing model derivatives by a "backwards finite difference")
- However you will miss a lot of the fun…

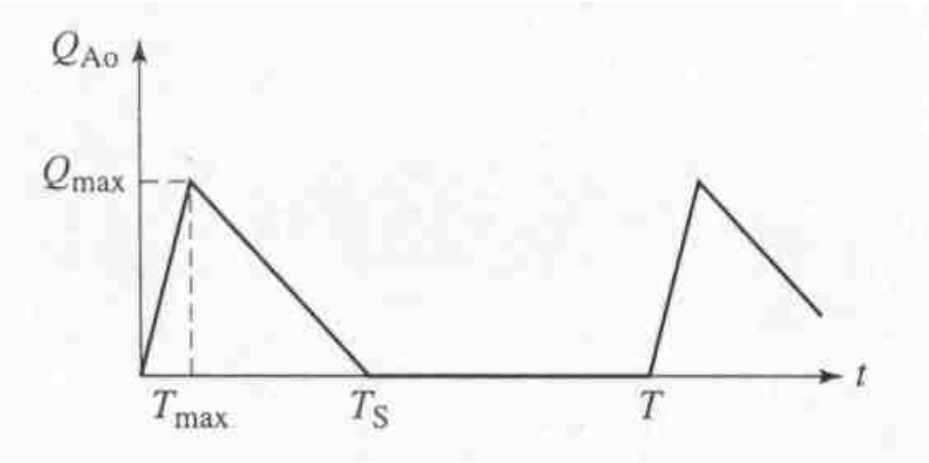
About the Simulator - Pros

- In real life people frequently use models developed by others. It is still very challenging when trying to modify parameters, or even certain expressions.
- Hoppensteadt-Peskin matlab programs are very well annotated!
- Programs are expandable – they allow the user the option of progressively make the model more and more complex.

Getting Around the Files after Downloading

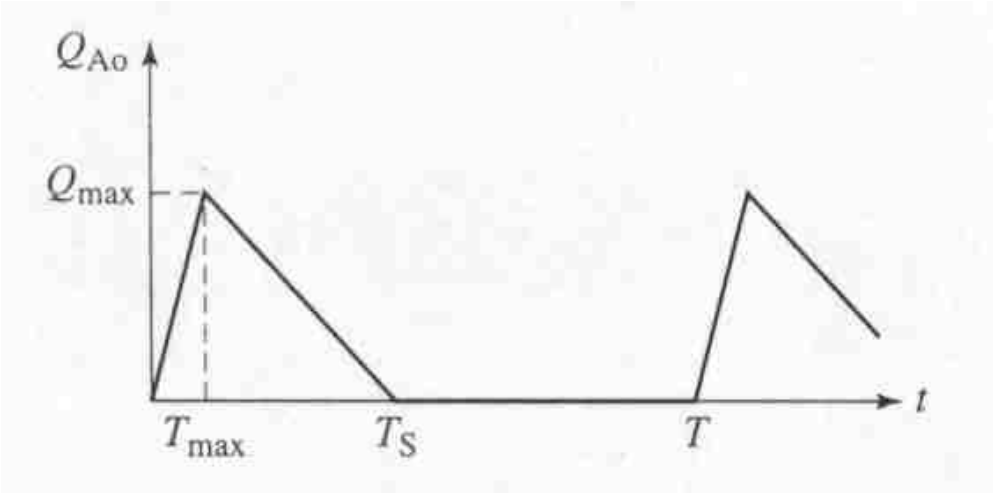
- Place all m files in one folder (your regular folder where all Matlab m files and Simulink models are).
- There are only three main files sa.m, LA_sa.m and cir.m in the simulator.
- All other files are functions ,used in the main files, and initializing files.
- One more file cir_out.m is an output file which plots the results of cir.m.

Let us simulate first only the systemic arterial pulse:



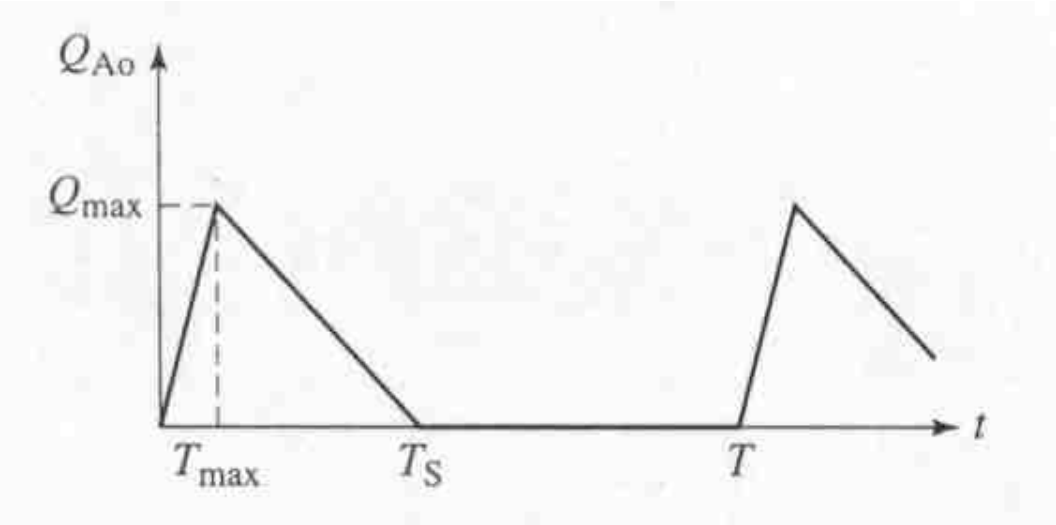
- That part of the simulator considers the cardiac output to be an external input signal.
- A simplified waveform of $Q(t)=Q_{A0}(t)$ is shown above.
- The systemic arteries are represented by C_{sa} and R_s.

Outflow from the left ventricle through the aortic valve into the systemic arterial tree:



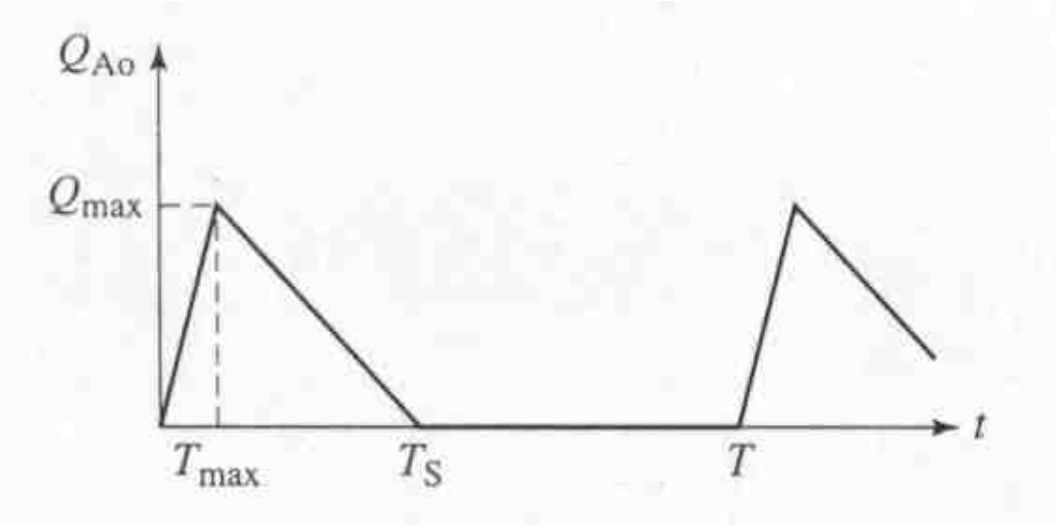
- $Q_{A0}(t)$ is a periodic triangular waveform.
- Q_{max} is the peak flow.
- T_{max} is the time of peak flow.
- T_S is the duration of the systole.
- T is the duration of the heart beat.

Stroke Volume and Cardiac Output



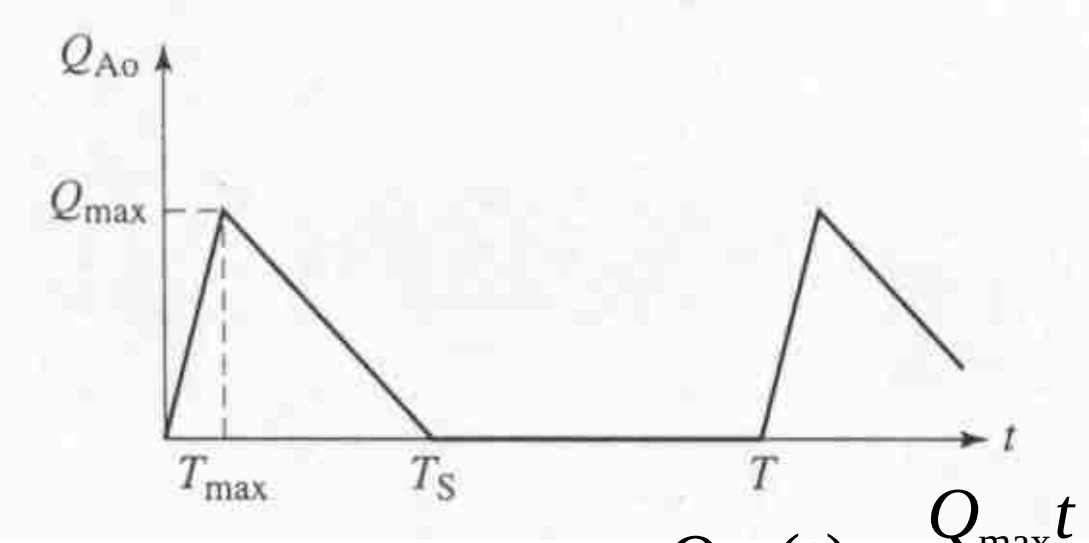
- Stroke Volume is the integral of $Q_{A0}(t)$ over one period T.
- Resulting Stroke Volume is $Q_{max}T_S/2$.
- Cardiac Output = Stroke Volume x (1/T) = $Q_{max}T_S/(2T)$.

Typical Parameter Values:



- T=0.0125 minutes.
- T_S=0.005 minutes.
- T_{max}=0.002 minutes.
- A choice of Q_{max}=28 liters/min gives a stroke volume of 0.07 liters and cardiac output = 5.6 liters/min

Algebraic Expression of $Q_{A0}(t)$:



$$Q_{A0}(t) = \frac{Q_{\max} t}{T_{\max}} \qquad 0 \le t \le T_{\max}$$

$$Q_{A0}(t) = \frac{Q_{\max}(T_S - t)}{T_S - T_{\max}} \qquad T_{\max} \le t \le T_S$$

$$Q_{A0}(t) = 0 \qquad T_S \le t \le T$$

Systemic Arterial pulse Dynamic Model

- Starting point is the volume change equation: $dV_{sa}(t)/dt=Q_{A0}(t)-Q_s(t)$.
- By the fixed arterial system compliance definition: $V_{sa}(t)=(V_{sa})_d+C_{sa}P_{sa}(t)$.
- The systemic resistance equation, if we neglect the impact of $P_{sv}(t)$: $Q_s(t)\approx P_{sa}(t)/R_s$
- It all becomes a single equation: $C_{sa}dP_{sa}(t)/dt=Q_{A0}(t)-P_{sa}(t)/R_s$. Note that it doesn't matter what is $(V_{sa})_d$

Simulator's Numerical Integration Explained:

- The derivative of $P_{sa}(t)$ is approximated by a backwards finite difference:
- $C_{sa}[(P_{sa}(t)-P_{sa}(t-\Delta t))/\Delta t]\approx Q_{A0}(t)-P_{sa}(t)/R_s$
- This leads to a simple recursion equation for computation of current variable value based on its previous value and the current input:

$$\boxed{P_{sa}(t)=[P_{sa}(t-\Delta t)+\Delta t Q_{A0}(t)/C_{sa}]/[1+\Delta t/(R_s C_{sa})]}$$

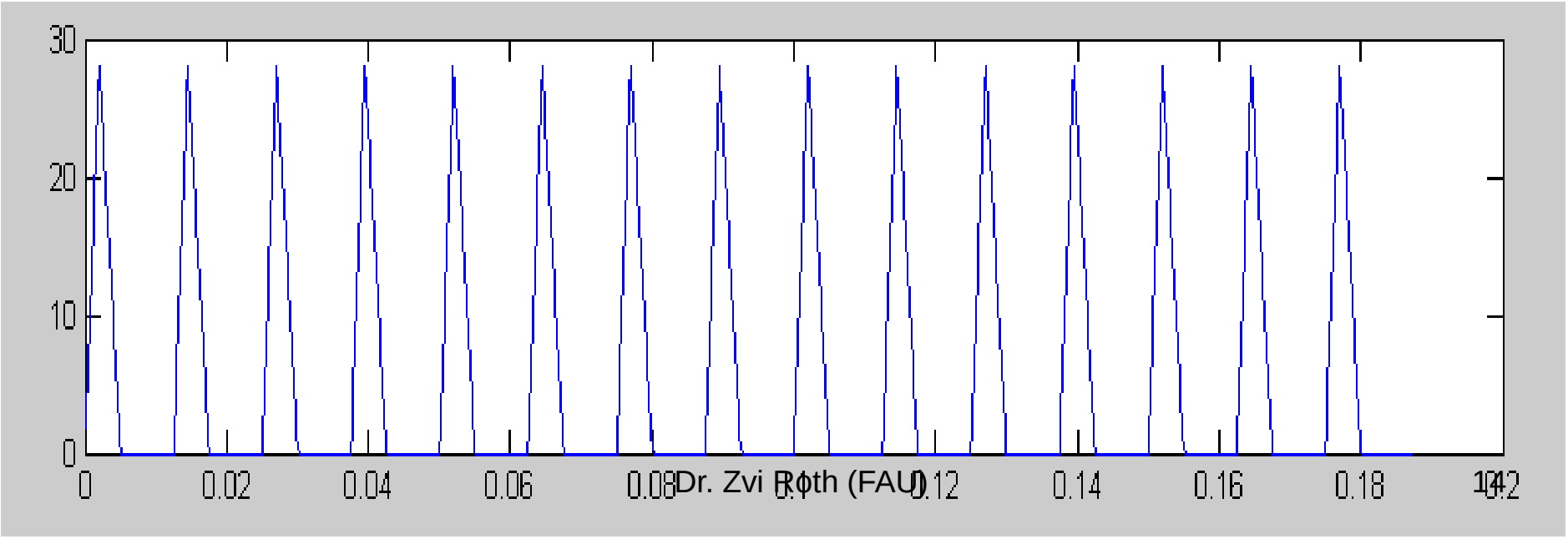
Simulator - Additional Numerical Issues

$P_{sa}(t)=[P_{sa}(t-\Delta t)+\Delta t Q_{A0}(t)/C_{sa}]/[1+\Delta t/(R_s C_{sa})]$

- The fixed step size Δt must be "small enough". Start with a reasonable value of $\Delta t=0.01T$. Check later if $\Delta t=0.005T$ produces the same or different results, etc.
- $P_{sa}(0)$ may be arbitrary. System eventually converges to the correct steady-state flow conditions (and then simply dismiss the transient due to the arbitrary initial condition).

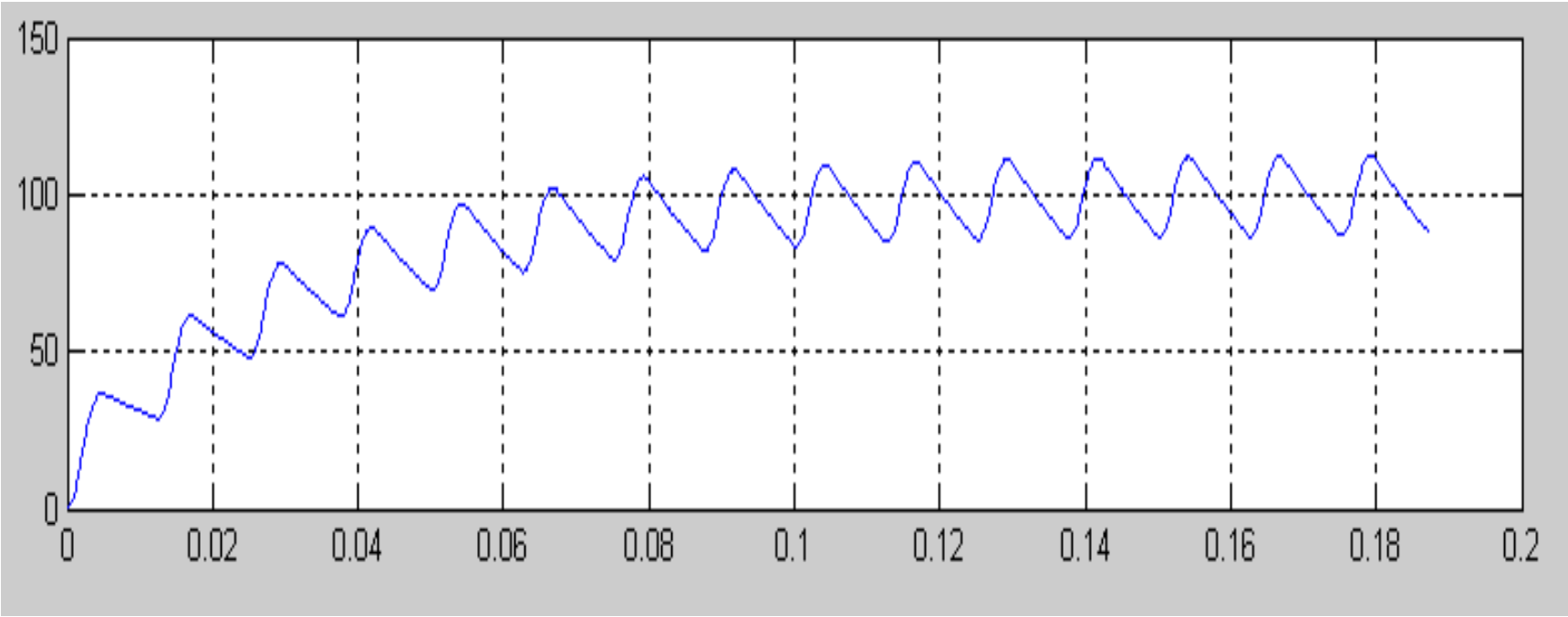
sa.m

- Model represents the systemic artery
- The other files used for this program are
  - in_sa.m – used for initialization
  - QAo_now – creates the aortic flow input
  - Psa_new.m – updates value according to the numerical recurssion discussed earlier.



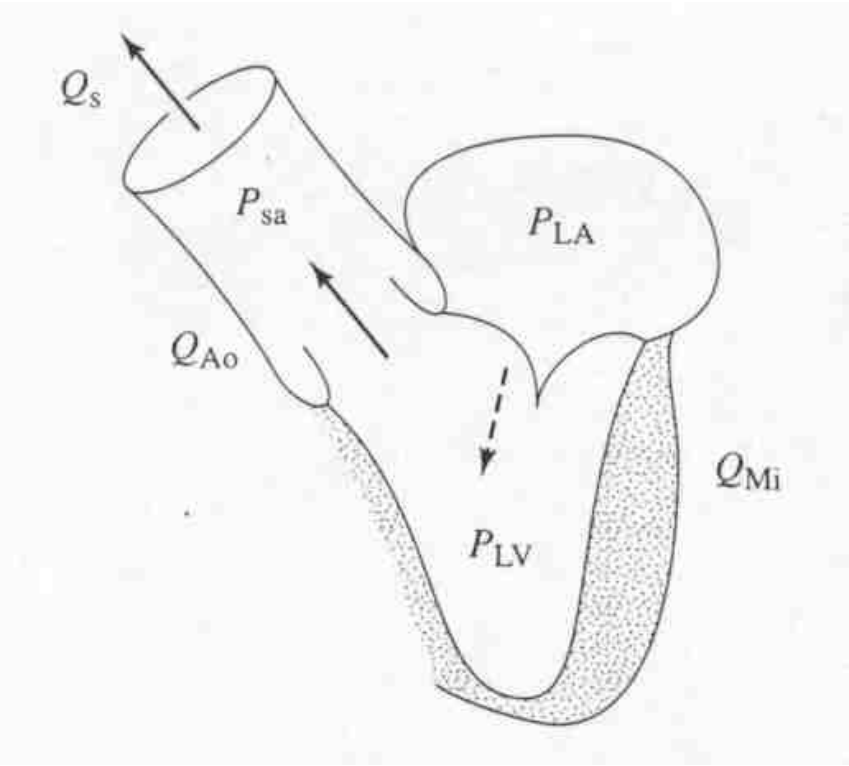
Systemic Arterial Pulse Simulation

- This part of the simulator uses four m files: sa.m is the main script. Also there are two functions and one initialization file. All files must be in the same directory.
- Launch sa.m, either from the command line writing >>sa, or from within the editor/debugger of sa.m.



- For arbitrary initialization the pressure comes to a steady state after some transient interval of time – the initial results are wrong in general and should be discarded.
- User can make changes to the initial pressure and other parameters, in the in_sa.m file.

Simulator's Phase II: Replacing the function $Q_{A0}(t)$ with a model of the left heart



- For the left ventricle we shall assume a periodic compliance:
- Input is now $C_{LV}(t)$.
- The Phase I input $Q_{A0}(t)$ will now be one of the outputs.
- Phase II must include models for the mitral and aortic valves.

Healthy vs. Diseased Valve Models

- A healthy valve is like a diode: It has very low resistance to forward flow and a very large resistance to back flow.
- Stenosis: Increased resistance to forward flow (due to narrowing of valve opening).
- Regurgitation: Leaky backflow, due to imperfect closure of the valve.

Left Heart and System model including Valve Models - Volumes

$$\frac{dV_{LV}(t)}{dt} = Q_{Mi}(t) - Q_{Ao}(t)$$

$$\frac{dV_{sa}(t)}{dt} = Q_{Ao}(t) - Q_{s}(t)$$

$$V_{LV}(t) = (V_{LV})_d + C_{LV}(t)P_{LV}(t)$$

$$V_{sa}(t) = (V_{sa})_d + C_{sa}P_{sa}(t)$$

- Q_{Mi},Q_{Ao} are the flows through the valves.
- “LV” index stands for “Left Ventricle”.

Left Heart and System model including Valve Models - Valves

- S_{Mi},S_{ao} are binary indicator functions.
- "LA" index stands for Left Atrium.
- R_{Mi},R_{Ao} are the <u>forward</u> resistances of the valves. Backward resistances don't matter due to the overriding action of the indicator functions.

$$Q_{Mi}(t) = S_{Mi}(t)\frac{P_{LA}(t) - P_{LV}(t)}{R_{Mi}}$$

$$Q_{Ao}(t) = S_{Ao}(t)\frac{P_{LV}(t) - P_{sa}(t)}{R_{Ao}}$$

$$S_{Mi}(t) = 1_if_P_{LA}(t) > P_{LV}(t)$$

$$S_{Mi}(t) = 0_if_P_{LA}(t) < P_{LV}(t)$$

$$S_{Ao}(t) = 1_if_P_{LV}(t) > P_{sa}(t)$$

$$S_{Ao}(t) = 0_if_P_{LV}(t) < P_{sa}(t)$$

Valve Model typical values

- $P_{LA}(t)$ is assumed constant, equal to 5 mmHg.
- Healthy valve resistances are very low (around 0.01 mmHg/(liters/min)).
- Valve stenosis is easy to model, simply increasing the forward resistance value.

$$Q_{Mi}(t) = S_{Mi}(t)\frac{P_{LA}(t) - P_{LV}(t)}{R_{Mi}}$$

$$Q_{Ao}(t) = S_{Ao}(t)\frac{P_{LV}(t) - P_{sa}(t)}{R_{Ao}}$$

$$S_{Mi}(t) = 1_if_P_{LA}(t) > P_{LV}(t)$$

$$S_{Mi}(t) = 0_if_P_{LA}(t) < P_{LV}(t)$$

$$S_{Ao}(t) = 1_if_P_{LV}(t) > P_{sa}(t)$$

$$S_{Ao}(t) = 0_if_P_{LV}(t) < P_{sa}(t)$$

Valve Regurgitation Modeling

$$Q_{Mi}(t) = \left(\frac{S_{Mi}(t)}{R_{MiF}} + \frac{1 - S_{Mi}(t)}{R_{MiB}}\right)(P_{LA}(t) - P_{LV}(t))$$

$$Q_{Ao}(t) = \left(\frac{S_{A0}(t)}{R_{AoF}} + \frac{1 - S_{Ao}(t)}{R_{AoB}}\right)(P_{LV}(t) - P_{sa}(t))$$

$$S_{Mi}(t) = 1 _ if _ P_{LA}(t) > P_{LV}(t)$$

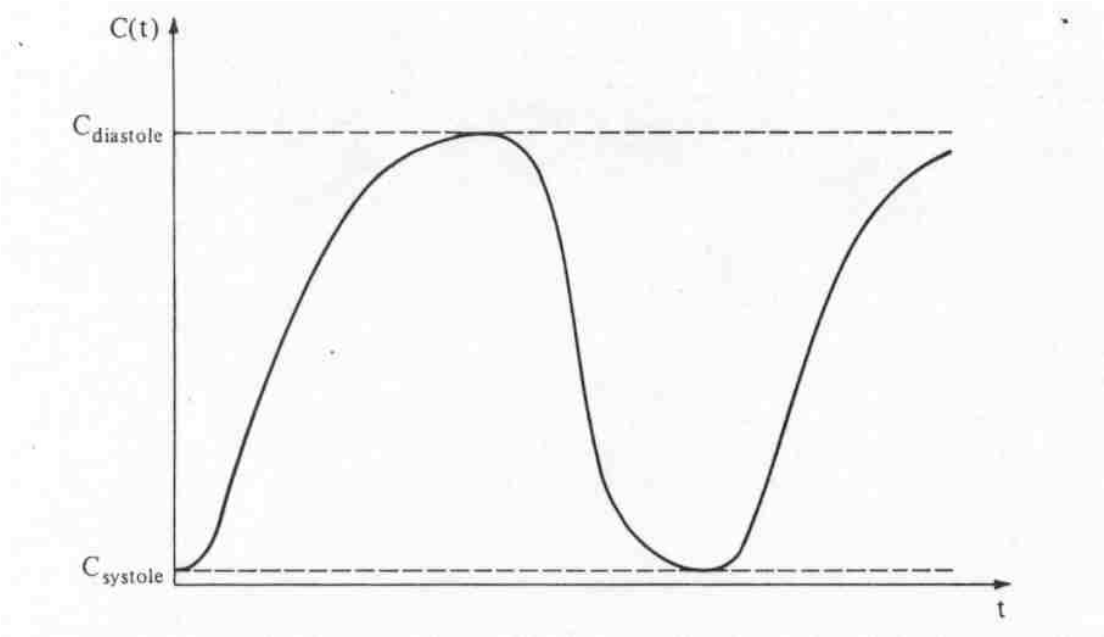
$$S_{Mi}(t) = 0 _ if _ P_{LA}(t) < P_{LV}(t)$$

$$S_{Ao}(t) = 1 _ if _ P_{LV}(t) > P_{sa}(t)$$

$$S_{Ao}(t) = 0 _ if _ P_{LV}(t) < P_{sa}(t)$$

- R_{MiF},R_{AoF} are the small forward resistances of the valves.
- R_{MiB},R_{AoB} are the large backward resistances of the valves. (These are always larger than the forward resistances).

Left heart, Valves and Systemic Model – What is the Input?



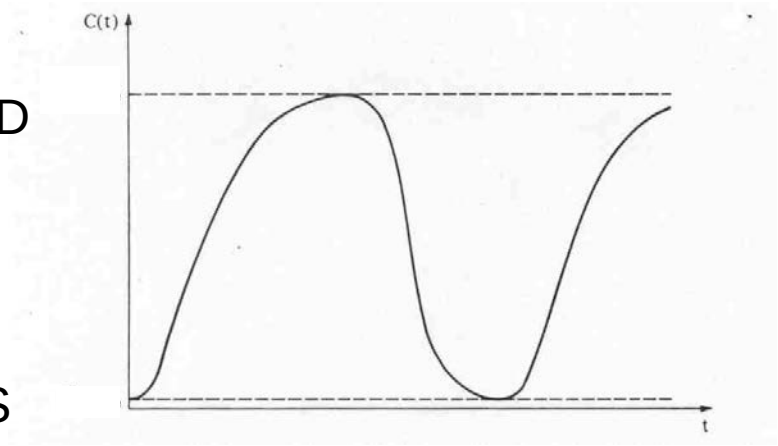
- The left ventricle's compliance $C_{LV}(t)$ is a periodic time function: $C_{LV}(t)=C_{LV}(t+T)$.
- Here we shall define t=0 as the end of the diastole (at point of maximum).

Left heart, Valves and Systemic Model – Analytic Function for $C_{LV}(t)$

CV_now.m

C_{LVD}

C_{LVS}



$$C_{LV}(t) = C_{LVD}\left(\frac{C_{LVS}}{C_{LVD}}\right)^{\frac{1-e^{-\frac{t}{\tau_S}}}{1-e^{-\frac{T_S}{\tau_S}}}} \quad _____ \quad 0 \le t \le T_S$$

$$C_{LV}(t) = C_{LVS}\left(\frac{C_{LVD}}{C_{LVS}}\right)^{\frac{1-e^{-\frac{t-T_S}{\tau_D}}}{1-e^{-\frac{T-T_S}{\tau_D}}}} \quad _____ \quad T_S \le t \le T$$

Analytic Function for $C_{LV}(t)$ (zoomed in)

$$C_{LV}(t) = C_{LVD}\left(\frac{C_{LVS}}{C_{LVD}}\right)^{\frac{1-e^{-\frac{t}{\tau_S}}}{1-e^{-\frac{T_S}{\tau_S}}}} \quad \text{_____} \quad 0 \le t \le T_S$$

$$C_{LV}(t) = C_{LVS}\left(\frac{C_{LVD}}{C_{LVS}}\right)^{\frac{1-e^{-\frac{t-T_S}{\tau_D}}}{1-e^{-\frac{T-T_S}{\tau_D}}}} \quad \text{_____} \quad T_S \le t \le T$$

Function of $C_{LV}(t)$ (typical values)

$$C_{LV}(t) = C_{LVD}\left(\frac{C_{LVS}}{C_{LVD}}\right)^{\frac{1-e^{-\frac{t}{\tau_S}}}{1-e^{-\frac{T_S}{\tau_S}}}} \quad _____ \quad 0 \le t \le T_S$$

$$C_{LV}(t) = C_{LVS}\left(\frac{C_{LVD}}{C_{LVS}}\right)^{\frac{1-e^{-\frac{t-T_S}{\tau_D}}}{1-e^{-\frac{T-T_S}{\tau_D}}}} \quad _____ \quad T_S \le t \le T$$

$$C_{LVD} \approx 0.0146 _[liters/mmHg]$$

$$C_{LVS} \approx 0.00003 _[liters/mmHg]$$

$$\tau_S \approx 0.0025 _[\min]$$

$$\tau_D \approx 0.0075 _[\min]$$

Dynamic Model for the Heart-Valves-System

$$\frac{d(C_{LV}(t)P_{LV}(t))}{dt} = Q_{Mi}(t) - Q_{Ao}(t) = S_{Mi}(t)\frac{P_{LA}(t) - P_{LV}(t)}{R_{Mi}} - S_{Ao}(t)\frac{P_{LV}(t) - P_{sa}(t)}{R_{Ao}}$$

$$\frac{d(C_{sa}P_{sa}(t))}{dt} = Q_{Ao}(t) - \frac{P_{sa}(t)}{R_s}$$

$$S_{Mi}(t) = 1_if_P_{LA}(t) > P_{LV}(t)$$

$$S_{Mi}(t) = 0_if_P_{LA}(t) < P_{LV}(t)$$

$$S_{Ao}(t) = 1_if_P_{LV}(t) > P_{sa}(t)$$

$$S_{Ao}(t) = 0_if_P_{LV}(t) < P_{sa}(t)$$

Again, a backwards finite difference approximates the derivative. File set_SMi_Sao.m determines the states of the valves.