BME 5742
Bio-Systems Modeling and Control

Lecture 38
Gas Exchange in the Lungs
Full Lung Model

Common features to all alveoli

- The inflowing venous blood is just partitioned among all alveoli – c_v is the same throughout the lungs.
- Inspired air composition (c_I) is constant throughout the lungs.
- Solubility of a gas (σ) is constant in all alveoli.
- <u>Assumption:</u> Temperature (T) is constant throughout the lungs.

Different features of different alveoli

- The perfusion (blood flow) Q is non-uniform: Due to gravity, lower parts of the lung have larger blood flow than the upper parts. It is attributed to higher hydrostatic pressure on the upper pulmonary vessels.
- Ventilation V_A is larger in the lower parts of the lung: Maximum alveolus volume (during inspiration) is more or less constant throughout. Minimum alveolus volume is smaller in the lower parts, due to weight effects of neighboring lung parts. V_A depends on the difference between maximum and minimum volumes of the alveolus.

Variations in the ventilation-perfusion ratio

- As V_A and Q vary from one alveolus to another, the ratio $r = V_A/Q$ is fluctuating about some mean value.
- Some pulmonary diseases may result in large variation of r, as air supply or blood supply to the lungs become compromised.
- As r varies, the variables c_A (air concentration), c_a (arterial blood concentration) and f (net transport) all vary among all alveoli.

Individual alveolus variables ($i=1,\ldots,3\cdot10^8$) :

Variables or coefficients that are not indexed by i are assumed constant in all alveoli.

$$(c_A)_i = \frac{r_i c_I + c_v}{r_i + \sigma KT}$$

$$(c_a)_i = \sigma KT \frac{r_i c_I + c_v}{r_i + \sigma KT}$$

$$r_i = \frac{(V_A)_i}{Q_i}$$

$$f_i = Q_i r_i \sigma \frac{P_I - P_v}{r_i + \sigma KT}$$

Overall net transport of gas in the lungs

$$(c_A)_i = \frac{r_i c_I + c_v}{r_i + \sigma KT}$$

$$(c_a)_i = \sigma KT \frac{r_i c_I + c_v}{r_i + \sigma KT}$$

$$r_i = \frac{(V_A)_i}{Q_i}$$

$$\boxed{f} = \sum_i f_i = \sum_i Q_i r_i \sigma \frac{P_I - P_v}{r_i + \sigma KT} \boxed{= \sigma (P_I - P_v) \sum_i \frac{Q_i r_i}{r_i + \sigma KT}}$$

Efficiency of Overall Net Transport of Gas in the Lungs

$$f = \sigma(P_I - P_v)\sum_i \frac{Q_i r_i}{r_i + \sigma KT} = \sigma(P_I - P_v)Q_0 E$$

$$E = \frac{1}{Q_0}\sum_i \frac{Q_i r_i}{r_i + \sigma KT} \qquad Q_0 = \sum_i Q_i$$

Total pulmonary blood flow

- The entity $\sigma(P_I - P_v)Q_0$ is the transport that would occur if venous blood were brought directly into contact with outside air.
- $0<E<1$ is an <u>efficiency</u> measure of gas transport. How large can it be?

Maximum Efficiency of Overall Net Transport of Gas in the Lungs

$$f = \sigma(P_I - P_v)\sum_i \frac{Q_i r_i}{r_i + \sigma KT} = \sigma(P_I - P_v)Q_0 E$$

$$E = \frac{1}{Q_0}\sum_i \frac{Q_i r_i}{r_i + \sigma KT} \qquad Q_0 = \sum_i Q_i$$

$$\boxed{E_{\max}} = \frac{1}{Q_0}\sum_i \frac{Q_i r_0}{r_0 + \sigma KT} = \boxed{\frac{r_0}{r_0 + \sigma KT}} \qquad r_0 = \frac{(V_A)_0}{Q_0}$$

- The total alveolar ventilation $(V_A)_0$ is the sum of all alveoli ventilations.
- <u>E is maximum if all alveoli have the same ventilation-perfusion ratio (r_0).</u> See Hoppensteadt-Peskin's book for a proof.

Actual Efficiency E

- There is no known physiological mechanism that helps to create a uniform ventilation-perfusion ratio throughout the lungs.
- Actual efficiency E is often way below the maximum value.
- The more random (and uncorrelated) the r_i ratios are, the smaller is the value of E.

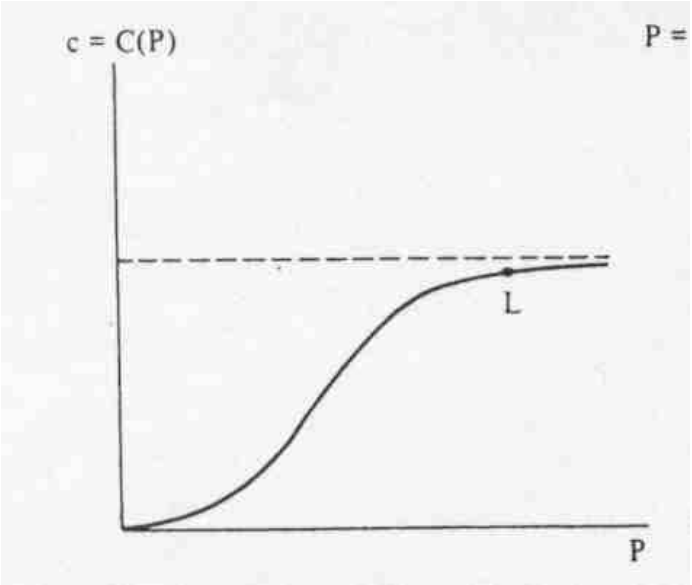
Mixed Alveolar Air and Arterial Blood

- In each individual alveolus there is an equilibrium between the expired air and the arterial blood.
- However, there is no such equilibrium between the total expired air and the total arterial blood.
- See Hoppensteadt-Peskin's book (section 2.5) for the mathematical details. The lack of uniformity in r_i prevents an overall equilibrium from being formed.

So far, the analysis held true for any gas going through the lungs system.

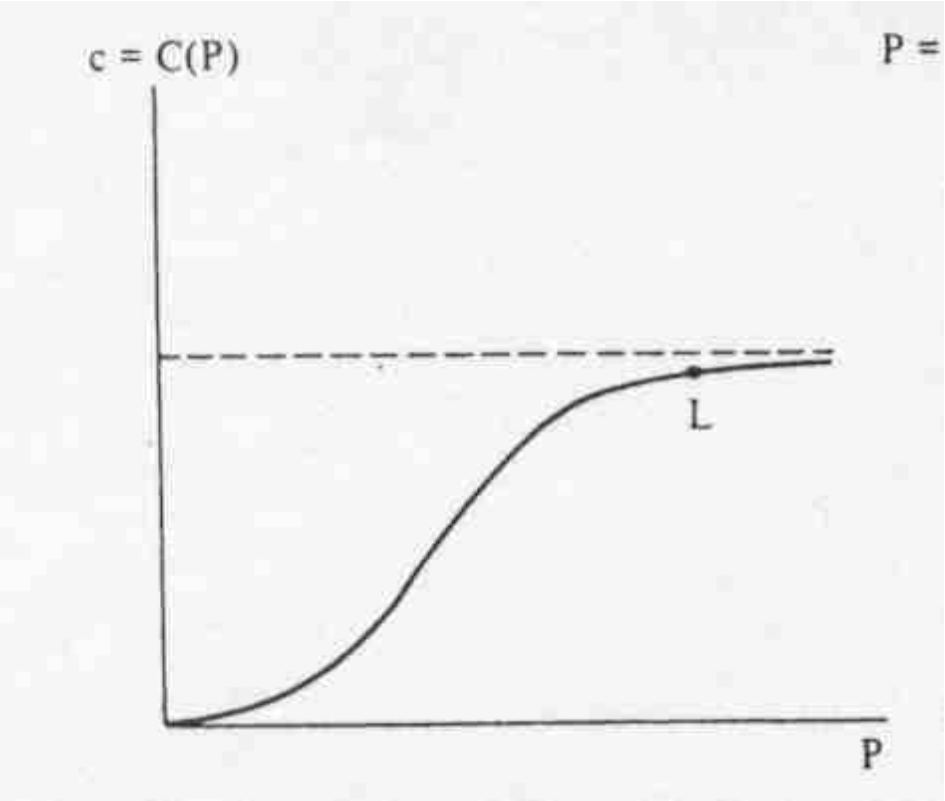
From now on, let us specialize the discussion to transport of O_2

Recall: O_2 blood partial pressure and air concentration are related via the O_2-Hb dissociation curve c=C(P)



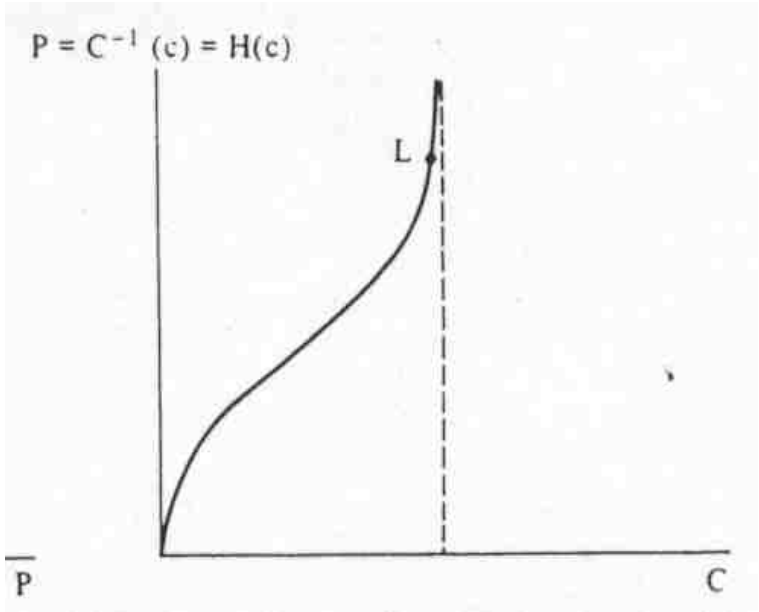
Oxygen concentration in the blood c relates to oxygen partial pressure in the lungs via the Hill equation (for enzymatic reaction with cooperation), shown above, arguing that much of the transport of O_2 is done by hemoglobin.

O_2-Hb dissociation curve c=C(P)



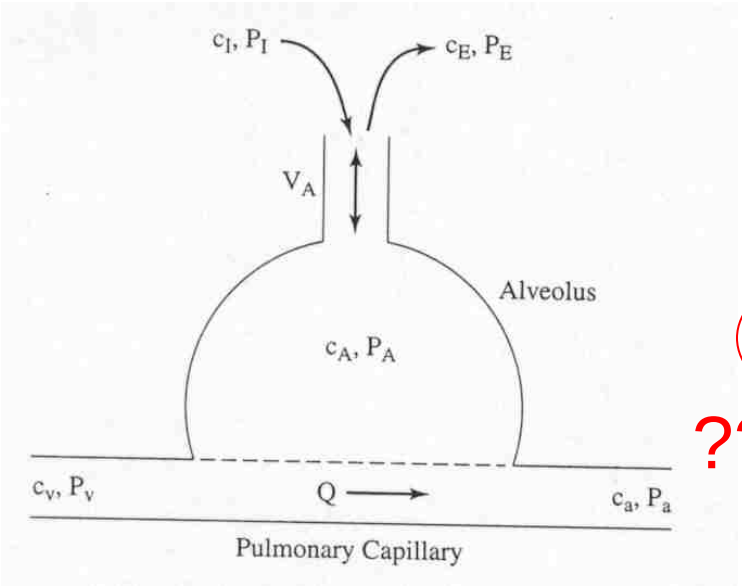
- Letter “L” stands for “lungs”. At the lungs Hb molecules are close to 97% saturated, and (c,P) is in the neighborhood of Point L.

The Inverse Dissociation Curve $P=C^{-1}(c)=H(c)$



- It is sometimes useful to use this inverse curve. For instance, if we want to represent P in terms of c.
- H() is a monotone increasing function with positive second derivative near Point L.

Recall: Equilibrium Solution for gas concentrations in alveolar air and arterial blood - Is it valid for O_2?



$$V_A(c_I - c_A) = Q(c_a - c_v)$$

$$c_a = \sigma KTc_A = \sigma KTP_A = \sigma KTP_a$$

???

Solution for c_a and c_A:

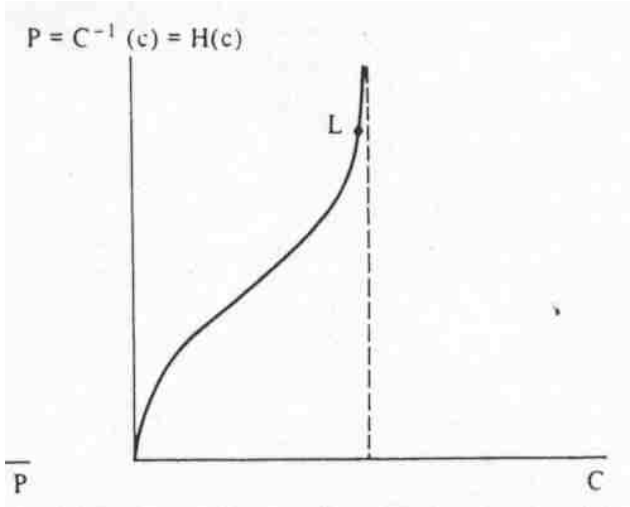
$$c_A = \frac{V_A c_I + Qc_v}{V_A + Q\sigma KT} = \frac{rc_I + c_v}{r + \sigma KT}$$

$$c_a = \sigma KT \frac{rc_I + c_v}{r + \sigma KT}$$

$$r = \frac{V_A}{Q}$$

Alveolar gas concentrations depend only on r, the ratio of ventilation to blood flow

Equilibrium of O_2 concentrations in alveolar air and arterial blood

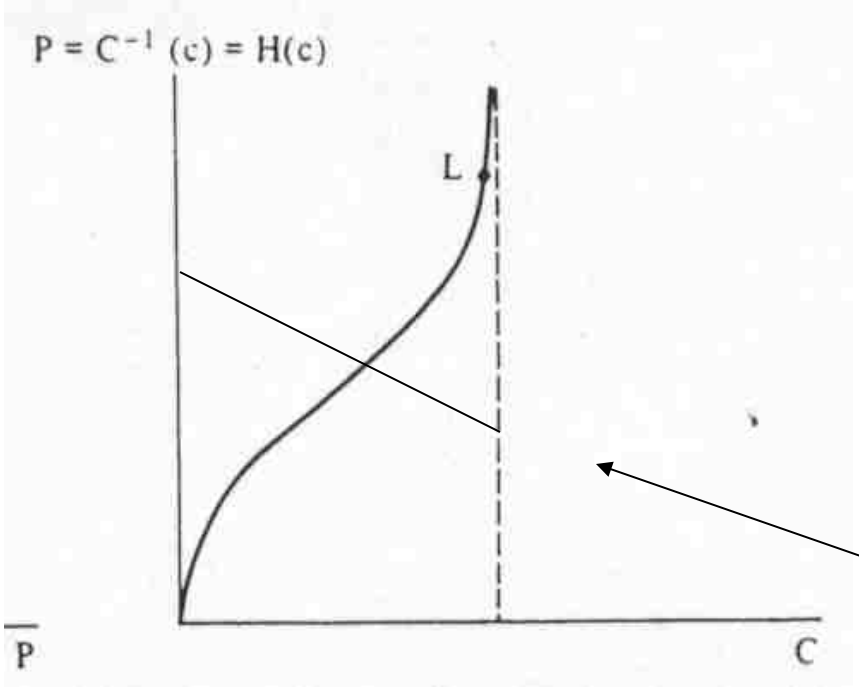


$$V_A(c_I - c_A) = Q(c_a - c_v)$$

$$P_a = P_A = KTc_A = H(c_a)$$

- Assuming that Q, V_A, c_I and c_v are all given, the two unknowns are c_A and c_a, now related nonlinearly.
- We can substitute for c_A in the upper equation, and obtain a single nonlinear equation for c_a:

Nonlinear Equation for c_a: O_2 concentrations in alveolar arterial blood



$$r(c_I - c_A) = c_a - c_v$$

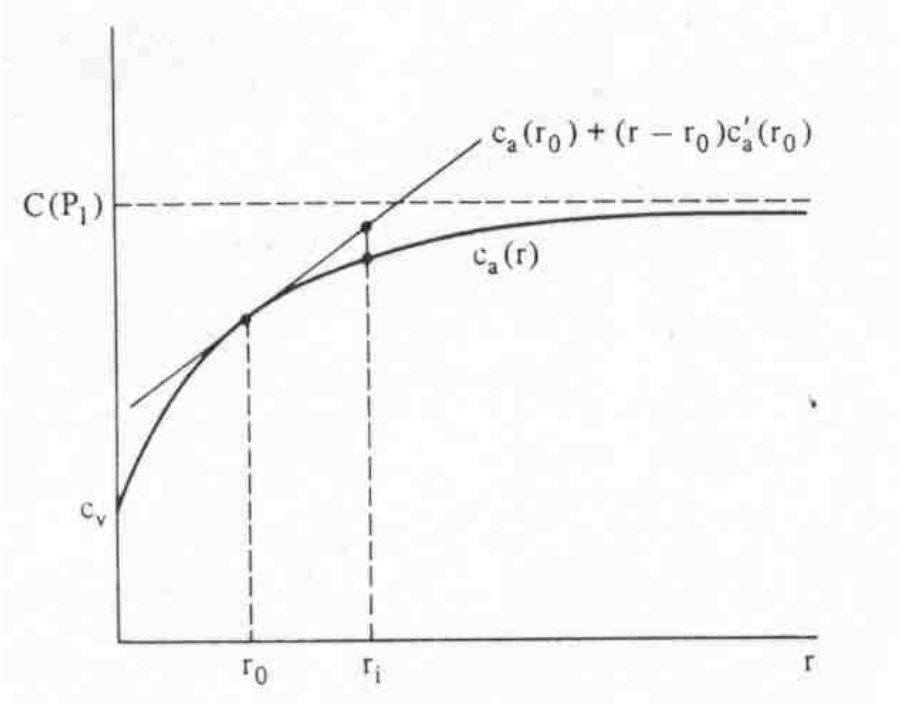
$$KTc_A = H(c_a)$$

$$\Rightarrow \boxed{r\left(c_I - \frac{H(c_a)}{kT}\right) = c_a - c_v}$$

$$\Leftrightarrow H(c_a) = KTc_I - \frac{KT}{r}(c_a - c_v)$$

- As can be seen from the graphic solution, solution for c_a always exists, and it is unique.
- If r is fixed, then as c_v increases, so does c_a

The solution c_a as function of r



- At the extreme ($r \rightarrow \infty$) $c_a \rightarrow C(P_l)$. This is the concentration of oxygen in blood that "equilibriates" directly with room air.
- As r increases, c_a increases with a smaller and smaller rate of increase.

Overall Net Transport of O_2 in Lungs

$$f = \sum_i Q_i (c_a(r_i) - c_v)$$

- We refer, as you recall, to f as the <u>flux of O_2</u>
- It can be proved (See Hoppensteadt-Peskin's book section 2.6) that the maximum value of f occurs if every r_i equals $r_0=(V_A)_0/Q_0$

Maximum O_2 Flux

$$f = \sum_i Q_i (c_a(r_i) - c_v) \leq f_0 = Q_0 (c_a(r_0) - c_v)$$

- If both c_l and c_v are given, the maximum O_2 flux is determined by the lung (r_i values in the lung).
- If f and c_l are given, then the above f restriction is actually a restriction on c_v. Venous O_2 concentration is maximized if r is uniform in the lungs.