Power-Efficient Resource Allocation for Time-Division Multiple Access Over Fading Channels

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***Abstract*—We investigate resource allocation policies for time-division multiple access (TDMA) over fading channels in the power-limited regime. For frequency-flat block-fading channels and transmitters having full channel state information (CSI), we first minimize power under a weighted sum average rate constraint and show that the optimal rate and time allocation policies can be obtained by a greedy water-filling approach with linear complexity in the number of users. Subsequently, we pursue power minimization under individual average rate constraints and establish that the optimal resource allocation also amounts to a greedy water-filling solution. Our approaches not only provide fundamental power limits when each user can support an infinite-size capacity-achieving codebook (continuous rates), but also yield guidelines for practical designs where users can only support a finite set of adaptive modulation and coding modes (discrete rates).**

***Index Terms*—Convex optimization, fading channel, time-division multiple access (TDMA), water-filling.**

I. Introduction

WITH battery-operated communicating nodes, power efficiency has emerged as a critical issue in both commercial and tactical radios designed to extend battery lifetime, especially for wireless networks of sensors equipped with nonrechargeable batteries. Capitalizing on the fact that transmit power is an increasing and strictly convex function of the transmission rate [1], power-efficient resource allocation has been pursued in [2]–[8]. Among them, [2]–[5] dealt with designs over additive white Gaussian noise (AWGN) channels whereas [6] and [7] considered power-efficient scheduling for time-division multi-access (TDMA) networks over fading channels; see also [8] where transmit power is minimized for orthogonal frequency-division multiplexing (OFDM) systems using quantized channel state information (CSI).

Resource allocation for fading channels also remains an active topic in information-theoretic studies, where most existing works aim at maximizing rate (achieve capacity) subject to average power constraints. Assuming that full CSI is available at both transmit and receive ends, the ergodic capacity region, the delay-limited capacity region, and optimal power allocation have been reported in [9] and [10] for fading multiple-access channels, and in [11], [12] for fading broadcast channels; see also [13] and [14] for characterization of the outage capacity regions for single-user and multiple-access fading channels, respectively.

In this paper, we reconsider these information-theoretic results pertaining to *rate* efficiency, and mainly investigate optimal resource allocation for fading channels from a *power* efficiency perspective. Specifically, we seek to minimize power cost under average rate constraints for fading TDMA systems, where successive decoding is not feasible due to practical restrictions. Although our framework is tailored for TDMA, it carries over to *any orthogonal* channelization including (orthogonal) frequency-division multiple access ((O)FDMA).

After modeling preliminaries described in Section II, we first study minimization of the total weighted power under an average sum–rate constraint in Section III. This problem is dual to [11], where rate was maximized under a sum–power constraint. Interestingly, we will see that the optimal resource allocation follows a novel greedy water-filling approach which can be implemented using a low-complexity algorithm. Besides continuous rates, an analogous algorithm is devised to minimize power when each user can only support adaptive modulation-coding (AMC) based discrete-rate transmissions. The second problem we consider is power minimization under individual rate constraints for multiple access (Section IV). Specifically, we formulate and solve power minimization under individual rate constraints when TDMA users rely on either continuous-, or, AMC-based discrete-rate transmissions. The related approaches can be also generalized to yield optimal capacity-achieving (rate-maximizing) resource allocation for orthogonal multiple-access fading channels as an important complement to the results for general multiple-access channels in

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[9]. The generalizations to the capacity-achieving resource allocation for time-division fading broadcast and multiple-access channels are outlined in Section V. Section VI provides numerical results, and Section VII concludes this paper.

II. Modeling Preliminaries

Consider K users in uplink TDMA communicating with an access point (AP) over wireless fading channels adhering to the following operating conditions:

(oc-1) The flat-fading channel coefficients

$$\left\{\sqrt{h_k}e^{j\theta_k}, h_k \geq 0\right\}_{k=1}^{K}$$

remain invariant during a block (assumed without loss of generality (w.l.o.g.) to have unit duration), but are allowed to vary from block-to-block (block-fading model). With ^{T} denoting transposition, the resultant $K \times 1$ vector of random channel gains $\boldsymbol{h} := [h_1, \ldots, h_K]^T$ is ergodic with continuous joint cumulative distribution function (cdf) $F(\boldsymbol{h})$ assumed known.

(oc-2) With full CSI, each user terminal $k = 1, \ldots, K$ transmits in a separate user-specific time slot with its transmission rate and power adapted to h_k per block.

Each user k can be scheduled by the AP to transmit per block over nonoverlapping fractions $\tau_k(\boldsymbol{h}) \geq 0$, $k = 1, \ldots, K$, with durations dependent on the fading state $\boldsymbol{h}$ and satisfying $\sum_{k=1}^{K} \tau_k(\boldsymbol{h}) \in [0, 1]\ \forall\, \boldsymbol{h}$. If a scheduled user k transmits over its fraction $\tau_k(\boldsymbol{h}) > 0$ with rate $\rho_k(\boldsymbol{h})$ and power $\pi_k(\boldsymbol{h})$, then clearly its overall transmission rate and power per block are $r_k(\boldsymbol{h}) = \tau_k(\boldsymbol{h})\rho_k(\boldsymbol{h})$ and $p_k(\boldsymbol{h}) = \tau_k(\boldsymbol{h})\pi_k(\boldsymbol{h})$, respectively. With full CSI available at the transmit end, the optimal resource allocation policies adapt $\boldsymbol{\tau}(\boldsymbol{h}) := [\tau_1(\boldsymbol{h}), \ldots, \tau_K(\boldsymbol{h})]^T$ and $\boldsymbol{r}(\boldsymbol{h}) := [r_1(\boldsymbol{h}) \ldots, r_K(\boldsymbol{h})]^T$ per channel realization $\boldsymbol{h}$ to minimize the total average power usage.

III. Average Sum–Rate Constraint

In this section, we consider the problem of minimizing total weighted power given a sum average rate constraint. Such a constraint may arise in a wireless (e.g., sensor) network, where the AP (fusion center) requires an aggregate rate $\check{r} > 0$ to perform a certain task (e.g., distributed estimation) using data from different users (sensors).

A. Average Power Region

With $\mathbb{R}_+^K$ standing for the K-dimensional space of nonnegative reals and $\mathbb{E}_{\boldsymbol{h}}$ for the expectation with respect to (w.r.t.) the vector of channel gains, let $\mathcal{F}(\check{r})$ denote the set of all feasible rate and time allocation policies satisfying the average rate constraint $\mathbb{E}_{\boldsymbol{h}}\left[\sum_{k=1}^{K} r_k(\boldsymbol{h})\right] \geq \check{r}$ with[1] $\sum_{k=1}^{K} \tau_k(\boldsymbol{h}) \leq 1, \forall\, \boldsymbol{h}$, and $(\boldsymbol{\tau}(\boldsymbol{h}), \boldsymbol{r}(\boldsymbol{h})) \in \mathbb{R}_+^K \times \mathbb{R}_+^K$. We assume henceforth w.l.o.g. that the system bandwidth $B = 1$ and the AWGN at the receiver has unit variance. Then using transmit power $\pi_k(\boldsymbol{h})$, user k can theoretically transmit with arbitrarily small error a number of bits per second per hertz (bits/s/Hz) up to the Shannon capacity

$$\rho_k(\boldsymbol{h}) = \log_2\left(1 + h_k\pi_k(\boldsymbol{h})\right);$$

hence, the (minimum) power required for a given rate is

$$\pi_k(\boldsymbol{h}) = (1/h_k)(2^{\rho_k(\boldsymbol{h})} - 1).$$

Taking into account the TDMA time fractions $\tau_k(\boldsymbol{h})$, the minimum power per block to attain rate $r_k(\boldsymbol{h}) = \tau_k(\boldsymbol{h})\rho_k(\boldsymbol{h})$ is

$$p_k(\boldsymbol{h}) = \tau_k(\boldsymbol{h})\pi_k(\boldsymbol{h}) = (\tau_k(\boldsymbol{h})/h_k)(2^{r_k(\boldsymbol{h})/\tau_k(\boldsymbol{h})} - 1)$$

for $\tau_k(\boldsymbol{h}) > 0$. When $\tau_k(\boldsymbol{h}) = 0$, this power is given by $p_k(\boldsymbol{h}) = \lim_{\tau_k(\boldsymbol{h})\to 0}(\tau_k(\boldsymbol{h})/h_k)(2^{r_k(\boldsymbol{h})/\tau_k(\boldsymbol{h})} - 1) = \infty$, if $r_k(\boldsymbol{h}) > 0$; and also if $\tau_k(\boldsymbol{h}) = r_k(\boldsymbol{h}) = 0$, then clearly $p_k(\boldsymbol{h}) = 0$. Summarizing, the instantaneous transmit power can be determined from the time fractions and rate per channel realization as shown in (1) at the bottom of the page. With $\bar{p}_k := \mathbb{E}_{\boldsymbol{h}}[p_k(\boldsymbol{h})]$, $\bar{\boldsymbol{p}} := [\bar{p}_1, \ldots, \bar{p}_K]^T$, and in accordance with the definition of the ergodic capacity region, it is possible to define a *power region* as follows.

Definition 1: The power region for achieving an average prescribed rate $\check{r}$ over a TDMA fading channel with transmit and receive ends having full CSI available, is given by

$$\mathcal{P}(\check{r}) := \bigcup_{(\boldsymbol{\tau}(\cdot), \boldsymbol{r}(\cdot)) \in \mathcal{F}(\check{r})} \{\bar{\boldsymbol{p}} \mid \bar{p}_k \geq \mathbb{E}_{\boldsymbol{h}}\left[P_k(\tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))\right], \forall\, K\}. \quad (2)$$

It will prove useful to establish that the power function in (1) and the region in (2) are convex.

Lemma 1: Function $P_k(\tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))$ is jointly convex in $\tau_k(\boldsymbol{h})$ and $r_k(\boldsymbol{h})$; and the power region $\mathcal{P}(\check{r})$ is a convex set of $\bar{\boldsymbol{p}}$ vectors.

Proof: See Appendix A. □

B. Continuous Rate Adaptation

Since $\mathcal{P}(\check{r})$ is convex, with nonnegative weights w_k collected in a vector $\boldsymbol{w} := [w_1, \ldots, w_K]^T$, each boundary point of the

[1]The trivial nonnegativity constraint on, e.g., the time fractions will be omitted when it is clear from the context.

$$p_k(\boldsymbol{h}) := P_k(\tau_k(\boldsymbol{h})r_k(\boldsymbol{h})) = \begin{cases} (\tau_k(\boldsymbol{h})/h_k)(2^{r_k(\boldsymbol{h})/\tau_k(\boldsymbol{h})} - 1), & \tau_k(\boldsymbol{h}) > 0 \\ \infty, & \tau_k(\boldsymbol{h}) = 0, r_k(\boldsymbol{h}) > 0 \\ 0, & \tau_k(\boldsymbol{h}) = r_k(\boldsymbol{h}) = 0. \end{cases} \quad (1)$$

power region can be obtained as the solution of the convex optimization problem:

$$\min_{\bar{\boldsymbol{p}}} \ \boldsymbol{w}^T\bar{\boldsymbol{p}}, \quad \text{subject to (s. to)} \quad \bar{\boldsymbol{p}} \in \mathcal{P}(\check{r}). \tag{3}$$

Solving (3) for *all* $\boldsymbol{w} \geq \mathbf{0}$ (inequalities for vectors are defined element-wise), yields all the boundary points, and thus the entire power region $\mathcal{P}(\check{r})$. When one or more of the entries of $\boldsymbol{w}$ are zero, the solution to (3) corresponds to an extreme point of the boundary surface of $\mathcal{P}(\check{r})$; see also [9]–[12], [14] for explicit characterization of extreme points in the capacity region boundary.

Since power is determined by time and rate [cf. (1)], the optimal power vector $\bar{\boldsymbol{p}}^* = \bar{\boldsymbol{p}}(\boldsymbol{\tau}^*, \boldsymbol{r}^*)$ for a given $\boldsymbol{w}$ is attained by a certain pair $(\boldsymbol{\tau}^*, \boldsymbol{r}^*) \in \mathcal{F}(\check{r})$. Finding this pair amounts to solving

$$\begin{cases} \min_{\boldsymbol{\tau}(\cdot),\boldsymbol{r}(\cdot)} & \sum_{k=1}^K w_k \mathbb{E}_{\boldsymbol{h}}\left[P_k(\tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))\right] \\ \text{s. to} & \mathbb{E}_{\boldsymbol{h}}\left[\sum_{k=1}^K r_k(\boldsymbol{h})\right] \geq \check{r} \\ & \sum_{k=1}^K \tau_k(\boldsymbol{h}) \leq 1, \forall\, \boldsymbol{h}. \end{cases} \tag{4}$$

As $P_k(\tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))$ is convex, it is easy to recognize (4) as a convex optimization problem, which can be efficiently solved using a dual-based approach that we pursue next.

With λ denoting the Lagrange multiplier corresponding to the sum–rate constraint, the Lagrangian of (4) without the time allocation constraint is

$$L(\lambda, \boldsymbol{\tau}, \boldsymbol{r}) = \sum_{k=1}^K w_k \mathbb{E}_{\boldsymbol{h}}\left[P_k(\tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))\right] - \lambda\left(\mathbb{E}_{\boldsymbol{h}}\left[\sum_{k=1}^K r_k(\boldsymbol{h})\right] - \check{r}\right). \tag{5}$$

The Lagrange dual function which includes the time allocation constraint is then given by

$$D(\lambda) = \min_{\boldsymbol{\tau},\boldsymbol{r}} \ L(\lambda, \boldsymbol{\tau}, \boldsymbol{r}), \quad \text{s. to} \sum_{k=1}^K \tau_k(\boldsymbol{h}) \leq 1, \quad \forall\, \boldsymbol{h} \tag{6}$$

and the dual problem of (4) is

$$\max_{\lambda \geq 0} D(\lambda). \tag{7}$$

Using standard results from convex optimization theory, it follows readily that the optimal value of (7) coincides with that of (4); i.e., there is no duality gap [15, p. 226].

To solve (7), we need to first find the dual function $D(\lambda)$ in (6). To this end, we treat λ as a rate reward weight and define a *net-cost* (power cost minus rate reward) function per user as

$$\varphi_k(\lambda, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h})) := w_k P_k(\boldsymbol{\tau}(\boldsymbol{h}), \boldsymbol{r}(\boldsymbol{h})) - \lambda r_k(\boldsymbol{h}). \tag{8}$$

Using (8), we can rewrite (5) as

$$L(\lambda, \boldsymbol{\tau}, \boldsymbol{r}) = \lambda\check{r} + \mathbb{E}_{\boldsymbol{h}}\left[\sum_{k=1}^K \varphi_k(\lambda, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))\right]. \tag{9}$$

Because channel gains are nonnegative, it follows from the definition of $\mathbb{E}_{\boldsymbol{h}}$ that minimizing $L(\lambda, \boldsymbol{\tau}, \boldsymbol{r})$ amounts to minimizing $\varphi_k(\lambda, \boldsymbol{\tau}(\boldsymbol{h}), \boldsymbol{r}(\boldsymbol{h}))$ per $\boldsymbol{h}$. This allows us to specify the optimum time and rate allocation in the dual function of (6) for any given value of the Lagrange multiplier λ, as summarized in the following lemma.

Lemma 2: For any fixed λ, if we define per $\boldsymbol{h}$ the link quality indicators $\forall\, k = 1, \ldots, K$

$$\varphi_k^*(\lambda, \boldsymbol{h}) := \begin{cases} \frac{\lambda}{\ln 2} - \frac{w_k}{h_k} - \lambda \log_2 \frac{\lambda h_k}{w_k \ln 2}, & h_k > \frac{w_k \ln 2}{\lambda} \\ 0, & h_k \leq \frac{w_k \ln 2}{\lambda}. \end{cases} \tag{10}$$

and select the (almost surely) unique user index

$$k^*(\lambda, \boldsymbol{h}) = \arg\min_k \ \varphi_k^*(\lambda, \boldsymbol{h}) \tag{11}$$

then the optimal resource allocation satisfying (6) is a greedy one assigning $([x]^+ := \max(0, x))$

$$\tau_{k^*}^*(\lambda, \boldsymbol{h}) = 1, \quad r_{k^*}^*(\lambda, \boldsymbol{h}) = [\log_2 \lambda - \log_2(w_{k^*} \ln 2/h_{k^*})]^+ \tag{12}$$

and $\tau_k^*(\lambda, \boldsymbol{h}) = r_k^*(\lambda, \boldsymbol{h}) = 0, \forall\, k \neq k^*(\lambda, \boldsymbol{h})$.

Proof: See Appendix B. □

Instead of water-filling the power as in rate-maximizing formulations, the power-minimizing allocation in(12) amounts to water-filling the optimal rate across fading states. (In both cases, however, the Lagrange multiplier determines the "water level.") The optimal time and rate allocations (which are in fact decoupled in Lemma 2) can be interpreted as follows. If terminal k transmits with rate $\rho_k(\boldsymbol{h}) = r_k(\boldsymbol{h})/\tau_k(\boldsymbol{h})$ over its time fraction $\tau_k(\boldsymbol{h}) > 0$, the corresponding net cost will be [cf. (1) and (8)]

$$\varphi_k(\lambda, \rho_k(\boldsymbol{h})) = (w_k/h_k)(2^{\rho_k(\boldsymbol{h})} - 1) - \lambda\rho_k(\boldsymbol{h}), \quad \tau_k(\boldsymbol{h}) > 0. \tag{13}$$

And the optimal rate allocation will be to water-fill its rate across $\boldsymbol{h}$ realizations; i.e., use

$$\rho_k^*(\boldsymbol{h}) = [\log_2 \lambda - \log_2(w_k \ln 2/h_k)]^+ \tag{14}$$

to attain its smallest net cost $\varphi_k^*(\lambda, \boldsymbol{h}) = \varphi_k(\lambda, \rho_k^*(\boldsymbol{h}))$. As $\varphi_k^*(\lambda, \boldsymbol{h})$ represents each user's link quality indicator (the smaller the better), we should then allow only the user k^* defined in (11) with the minimal net cost to transmit since this user can utilize the time slot in the most power-efficient manner per channel realization $\boldsymbol{h}$. Therefore, the optimal allocation is to assign $\tau_{k^*}^*(\lambda, \boldsymbol{h}) = 1$, $r_{k^*}^*(\lambda, \boldsymbol{h}) = \tau_{k^*}^*(\lambda, \boldsymbol{h})\rho_k^*(\boldsymbol{h}) = [\log_2 \lambda - \log_2(w_{k^*} \ln 2/h_{k^*})]^+$, and let all other users defer their transmissions.

A couple of remarks are now due to clarify the almost sure uniqueness alluded to before (11).

Remark 1: The obvious setting where a single "winner user" is impossible is when $h_k \leq w_k \ln 2/\lambda, \forall\, k$; that is, when all users have identical $\varphi_k^*(\lambda, \boldsymbol{h}) = 0$. However, (12) confirms that when $h_k \leq w_k \ln 2/\lambda, \forall\, k$, the optimal $r_{k^*}^*(\lambda, \boldsymbol{h}) = 0$ and thus whichever terminal we select as the single winner is irrelevant because no one will waste rate or power resources anyway. (This case corresponds to a setting where all channels undergo deep fading and the *unique* most power-efficient act is for all users to defer in such channel realizations).

Remark 2: Going beyond the case of the previous remark, for two users k and $k' \neq k$ to have identical but nonzero net costs it must hold that

$$\frac{\lambda}{\ln 2} - \frac{w_k}{h_k} - \lambda \log_2 \frac{\lambda h_k}{w_k \ln 2} - \left(\frac{\lambda}{\ln 2} - \frac{w_{k'}}{h_{k'}} - \lambda \log_2 \frac{\lambda h_{k'}}{w_{k'} \ln 2} \right) := d_{k,k'}(\varphi^*) \equiv 0.$$

But since the cdf of the fading process is assumed continuous under (oc-1), the event $\{d_{k,k'}(\varphi^*) \equiv 0\}$ has probability measure zero. Likewise, having more than two "winner users" tie is a measure-zero event for ergodic fading channels with continuous cdf. This explains why an almost surely unique winner was asserted in (11).

With Lemma 2 providing the optimal rate and power allocation for each λ and channel realization $\boldsymbol{h}$, we can proceed to show that $\check{r} - \mathbb{E}_{\boldsymbol{h}}[r_{k^*}(\lambda, \boldsymbol{h})]$ is a subgradient of the dual function $D(\lambda)$ at λ; see, e.g., [16, p. 604] from the definition of the subgradient. This fact ensures that the optimal λ^* for the dual problem (7) can be obtained using a subgradient iteration (indexed by i)

$$\lambda^{(i+1)} = \left[\lambda^{(i)} + \beta \left(\check{r} - \mathbb{E}_{\boldsymbol{h}} \left[r^*_{k^*}(\lambda^{(i)}, \boldsymbol{h}) \right] \right) \right]^+ \tag{15}$$

where β denotes a positive step size. With sufficiently small β, geometric convergence of the iteration in (15) to λ^* is guaranteed from any initial $\lambda^{(0)} \geq 0$, thanks to convexity [16, p. 621].

Summarizing, the dual problem (7) can be solved with the following algorithm.

Algorithm 1: *Subgradient projection iterations*
initialize with any $\lambda^{(0)} \geq 0$; and

repeat: with $\lambda^{(i)}$ available from the previous iteration, find the winner user $k^*(\lambda^{(i)}, \boldsymbol{h})$ via (11) and optimal rate $r_{k^*}(\lambda^{(i)}, \boldsymbol{h})$ via (12) per $\boldsymbol{h}$; and update $\lambda^{(i+1)}$ as in (15);

stop the iterations when $|\lambda^{(i+1)} - \lambda^{(i)}| < \varepsilon$ for a preselected tolerance ε.

With ensured convergence of Algorithm 1 to λ^*, we are ready to state our first basic result.

Theorem 1: The (almost surely) optimal time and rate allocation for (4) is given by $\boldsymbol{\tau}^*(\lambda^*, \boldsymbol{h})$ and $\boldsymbol{r}^*(\lambda^*, \boldsymbol{h})$ in Lemma 2, where the optimal λ^* is obtained via Algorithm 1 and satisfies $\lambda^* > 0$ and $\mathbb{E}_{\boldsymbol{h}}[r^*_{k^*}(\lambda^*, \boldsymbol{h})] = \check{r}$.

Proof: See Appendix C. □

It is worth to emphasize that *complexity* of the power-minimizing resource allocation scheme implied by Theorem 1 is *linear* in the number of users K. This is because both Algorithm 1 employed to find λ^* has geometric (i.e., linear) convergence; and also the greedy policy of assigning the entire block to a single winner user involves just K comparisons of the link indicators $\varphi_k^*(\lambda, \boldsymbol{h})$ per realization $\boldsymbol{h}$ [cf. Lemma 2]. Using such a computationally efficient scheme, the optimal allocation for (4) and, subsequently, every boundary point of $\mathcal{P}(\check{r})$ can be obtained by solving (3).

Remark 3: Almost sure optimality of the allocation claimed in Theorem 1 follows from the almost sure uniqueness of the winner user in Lemma 2 argued under Remark 2. Since having multiple winners corresponds to a measure-zero event, choosing at random a single winner for each channel realization over which net costs of more than one user tie, has measure-zero effect to Theorem 1's allocation scheme that minimizes *average* weighted sum–power subject to an *average* sum–rate constraint; see also [9, Lemma 3.15], where almost sure optimality is asserted in the rate-maximizing context. In both contexts, almost sure optimality of the greedy allocation is ensured for channels with continuous cdf. Notwithstanding, if $\boldsymbol{h}$ is deterministic or drawn from a discontinuous cdf $F(\boldsymbol{h})$, the optimal sharing of the bock by multiple winners must be determined to satisfy the average rate constraint with equality; and the optimal allocation, in general, will not be greedy.

C. AMC-Based Discrete Rate Adaptation

User terminals in practice will not be able to support the continuous rates implied by an infinite-size codebook. Moreover, the codewords in use may not be capacity achieving. These considerations motivate the power-efficient resource allocation pursued in this section, where each user k can only support a finite number of, say M_k, discrete rates obtained through the use of distinct AMC modes [21]. If scheduled, each user k will select a (modulation, channel code) pair to transmit over its active time fraction $\tau_k(\boldsymbol{h}) > 0$ with an AMC rate $\rho_{k,m}$. In addition to $\{\rho_{k,m}\}_{m=1}^{M_k}$ nonzero rates (AMC modes) that can differ per user $k = 1, \ldots, K$, let $\rho_{k,0} = 0$ denote the zero "defer-rate" (cf. Remark 1). Clearly, for the prescribed rate $\check{r}$ to be feasible it must satisfy $\check{r} \in [0, \max_k \rho_{k,M_k}]$. To minimize power with discrete rates, we first need to replace the power function in(1) with one relating transmit power (π_k) not only with AMC rates $(\rho_{k,m})$ and time fractions but also with the bit-error rate (BER ϵ_k) per channel realization (h_k). For constellation- and code-specific constants κ_1 and κ_2, the BER can be accurately approximated as [18]

$$\epsilon_k(h_k \pi_k, \rho_{k,m}) = \kappa_1 \exp\left(-\frac{\kappa_2 h_k \pi_k}{2^{\rho_{k,m}} - 1} \right). \tag{16}$$

Given h_k and a maximum allowable BER$\check{\epsilon}_k$, the minimum transmit power $\pi_{k,m}$ can be determined for each AMC mode $\rho_{k,m}$ such that $\epsilon_k(h_k \pi_k, \rho_{k,m}) = \check{\epsilon}_k$. From (16), this power is given by

$$\pi_{k,m}(h_k) = (\Gamma / h_k)(2^{\rho_{k,m}} - 1)$$

where $\Gamma := \kappa_2^{-1} \ln(\kappa_1 / \check{\epsilon}_k) \geq 1$ denotes the excess signal-to-noise ratio (SNR) required for the AMC-based system to attain the same transmission rate as the capacity-achieving system.

Connecting with straight-line segments the M_k points representing the AMC pairs $(\rho_{k,m}, \pi_{k,m}(h_k))$, we obtain the *piecewise linear* power–rate function $\Pi_k(\rho_k(\boldsymbol{h}))$, depicted in

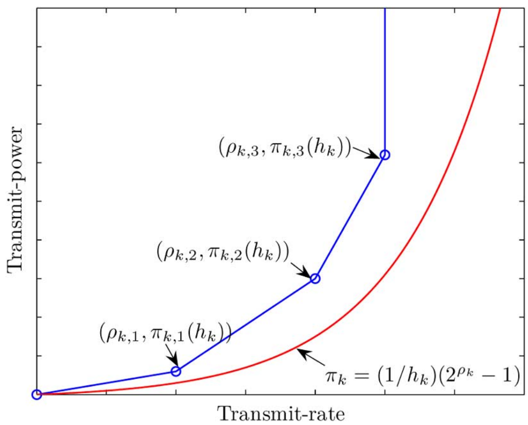

Fig. 1. Piecewise linear function $\Pi_k(\rho_k(\boldsymbol{h}))$ relating transmit-rates and powers.

Fig. 1, which is also convex [2]. Note that as $M_k \to \infty$ and the SNR gap $\Gamma \to 1$, $\Pi_k(\rho_k(\boldsymbol{h}))$ approaches Shannon's continuous power-rate function, i.e., $\pi_k(\boldsymbol{h}) = (1/h_k)\left(2^{\rho_k(\boldsymbol{h})} - 1\right)$, which is also plotted in Fig. 1. Points on any linear segment of $\Pi_k(\rho_k(\boldsymbol{h}))$ can be attained via time sharing. Specifically, using the mode m over $\alpha_{k.m}$ percentage of the τ_k fraction and letting $\tau_{k,m} := \alpha_{k,m}\tau_k$, user k can transmit with rate and power given, respectively, by

$$r_k(\boldsymbol{h}) = \tau_k(\boldsymbol{h})\rho_k(\boldsymbol{h}) = \sum_{m=0}^{M_k} \tau_{k,m}(\boldsymbol{h})\rho_{k,m} \quad (17)$$

$$p_k(\boldsymbol{h}) = \tau_k(\boldsymbol{h})\pi_k(\boldsymbol{h}) = \sum_{m=0}^{M_k} \tau_{k,m}(\boldsymbol{h})\pi_{k,m}(h_k). \quad (18)$$

Since each $\rho_{k,m}$ denotes a known AMC rate and the $\pi_{k,m}(h_k)$ required to attain this rate with the prescribed BER$\check{\epsilon}_k$ is also known, the only variables to minimize over the average power are the time fractions collected in the vector $\boldsymbol{\tau}(\boldsymbol{h}) := \{\tau_{k,m}(\boldsymbol{h}),\ m = 0, \ldots, M_k, k = 1, \ldots, K\}$. To formulate the power minimization problem in this discrete rate setup, let $\mathcal{F}'(\check{r})$ denote a set including all the feasible $\boldsymbol{\tau}(\cdot)$ satisfying the average rate constraint $\mathbb{E}_{\boldsymbol{h}}\left[\sum_{k=1}^{K}\sum_{m=0}^{M_k}\tau_{k,m}(\boldsymbol{h})\rho_{k,m}\right] \geq \check{r}$ and $\sum_{k=1}^{K}\sum_{m=0}^{M_k}\tau_{k,m}(\boldsymbol{h}) \leq 1\ \forall\ \boldsymbol{h}$. Then the average power region can be defined as [cf. (2)]

$$\mathcal{P}'(\check{r}) = \bigcup_{\boldsymbol{\tau}(\cdot)\in\mathcal{F}'(\check{r})} \left\{\bar{\boldsymbol{p}} \mid \bar{p}_k \geq \mathbb{E}_{\boldsymbol{h}}\left[\sum_m \tau_{k,m}(\boldsymbol{h})\pi_{k,m}(h_k)\right],\ \forall k\right\}. \quad (19)$$

As in Lemma 1, it is possible to show that $\mathcal{P}'(\check{r})$ in (19) is a convex set of $\bar{\boldsymbol{p}}$ vectors, and each boundary point of $\mathcal{P}'(\check{r})$ solves for a weight vector $\boldsymbol{w} \geq \boldsymbol{0}$ the problem

$$\min_{\bar{\boldsymbol{p}}} \boldsymbol{w}^T\bar{\boldsymbol{p}},\ \text{s. to}\ \bar{\boldsymbol{p}} \in \mathcal{P}'(\check{r}). \quad (20)$$

This amounts to finding the optimal time allocation vector by solving [cf. (4)]

$$\begin{cases} \min_{\boldsymbol{\tau}(\cdot)} & \sum_{k=1}^{K} w_k \mathbb{E}_{\boldsymbol{h}}\left[\sum_{m=0}^{M_k}\tau_{k,m}(\boldsymbol{h})\pi_{k,m}(h_k)\right] \\ \text{s. to} & \mathbb{E}_{\boldsymbol{h}}\left[\sum_{k=1}^{K}\sum_{m=0}^{M_k}\tau_{k,m}(\boldsymbol{h})\rho_{k,m}\right] \geq \check{r} \\ & \sum_{k=1}^{K}\sum_{m=0}^{M_k}\tau_{k,m}(\boldsymbol{h}) \leq 1, \forall\ \boldsymbol{h}. \end{cases} \quad (21)$$

Clearly, (21) is a convex optimization problem (in fact a linear program), which can be solved using a dual-based approach analogous to that detailed in Section III-B. To outline the basic steps, the Lagrangian is now given by

$$L(\lambda, \boldsymbol{\tau}) = \sum_{k=1}^{K} w_k \mathbb{E}_{\boldsymbol{h}}\left[\sum_{m=0}^{M_k}\tau_{k,m}(\boldsymbol{h})\pi_{k,m}(h_k)\right] - \lambda\left(\mathbb{E}_{\boldsymbol{h}}\left[\sum_{k=1}^{K}\sum_{m=0}^{M_k}\tau_{k,m}(\boldsymbol{h})\rho_{k,m}\right] - \check{r}\right). \quad (22)$$

The Lagrange dual function is

$$D(\lambda) = \min_{\boldsymbol{\tau}} L(\lambda, \boldsymbol{\tau}),\ \text{s. to}\ \sum_{k=1}^{K}\sum_{m=0}^{M_k}\tau_{k,m}(\boldsymbol{h}) \leq 1, \quad \forall\, \boldsymbol{h} \quad (23)$$

and the dual problem of (21) is

$$\max_{\lambda\geq 0} D(\lambda). \quad (24)$$

Upon defining the net-cost per user-mode pair (k, m) as

$$\varphi_{k,m}(\lambda, \boldsymbol{h}) := w_k\pi_{k,m}(h_k) - \lambda\rho_{k,m} \quad (25)$$

it is possible to rewrite (22) as

$$L(\lambda, \boldsymbol{\tau}) = \lambda\check{r} + \mathbb{E}_{\boldsymbol{h}}\left[\sum_{k=1}^{K}\tau_{k,m}(\boldsymbol{h})\varphi_{k,m}(\lambda, \boldsymbol{h})\right].$$

Based on the latter, we have established the following counterpart of Lemma 2 for discrete rates.

Lemma 3: For any fixed λ, the optimal time allocation solving (23) is a greedy one assigning each block to a single user-mode pair $(k^*, m^*_{k^*})$ per channel gain realization $\boldsymbol{h}$; i.e.,

$$\tau^*_{k,m}(\lambda, \boldsymbol{h}) = \begin{cases} 1, & \text{if } (k, m) = (k^*, m^*_{k^*}) \\ 0, & \text{if } (k, m) \neq (k^*, m^*_{k^*}) \end{cases} \quad (26)$$

where the winner user-mode pair is found using (23) as

$$(k^*(\lambda, \boldsymbol{h}), m^*_{k^*}(\lambda, \boldsymbol{h})) = \arg\min_{(k,m)} \varphi_{k,m}(\lambda, \boldsymbol{h}). \quad (27)$$

Proof: See Appendix D. □

With the optimal allocation established in Lemma 3, we can obtain the optimal λ^* for the dual problem (24) using the subgradient iterations (indexed by i)

$$\lambda^{(i+1)} = \left[\lambda^{(i)} + \beta\left(\check{r} - \mathbb{E}_{\boldsymbol{h}}\left[\rho_{k^*,m^*_{k^*}}(\lambda^{(i)}, \boldsymbol{h})\right]\right)\right]^+. \quad (28)$$

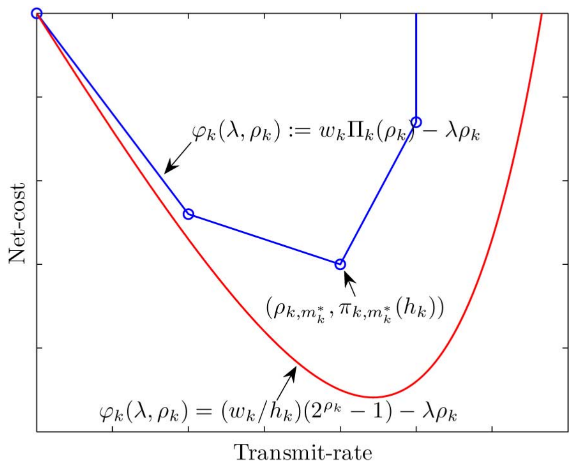

Fig. 2. Piecewise linear net-cost.

Following the steps used to prove Theorem 1, we can subsequently establish the following optimal allocation result for discrete rates.

Theorem 2: The (almost surely) optimal time allocation for (21) is given by $\boldsymbol{\tau}^*(\lambda^*, \boldsymbol{h})$ in Lemma 3, where the optimal λ^* can be obtained via subgradient iterations (28) and satisfies $\lambda^* > 0$ and $\mathbb{E}_{\boldsymbol{h}}\left[\rho_{k^*,m^*_{k^*}}(\lambda^*, \boldsymbol{h})\right] = \check{r}$.

Remarks 1 and 2 on the possibility of all users deferring and almost sure optimality of the power-efficient allocation with continuous rates carry over to the discrete-rate allocation of Theorem 2. Likewise, the complexity in determining each boundary point of $\mathcal{P}^\circ(\check{r})$ is linear, i.e., $\mathcal{O}\left(\sum_{k=1}^K M_k\right)$.

Remark 4: A water-filling interpretation analogous to that described after Lemma 2 for continuous-rate transmissions can be also provided for the optimum allocation of Lemma 3 with discrete-rate transmissions. To illustrate this, consider the net-cost [cf. (13)] $\varphi_k(\lambda, \rho_k(\boldsymbol{h})) := w_k\Pi_k(\rho_k(\boldsymbol{h})) - \lambda\rho_k(\boldsymbol{h})$, where $\Pi_k(\rho_k(\boldsymbol{h}))$ is the piecewise linear power–rate function of Fig. 1, with its M_k line segments having slopes denoted by $\{s_{k,m}\}_{m=1}^{M_k}$. This net-cost depicted in Fig. 2 is a piecewise linear and convex function of the rate $\rho_k(\boldsymbol{h})$. Its minimum must occur at a vertex ρ_{k,m_k^*}, which corresponds to a corner point $\left(\rho_{k,m_k^*}, \pi_{k,m_k^*}(h_k)\right)$ of the power–rate function Π_k. For rates to the left of this point, the derivative of $\varphi_k(\lambda, \rho_k(\boldsymbol{h}))$ w.r.t. ρ_k must be negative and for rates to the right of this point it should be positive; hence

$$w_k s_{k,m_k^*} - \lambda \le 0 < w_k s_{k,m_k^*+1} - \lambda. \tag{29}$$

Recall that with continuous rates too, the derivative of $\varphi_k(\lambda, \boldsymbol{h})$ changes sign before and after the optimal $\rho_k^*(\boldsymbol{h})$, which turns out (after equating this derivative to zero) to obey the *continuous* water-filling principle [cf. (14)]. Finding the mode m_k for which the before-and-after slopes satisfy (29) constitutes a *discrete* water-filling approach to determining the optimal $\rho_k^*(\boldsymbol{h}) = \rho_{k,m_k^*}$. In fact, this approach is equivalent to that in Lemma 3, where the optimization in (27) is performed over the vertices of the net cost; i.e., $\arg\min_{\rho_k(\boldsymbol{h})} \varphi_k(\lambda, \rho_k(\boldsymbol{h})) \equiv \arg\min_{(k,m)} \varphi_{k,m}(\lambda, \boldsymbol{h})$.

Remark 5: Instead of the average sum–rate constraint, the formulation in this and the previous sections can be generalized to an average *weighted*-sum–rate constraint, namely, $\mathbb{E}_{\boldsymbol{h}}\left[\sum_{k=1}^K w_k^r r_k(\boldsymbol{h})\right] \ge \check{r}$, where w_k^r can be viewed as a rate–reward weight for user $k = 1, \ldots, K$. To accommodate this generalization, it suffices to replace the net cost for continuous rates in (8) with

$$\varphi_k(\lambda, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h})) := w_k P_k(\boldsymbol{\tau}(\boldsymbol{h}), \boldsymbol{r}(\boldsymbol{h})) - \lambda w_k^r r_k(\boldsymbol{h}) \tag{30}$$

and the net-cost for discrete rates in (25) with

$$\varphi_{k,m}(\lambda, \boldsymbol{h}) := w_k \pi_{k,m}(h_k) - \lambda w_k^r \rho_{k,m}. \tag{31}$$

With these modifications the optimal policies are still given by Theorems 1 or 2.

IV. Average Individual-Rate Constraints

In this section, power minimization is pursued for multiple-access orthogonal (TDMA) channels under individual average rate requirements $\check{\boldsymbol{r}} := [\check{r}_1, \ldots, \check{r}_K]^T$. The set $\mathcal{F}(\check{\boldsymbol{r}})$ here includes all feasible rate and time allocation policies satisfying the individual rate constraints $\mathbb{E}_{\boldsymbol{h}}[\tau_k(\boldsymbol{h}) r_k(\boldsymbol{h})] \ge \check{r}_k, k = 1, \ldots, K$ and $\sum_{k=1}^K \tau_k(\boldsymbol{h}) \in [0, 1], \forall\, \boldsymbol{h}$. And the average power region is correspondingly given by [cf. (2)]

$$\mathcal{P}(\check{\boldsymbol{r}}) = \bigcup_{(\boldsymbol{\tau}(\cdot), \boldsymbol{r}(\cdot)) \in \mathcal{F}(\check{\boldsymbol{r}})} \{\bar{\boldsymbol{p}} \mid \bar{p}_k \ge \mathbb{E}_{\boldsymbol{h}}\left[P_k(\tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))\right], \forall\, k\}. \tag{32}$$

A. Continuous Rate Adaptation

As in Lemma 1, $\mathcal{P}(\check{\boldsymbol{r}})$ is a convex region of $\bar{\boldsymbol{p}}$ vectors; and thus, each boundary point of $\mathcal{P}(\check{\boldsymbol{r}})$ minimizes a weighted sum of average powers; i.e.,

$$\min_{\bar{\boldsymbol{p}}} \boldsymbol{w}^T \bar{\boldsymbol{p}}, \text{ s. to } \bar{\boldsymbol{p}} \in \mathcal{P}(\check{\boldsymbol{r}}). \tag{33}$$

By solving (33) for all $\boldsymbol{w} \ge \boldsymbol{0}$, we determine all the boundary points, and hence the entire region $\mathcal{P}(\check{\boldsymbol{r}})$. Again, we will explicitly characterize the optimal resource allocation policies and the resultant boundary point for any $\boldsymbol{w} > \boldsymbol{0}$.

At a boundary point associated with a weight vector $\boldsymbol{w}$, we must have the optimal power vector $\bar{\boldsymbol{p}}^* = \bar{\boldsymbol{p}}(\boldsymbol{\tau}^*, \boldsymbol{r}^*)$ for a certain $(\boldsymbol{\tau}^*, \boldsymbol{r}^*)$, which can be found by solving

$$\begin{cases} \min_{\boldsymbol{\tau}(\cdot), \boldsymbol{r}(\cdot)} & \sum_{k=1}^K w_k \mathbb{E}_{\boldsymbol{h}}\left[P_k(\tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))\right] \\ \text{s. to} & \mathbb{E}_{\boldsymbol{h}}\left[r_k(\boldsymbol{h})\right] \ge \check{r}_k, k = 1, \ldots, K \\ & \sum_{k=1}^K \tau_k(\boldsymbol{h}) \le 1, \forall\, \boldsymbol{h}. \end{cases} \tag{34}$$

It is easy to see that (34) is a convex optimization problem and similar to (4) it can be solved using a dual-based approach. The individual average rate constraints give rise to a vector of Lagrange multipliers $\boldsymbol{\lambda} := [\lambda_1, \ldots, \lambda_K]^T$ with λ_k associated with $\mathbb{E}_{\boldsymbol{h}}\left[r_k(\boldsymbol{h})\right] \ge \check{r}_k, \forall\, k$. The net-cost per user k is now defined as

$$\varphi_k(\lambda_k, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h})) := w_k P_k(\boldsymbol{\tau}(\boldsymbol{h}), \boldsymbol{r}(\boldsymbol{h})) - \lambda_k r_k(\boldsymbol{h}). \tag{35}$$

Different from (8), here $\varphi_k(\lambda_k, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))$ depends only on its own rate reward weight λ_k.

Based on $\varphi_k(\lambda_k, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))$, the link quality indicator per $\boldsymbol{h}$ is defined as

$$\varphi_k^*(\lambda_k, \boldsymbol{h}) := \begin{cases} \frac{\lambda_k}{\ln 2} - \frac{w_k}{h_k} - \lambda_k \log_2 \frac{\lambda_k h_k}{w_k \ln 2}, & h_k > \frac{w_k \ln 2}{\lambda_k} \\ 0, & h_k \leq \frac{w_k \ln 2}{\lambda_k} \end{cases} \tag{36}$$

and the winner user index is selected according to

$$k^*(\boldsymbol{\lambda}, \boldsymbol{h}) = \arg\min_k \varphi_k^*(\lambda_k, \boldsymbol{h}). \tag{37}$$

The optimal resource allocation for a given $\boldsymbol{\lambda}$ is again a greedy one assigning [cf. Lemma 2], see equation (38) at the bottom of the page.

As with (15), we can show that the optimal $\boldsymbol{\lambda}^*$ for the dual problem of (34) can be iteratively obtained using subgradient iterations

$$\lambda_k^{(i+1)} = \left[\lambda_k^{(i)} + \beta\left(\check{r}_k - \mathbb{E}_{\boldsymbol{h}}\left[r_k^*(\boldsymbol{\lambda}^{(i)}, \boldsymbol{h})\right]\right)\right]^+, \quad \forall\, k. \tag{39}$$

Convexity again guarantees that iterates in (39) converge to $\boldsymbol{\lambda}^*$ from any initial $\boldsymbol{\lambda}^{(0)} \geq \mathbf{0}$.

The counterpart of Theorem 1 is as follows.

Theorem 3: The (almost surely) optimal time and rate allocation for (44) is given by $\boldsymbol{\tau}^*(\boldsymbol{\lambda}^*, \boldsymbol{h})$ and $\boldsymbol{r}^*(\boldsymbol{\lambda}^*, \boldsymbol{h})$ in (46), where the optimal $\boldsymbol{\lambda}^*$ is obtained via iterations (48) and satisfies $\boldsymbol{\lambda}^* > \mathbf{0}$ and $\mathbb{E}_{\boldsymbol{h}}\left[r_k^*(\boldsymbol{\lambda}^*, \boldsymbol{h})\right] = \check{r}_k,\ k = 1, \ldots, K$.

Clearly, the power-efficient resource allocation under individual rate constraints also obeys a greedy water-filling principle and its solution incurs linear complexity $\mathcal{O}(K)$. As with Theorem 1, the winner-takes-all policy in Theorem 3 is almost surely optimal for reasons analogous to those elaborated in Remarks 2 and 3.

To gain further insight, consider a special case where the fading channels are independent. Let $F_k(\cdot)$ stand for the cdf of user k's fading channel, and $\varphi_i^{-1}(\varphi_k(h_k))$ for the value of h_i which satisfies[2] $\varphi_i^*(\lambda_i^*, h_i) = \varphi_k^*(\lambda_k^*, h_k)$. We can then establish the following corollary of Theorem 3:

Corollary 1: If the fading channels across users are independent, the optimal solution $\bar{\boldsymbol{p}}^*$ to (33) for a given $\boldsymbol{w} > \mathbf{0}$ can be obtained in closed form as: $\forall\, k$

$$\bar{p}_k^* = \int_{\frac{w_k \ln 2}{\lambda_k^*}}^{\infty} \left(\frac{\lambda_k^*}{w_k \ln 2} - \frac{1}{h_k}\right) \times \prod_{i \neq K} F_i(\varphi_i^{-1}(\varphi_k(h_k))) dF_k(h_k) \tag{40}$$

where each entry λ_k^* of the vector $\boldsymbol{\lambda}^*$ is the unique solution of the equations:$\forall\, k$

$$\int_{\frac{w_k \ln 2}{\lambda_k^*}}^{\infty} \log_2\left(\frac{\lambda_k^* h_k}{w_k \ln 2}\right) \prod_{i \neq k} F_i(\varphi_i^{-1}(\varphi_k(h_k))) dF_k(h_k) = \check{r}_k \tag{41}$$

Proof: See Appendix E. □

Independence of channel gains led to the analytical expressions (40) and (41) for the optimum Lagrange multiplier and the minimum power. However, even when channels are correlated it is possible to obtain the expected values needed to carry out the subgradient iterations using a Monte Carlo approach, so long as the joint cdf $F(\boldsymbol{h})$ is known. Specifically, $\mathbb{E}_{\boldsymbol{h}}\left[r_k^*(\boldsymbol{\lambda}^{(i)}, \boldsymbol{h})\right]$ in (39) can be replaced by $N^{-1}\sum_{n=1}^{N} r_k^*(\boldsymbol{\lambda}^{(i)}, \boldsymbol{h}[n])$, where $\{\boldsymbol{h}[n]\}_{n=1}^{N}$ are realizations of $\boldsymbol{h}$ generated from $F(\boldsymbol{h})$.

B. Discrete Rate Adaptation

Here we consider individual average rate constraints when each user can only support a finite number of AMC modes. As with Section III-C, finding power-efficient resource allocation with a finite number of AMC modes amounts to optimizing over the user–mode time allocation $\boldsymbol{\tau}(\boldsymbol{h}) := \{\tau_{k,m}(\boldsymbol{h}),\ m = 0, \ldots, M_k, k = 1, \ldots, K\}$. Let $\mathcal{F}'(\check{\boldsymbol{r}})$ denote a set including all the feasible $\boldsymbol{\tau}(\cdot)$ vectors satisfying the average individual rate constraints $\mathbb{E}_{\boldsymbol{h}}\left[\sum_{k=1}^{K}\sum_{m=0}^{M_k} \tau_{k,m}(\boldsymbol{h})\rho_{k,m}\right] \geq \check{r}_k,\ k = 1, \ldots, K$, and $\sum_{k=1}^{K}\sum_{m=0}^{M_k} \tau_{k,m}(\boldsymbol{h}) \in [0, 1]\ \forall\, \boldsymbol{h}$. The average power region is then [cf. (19)]

$$\mathcal{P}'(\check{\boldsymbol{r}}) = \bigcup_{\boldsymbol{\tau}(\cdot) \in \mathcal{F}'(\check{\boldsymbol{r}})} \left\{\bar{\boldsymbol{p}} \mid \bar{p}_k \geq \mathbb{E}_{\boldsymbol{h}}\left[\sum_m \tau_{k,m}(\boldsymbol{h})\pi_{k,m}(h_k)\right],\ \forall\, k\right\}. \tag{42}$$

For this convex region $\mathcal{P}'(\check{\boldsymbol{r}})$, each boundary point solves for a weight vector $\boldsymbol{w}$ the problem

$$\min_{\bar{\boldsymbol{p}}} \boldsymbol{w}^T \bar{\boldsymbol{p}}, \quad \text{s. to } \bar{\boldsymbol{p}} \in \mathcal{P}'(\check{\boldsymbol{r}}) \tag{43}$$

which amounts to finding the optimal time allocation as [cf. (21)]

$$\begin{cases} \min_{\boldsymbol{\tau}(\cdot)} & \sum_{k=1}^{K} w_k \mathbb{E}_{\boldsymbol{h}}\left[\sum_{m=0}^{M_k} \tau_{k,m}(\boldsymbol{h})\pi_{k,m}(h_k)\right] \\ \text{s. to} & \mathbb{E}_{\boldsymbol{h}}\left[\sum_{m=0}^{M_k} \tau_{k,m}(\boldsymbol{h})\rho_{k,m}\right] \geq \check{r}_k, k = 1, \ldots, K \\ & \sum_{k=1}^{K}\sum_{m=0}^{M_k} \tau_{k,m}(\boldsymbol{h}) \leq 1, \forall\, \boldsymbol{h}. \end{cases} \tag{44}$$

[2]Notice that the value of $\varphi_k^*(\lambda_k^*, \boldsymbol{h})$ in (36) indeed depends on h_k only.

$$\begin{cases} \tau_k^*(\boldsymbol{\lambda}, \boldsymbol{h}) = 1, & r_k^*(\boldsymbol{\lambda}, \boldsymbol{h}) = [\log_2 \lambda_k - \log_2(w_k \ln 2/h_k)]^+, k = k^*(\boldsymbol{\lambda}, \boldsymbol{h}); \\ \tau_k^*(\boldsymbol{\lambda}, \boldsymbol{h}) = 0, & r_k^*(\boldsymbol{\lambda}, \boldsymbol{h}) = 0, k \neq k^*(\boldsymbol{\lambda}, \boldsymbol{h}). \end{cases} \tag{38}$$

With $\boldsymbol{\lambda} := [\lambda_1, \ldots, \lambda_K]^T$ denoting the Lagrange multipliers associated with $\mathbb{E}_{\boldsymbol{h}}\left[\sum_{m=0}^{M_k} \tau_{k,m}(\boldsymbol{h})\rho_{k,m}\right] \geq \check{r}_k, \forall\, k$, the net-cost depends on individual rate reward weights λ_k per user k

$$\varphi_{k,m}(\lambda_k, \boldsymbol{h}) := w_k \pi_{k,m}(h_k) - \lambda_k \rho_{k,m}. \tag{45}$$

The optimal time allocation for a given $\boldsymbol{\lambda}$ is again to assign each block to a single user–mode pair $(k^*, m^*_{k^*})$ per $\boldsymbol{h}$; i.e.,

$$\tau^*_{k,m}(\boldsymbol{\lambda}, \boldsymbol{h}) = \begin{cases} 1, & \text{if } (k,m) = (k^*, m^*_{k^*}) \\ 0, & \text{if } (k,m) \neq (k^*, m^*_{k^*}) \end{cases} \tag{46}$$

where the winner user–mode pair is now found using (45) as

$$(k^*(\boldsymbol{\lambda}, \boldsymbol{h}), m^*_{k^*}(\boldsymbol{\lambda}, \boldsymbol{h})) = \arg \min_{(k,m)} \varphi_{k,m}(\lambda_k, \boldsymbol{h}). \tag{47}$$

As with (28), the optimal $\boldsymbol{\lambda}^*$ for the dual problem of (44) can be iteratively obtained using subgradient iterations: $\forall\, k$

$$\lambda_k^{(i+1)} = \left[\lambda_k^{(i)} + \beta\left(\check{r}_k - \mathbb{E}_{\boldsymbol{h}}\left[\sum_{m=0}^{M_k} \tau^*_{k,m}(\boldsymbol{\lambda}^{(i)}, \boldsymbol{h})\rho_{k,m})\right]\right)\right]^+. \tag{48}$$

Again, the convergence of iterations in (48) to $\boldsymbol{\lambda}^*$ is guaranteed from any initial $\boldsymbol{\lambda}^{(0)} \geq \mathbf{0}$.

The counterpart of Theorem 2 is as follows.

Theorem 4: The (almost surely) optimal time allocation for (44) is given by $\boldsymbol{\tau}^*(\boldsymbol{\lambda}^*, \boldsymbol{h})$ in (46), where the optimal $\boldsymbol{\lambda}^*$ is obtained via subgradient iterations (48) and satisfies $\boldsymbol{\lambda}^* > \mathbf{0}$ and $\mathbb{E}_{\boldsymbol{h}}\left[\sum_{m=0}^{M_k} \tau^*_{k,m}(\boldsymbol{\lambda}^*, \boldsymbol{h})\rho_{k,m})\right] = \check{r}_k, k = 1, \ldots, K$.

Once more, the complexity is linear in the number of user–modes, i.e., $\mathcal{O}\left(\sum_{k=1}^K M_k\right)$. Using the resource allocation policy in Theorem 4, all the boundary points of $\mathcal{P}'(\check{\boldsymbol{r}})$ and hence the entire power region can be determined.

Remark 6: Theorems 1–4 can be extended to frequency-selective fading channels, which are often encountered in wideband communication systems; see similar extensions in [11], [9], [20], [21], and references therein. If the channel remains invariant over the multipath delay spread, it can be cast in the frequency domain as a set of parallel time-invariant Gaussian multiple-access subchannels [19]. Consider such a K-user spectral Gaussian block-fading TDMA channel with continuous fading spectra $H_1(f, \boldsymbol{\omega}), H_2(f, \boldsymbol{\omega}), \ldots, H_K(f, \boldsymbol{\omega})$, where frequency f ranges over the system bandwidth and $\boldsymbol{\omega}$ is the fading state at a given time block. Then the optimal resource allocation strategies can be obtained from Theorems 1-4 by replacing the fading state $\boldsymbol{h}$ with the frequency and fading state pair $(f, \boldsymbol{\omega})$ to determine power regions for frequency-selective channels.

V. Capacity-Achieving Resource Allocation

The novel greedy water-filling algorithms developed in Sections III and IV for power-minimizing resource allocation can be generalized to provide rate-maximizing resource allocation schemes for time-division fading broadcast or multiple-access channels. These schemes complement rather nicely the available capacity-achieving policies in [11], [17] and [9].

A. Time-Division Broadcast Channel

In the time-division broadcast setting, rate maximization is sought under an average sum–power constraint $\mathbb{E}_{\boldsymbol{h}}\left[\sum_{k=1}^K p_k(\boldsymbol{h})\right] \leq \check{p}$. Given $\tau_k(\boldsymbol{h})$ and $p_k(\boldsymbol{h})$, Shannon's capacity formula dictates that the maximum achievable rate per user k is

$$\begin{aligned} r_k(\boldsymbol{h}) &:= R_k(\tau_k(\boldsymbol{h}), p_k(\boldsymbol{h})) \\ &= \begin{cases} \tau_k(\boldsymbol{h}) \log_2(1 + h_k p_k(\boldsymbol{h})/\tau_k(\boldsymbol{h})), & \tau_k(\boldsymbol{h}) > 0 \\ 0, & \tau_k(\boldsymbol{h}) = 0. \end{cases} \end{aligned} \tag{49}$$

With $\tau_k(\boldsymbol{h}) = 0$ and $p_k(\boldsymbol{h}) > 0$, (49) yields the asymptotic rate $r_k(\boldsymbol{h}) = \lim_{\tau_k(\boldsymbol{h}) \to 0} \tau_k(\boldsymbol{h}) \log_2(1 + h_k p_k(\boldsymbol{h})\ /\tau_k(\boldsymbol{h})) = 0$. On the other hand, $r_k(\boldsymbol{h}) = 0$, if $\tau_k(\boldsymbol{h}) = p_k(\boldsymbol{h}) = 0$.

With $\mathcal{F}(\check{p})$ denoting the set of all feasible time and power allocations satisfying the sum average power constraint, the capacity region is defined as [11] follows.

Definition 2: The capacity region for the time-division broadcast fading channel under a prescribed power $\check{p}$ and with transmit/receive ends having full CSI, is given by

$$\mathcal{C}(\check{p}) := \bigcup_{(\boldsymbol{\tau}(\cdot), \boldsymbol{p}(\cdot)) \in \mathcal{F}(\check{p})} \{\bar{\boldsymbol{r}} \mid 0 \leq \bar{r}_k \leq \mathbb{E}_{\boldsymbol{h}}\left[R_k(\tau_k(\boldsymbol{h}), p_k(\boldsymbol{h}))\right], \forall\, k\}$$

Analogous to Lemma 1, $\mathcal{C}(\check{p})$ is convex because the rate–power function $R_k(\tau_k(\boldsymbol{h}), p_k(\boldsymbol{h}))$ in (49) is jointly concave in $\tau_k(\boldsymbol{h})$ and $p_k(\boldsymbol{h})$. Each boundary point of $\mathcal{C}(\check{p})$ now maximizes a weighted sum of average rates; i.e., it solves the convex optimization problem

$$\max_{\bar{\boldsymbol{r}}} \boldsymbol{w}^T \bar{\boldsymbol{r}}, \quad \text{s. to } \bar{\boldsymbol{r}} \in \mathcal{C}(\check{p}). \tag{50}$$

The optimal $\bar{\boldsymbol{r}}^*$ in (50) can be determined by finding the optimal solution of

$$\begin{cases} \max_{\boldsymbol{\tau}(\cdot), \boldsymbol{p}(\cdot)} & \sum_{k=1}^K w_k \mathbb{E}_{\boldsymbol{h}}\left[R_k(\tau_k(\boldsymbol{h}), p_k(\boldsymbol{h}))\right] \\ \text{s. to} & \mathbb{E}_{\boldsymbol{h}}\left[\sum_{k=1}^K p_k(\boldsymbol{h})\right] \leq \check{p}, \\ & \sum_{k=1}^K \tau_k(\boldsymbol{h}) \leq 1, \forall\, \boldsymbol{h}. \end{cases} \tag{51}$$

With λ denoting the Lagrange multiplier corresponding to the sum–power constraint, we can in a dual fashion define a *net-reward* (rate reward minus power cost) function per user k as

$$\phi_k(\lambda, \tau_k(\boldsymbol{h}), p_k(\boldsymbol{h})) := w_k R_k(\boldsymbol{\tau}(\boldsymbol{h}), \boldsymbol{p}(\boldsymbol{h})) - \lambda p_k(\boldsymbol{h}). \tag{52}$$

The optimal capacity-achieving resource allocation for a fixed λ can then be obtained via a greedy water-filling approach as follows. With the whole time block assigned to user k, i.e., $\tau_k(\boldsymbol{h}) = 1$, optimal allocation comprises power water-filling across fading states

$$p_k^*(\boldsymbol{h}) = \left[\frac{w_k}{\lambda \ln 2} - \frac{1}{h_k}\right]^+. \tag{53}$$

Accordingly, we define the link quality indicator (the larger the better)

$$\phi_k^*(\lambda,\boldsymbol{h}) := \phi_k(\lambda,1,p_k^*(\boldsymbol{h})) = \begin{cases} \frac{w_k}{\ln 2}\ln\left(\frac{w_k h_k}{\lambda \ln 2}\right) - \frac{w_k}{\ln 2} + \frac{\lambda}{h_k}, & h_k > \frac{\lambda \ln 2}{w_k} \\ 0, & h_k \le \frac{\lambda \ln 2}{w_k} \end{cases} \quad (54)$$

which is the maximum net-reward user k can obtain. The optimal resource allocation then assigns the entire time block to the best user link with largest net-reward at $\boldsymbol{h}$; i.e.,

$$\begin{cases} \tau_k^*(\lambda,\boldsymbol{h}) = 1, & p_k^*(\lambda,\boldsymbol{h}) = \left[\frac{w_k}{\lambda \ln 2} - \frac{1}{h_k}\right]^+, k = k^*(\lambda,\boldsymbol{h}) \\ \tau_k^*(\lambda,\boldsymbol{h}) = 0, & p_k^*(\lambda,\boldsymbol{h}) = 0, k \ne k^*(\lambda,\boldsymbol{h}) \end{cases} \quad (55)$$

where the winner user index is given by

$$k^*(\lambda,\boldsymbol{h}) = \arg\max_k \ \phi_k^*(\lambda,\boldsymbol{h}) \quad (56)$$

Using the resource allocation policy in (55), the optimal power price λ^* can then be iteratively obtained using a subgradient iteration (indexed by i)

$$\lambda^{(i+1)} = \left[\lambda^{(i)} + \beta\left(\mathbb{E}_{\boldsymbol{h}}\left[p_{k^*}^*(\lambda^{(i)},\boldsymbol{h})\right] - \check{p}\right)\right]^+. \quad (57)$$

With sufficiently small β, convexity ensures that iterations in (57) will converge geometrically to λ^* from any nonnegative $\lambda^{(0)}$. Summarizing, we have the following.

Theorem 5: The (almost surely) optimal time and power allocation for (51) is given by $\boldsymbol{\tau}^*(\lambda^*,\boldsymbol{h})$ and $\boldsymbol{p}^*(\lambda^*,\boldsymbol{h})$ in (51), where the optimal λ^* is obtained using (55) and satisfies $\lambda^* > 0$ and $\mathbb{E}_{\boldsymbol{h}}\left[p_{k^*}^*(\lambda^*,\boldsymbol{h})\right] = \check{p}$.

Theorem 5 establishes that the capacity-achieving resource allocation for time-division broadcast transmissions should assign the entire block to the (almost surely) single user per $\boldsymbol{h}$. Notice that over deep fades, i.e., when $h_k \le \frac{\lambda^* \ln 2}{w_k}\ \forall\, k$, the "winner" user k^* will transmit with power

$$p_{k^*}^*(\lambda^*,\boldsymbol{h}) = \left[\frac{w_{k^*}}{\lambda \ln 2} - \frac{1}{h_{k^*}}\right]^+ = 0.$$

In this case, all users will do what is intuitively reasonable, namely, defer their transmissions.

B. Time-Division Multiple-Access Channel

In a TDMA setup, users transmit to the AP subject to individual average power constraints $\check{\boldsymbol{p}} := [\check{p}_1,\ldots,\check{p}_K]^T$; i.e., $\mathbb{E}_{\boldsymbol{h}}[p_k(\boldsymbol{h})] \le \check{p}_k$. With $\mathcal{F}(\check{\boldsymbol{p}})$ denoting the set of all feasible time and power allocation options satisfying these constraints, the achievable capacity region is

$$\mathcal{C}(\check{\boldsymbol{p}}) := \bigcup_{(\boldsymbol{\tau}(\cdot),\boldsymbol{p}(\cdot))\in\mathcal{F}(\check{\boldsymbol{p}})} \{\bar{\boldsymbol{r}} \mid 0 \le \bar{r}_k \le \mathbb{E}_{\boldsymbol{h}}[R_k(\tau_k(\boldsymbol{h}),p_k(\boldsymbol{h}))],\ \forall\, k\}.$$

Each boundary point of $\mathcal{C}(\check{\boldsymbol{p}})$ then maximizes

$$\max_{\bar{\boldsymbol{r}}} \boldsymbol{w}^T\bar{\boldsymbol{r}}, \quad \text{s. to } \bar{\boldsymbol{r}} \in \mathcal{C}(\check{\boldsymbol{p}}) \quad (58)$$

and finding the optimal $\bar{\boldsymbol{r}}^*$ amounts to solving

$$\begin{cases} \max_{\boldsymbol{\tau}(\cdot),\boldsymbol{p}(\cdot)} & \sum_{k=1}^K w_k \mathbb{E}_{\boldsymbol{h}}[R_k(\tau_k(\boldsymbol{h}),p_k(\boldsymbol{h}))] \\ \text{s. to} & \mathbb{E}_{\boldsymbol{h}}[p_k(\boldsymbol{h})] \le \check{p}_k,\ k=1,\ldots,K, \\ & \sum_{k=1}^K \tau_k(\boldsymbol{h}) \le 1,\ \forall\, \boldsymbol{h}. \end{cases} \quad (59)$$

With $\boldsymbol{\lambda} := [\lambda_1,\ldots,\lambda_K]^T$ denoting the Lagrange multipliers associated with the individual power constraints, the *net-reward* per user is now given by

$$\phi_k(\lambda_k,\tau_k(\boldsymbol{h}),p_k(\boldsymbol{h})) := w_k R_k(\boldsymbol{\tau}(\boldsymbol{h}),\boldsymbol{p}(\boldsymbol{h})) - \lambda_k p_k(\boldsymbol{h}). \quad (60)$$

Upon defining the link quality indicators

$$\phi_k^*(\lambda_k,\boldsymbol{h}) : \begin{cases} \frac{w_k}{\ln 2}\ln\left(\frac{w_k h_k}{\lambda_k \ln 2}\right) - \frac{w_k}{\ln 2} + \frac{\lambda_k}{h_k}, & h_k > \frac{\lambda_k \ln 2}{w_k} \\ 0, & h_k \le \frac{\lambda_k \ln 2}{w_k} \end{cases} \quad (61)$$

the optimal resource allocation then assigns

$$\begin{cases} \tau_k^*(\boldsymbol{\lambda},\boldsymbol{h}) = 1, & p_k^*(\boldsymbol{\lambda},\boldsymbol{h}) = \left[\frac{w_k}{\lambda_k \ln 2} - \frac{1}{h_k}\right]^+, k = k^*(\boldsymbol{\lambda},\boldsymbol{h}) \\ \tau_k^*(\boldsymbol{\lambda},\boldsymbol{h}) = 0, & p_k^*(\boldsymbol{\lambda},\boldsymbol{h}) = 0, k \ne k^*(\boldsymbol{\lambda},\boldsymbol{h}) \end{cases} \quad (62)$$

where the winner user index is

$$k^*(\boldsymbol{\lambda},\boldsymbol{h}) = \arg\max_k \phi_k^*(\lambda_k,\boldsymbol{h}). \quad (63)$$

The optimal $\boldsymbol{\lambda}^*$ is found with properly modified subgradient iterations

$$\lambda_k^{(i+1)} = \left[\lambda_k^{(i)} + \beta\left(\mathbb{E}_{\boldsymbol{h}}\left[p_{k^*}^*(\boldsymbol{\lambda}^{(i)},\boldsymbol{h})\right] - \check{p}_k\right)\right]^+,\ k=1,\ldots,K. \quad (64)$$

Summarizing, we have established the following.

Theorem 6: The (almost surely) optimal time and power allocation for (59) is given by $\boldsymbol{\tau}^*(\boldsymbol{\lambda}^*,\boldsymbol{h})$ and $\boldsymbol{p}^*(\boldsymbol{\lambda}^*,\boldsymbol{h})$ in (55), where the optimal $\boldsymbol{\lambda}^*$ is obtained using (64) and satisfies $\boldsymbol{\lambda}^* > \boldsymbol{0}$ and $\mathbb{E}_{\boldsymbol{h}}\left[p_k^*(\boldsymbol{\lambda}^*,\boldsymbol{h})\right] = \check{p}_k,\ k=1,\ldots,K$.

C. Comparison With Existing Results

For time-division fading broadcast channels, [11] and [17] have shown that the capacity-achieving resource allocation policies can be obtained by a "water-filling over concave envelopes" procedure. However, determining concave envelopes requires numerical solution of as many as $K(K-1)/2$ nonlinear equations per fading state $\boldsymbol{h}$, and is only computationally tractable when the number of users K is small. More importantly, optimality of the resource allocation schemes in [11], [17] has not been proved for the general case ($K > 2$). Theorem 5 provides a provably (almost surely) optimal time and power allocation solution implemented through a low-complexity greedy water-filling approach. Having obtained the optimal power price λ^*, this solution maximizes a net-reward per $\boldsymbol{h}$ by scheduling transmission to a single user k^*. Relying on convex optimization tools, a subgradient algorithm with linear complexity $\mathcal{O}(K)$ and fast convergence is also developed to determine λ^*. The novel greedy water-filling approach can be further generalized

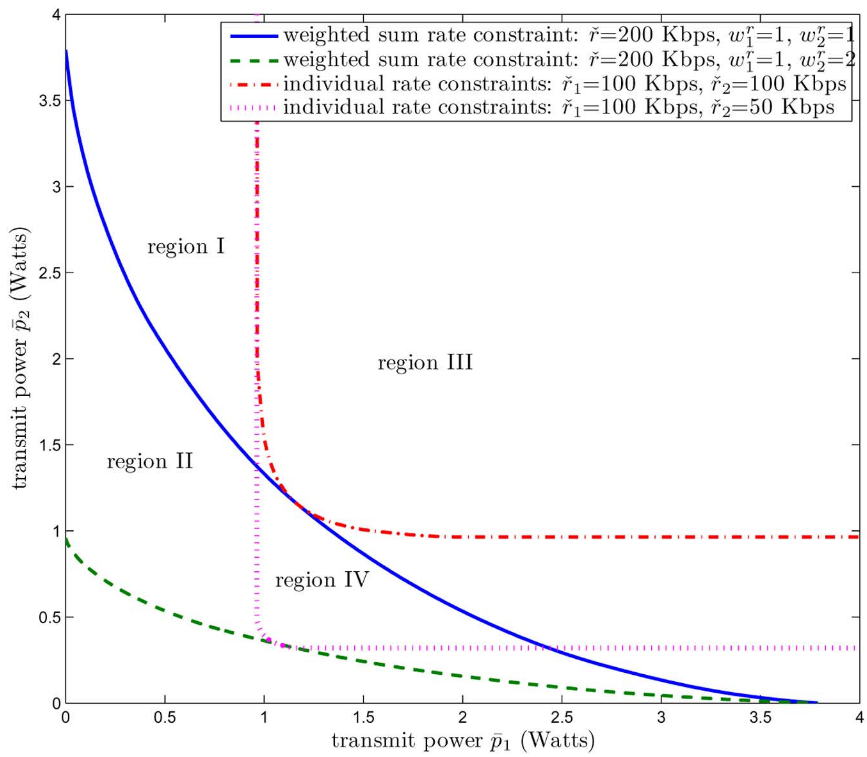

Fig. 3. Power regions for the infinite-size codebook case when two users have identical average normalized SNRs: $\bar{h}_1/(N_0B) = \bar{h}_2/(N_0B) = 0$ dBW.

to develop optimal resource allocation schemes for the fading TDMA channel.

Rate-maximizing resource allocation under individual power constraints for fading multiple-access channels was dealt with in [9], where it was shown that superposition coding and successive decoding should be employed to achieve capacity using a greedy algorithm based on a polymatroid structure. In the TDMA setting considered here, there is no need for such a polymatroid structure since Theorem 6 shows that a greedy water-filling based winner-takes-all policy is almost surely optimal. This is an important complement to the results for general multiple-access channels as many wireless standards typically rely on orthogonal access schemes for multiple-user communications.

In a nutshell, this paper has developed a unified framework providing the capacity-achieving resource allocation schemes for orthogonal (time-division as a special case) fading broadcast and multiple-access channels. In fact, based on greedy discrete-water-filling approaches dual to those given by Theorems 2 and 4, rate-maximizing resource allocation strategies can be also developed for adaptive transmissions relying on discrete AMC modes, a topic not considered in [11] and [9].

VI. Numerical Results

In this section, we present numerical tests of our power-efficient resource allocation for a two-user Rayleigh flat-fading TDMA channel. The available system bandwidth is $B = 100$ kHz , and the AWGN has two-sided power spectral density $N_0/2$ W/Hz . The user fading processes are independent and h_k, $k = 1, 2$, are generated from a Rayleigh distribution with variance $\bar{h}_k$. The average normalized SNR for user k is $\bar{h}_k/(N_0B)$. (The receive SNR is $\bar{h}_k/(N_0B)$ dBW multiplied by the transmit power p_k measured in Watts.)

Supposing $\bar{h}_k/(N_0B) = 0$ dBW for $k = 1, 2$, we test power-efficient resource allocation under a *weighted* sum average rate constraint $\mathbb{E}_{\boldsymbol{h}}\left[w_1^r r_1(\boldsymbol{h}) + w_2^r r_2(\boldsymbol{h})\right] \leq \check{r} = 200$ kbits/second (kbps) [cf. Remark 5], for two sets of weights: i) $w_1^r = 1$, $w_2^r = 1$, and ii) $w_1^r = 1$, $w_2^r = 2$; and for two sets of individual rate constraints: iii) $\check{r}_1 =$100 kbps, $\check{r}_2 = 100$ kbps, and iv) $\check{r}_1 =$100 kbps, $\check{r}_2 =$50 kbps. Fig. 3 depicts the power regions of the Rayleigh-fading TDMA channels for the infinite-size codebook case. It is seen that power regions I and III under the weighted sum rate constraint i) and under the individual rate constraints iii) are symmetric with respect to the line $\bar{p}_2 = \bar{p}_1$. Since the individual rate constraints can be seen as a realization of the weighted sum–rate constraint, i.e., $w_1^r\check{r}_1 + w_2^r\check{r}_2 = \check{r}$, the power region I contains power region III. It is clear that when $w_1 = w_2$, due to the symmetry in channel quality and rate-reward weights between the two users, the resultant optimal resource allocation under the weighted sum rate constraint should coincide with that under $\check{r}_1 = \check{r}_2$. For this reason, the two power regions touch each other in this case. Power regions II and IV under the weighted sum average rate constraint ii) and under individual rate constraints iv), are similarly related. They are not symmetric with respect to $\bar{p}_2 = \bar{p}_1$ because rate-reward weights or individual rate constraints are unequal. Power region II contains power region IV, and the two regions touch each other at one point.

To test our finite rate allocation schemes, we consider that each user supports three uncoded M-QAM modes: 4-QAM,

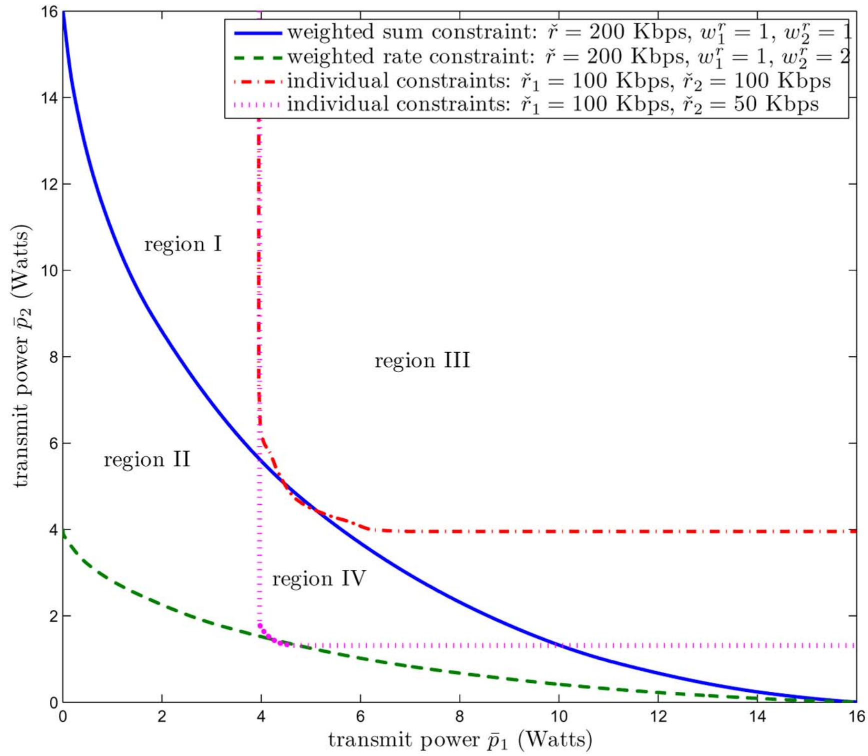

Fig. 4. Power regions for the finite rate set case when two users have identical average normalized SNRs: $\bar{h}_1/(N_0B) = \bar{h}_2/(N_0B) = 0$ dBW.

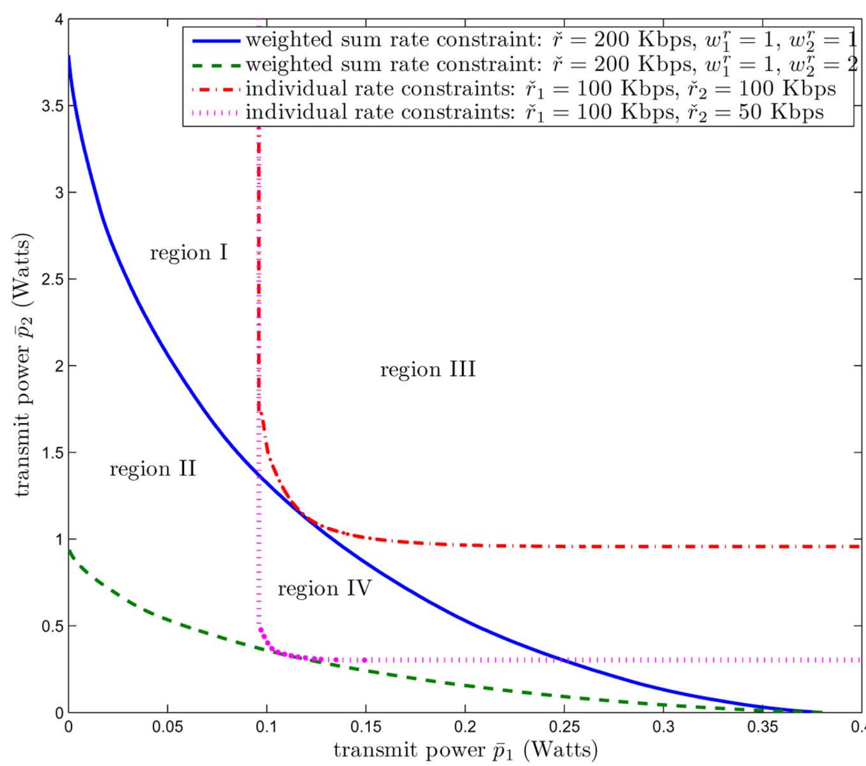

Fig. 5. Power regions for the infinite-size codebook case when two users have 10-dB difference in average normalized SNRs: $\bar{h}_1/(N_0B) = 10$ dBW, and $\bar{h}_2/(N_0B) = 0$ dBW.

16-QAM, and 64-QAM ($\rho_{k,m} = 2, 4,$ or 6). For these rectangular signal constellations, the BER is given by (16) with $\kappa_1 = 0.2$ and $\kappa_2 = 1.5$ [18], from which we can determine the rate–power pairs $\{(\rho_{k,m}, \pi_{k,m}(h_k)),\ m = 1, 2, 3\}$ for $k = 1, 2$.

The corresponding power regions I–IV under constraints i)–iv) for this discrete rate case with prescribed $BER = 10^{-3}$ are depicted in Fig. 4. Trends similar to those in Fig. 3 are observed. However, the power regions shrink since more power is required

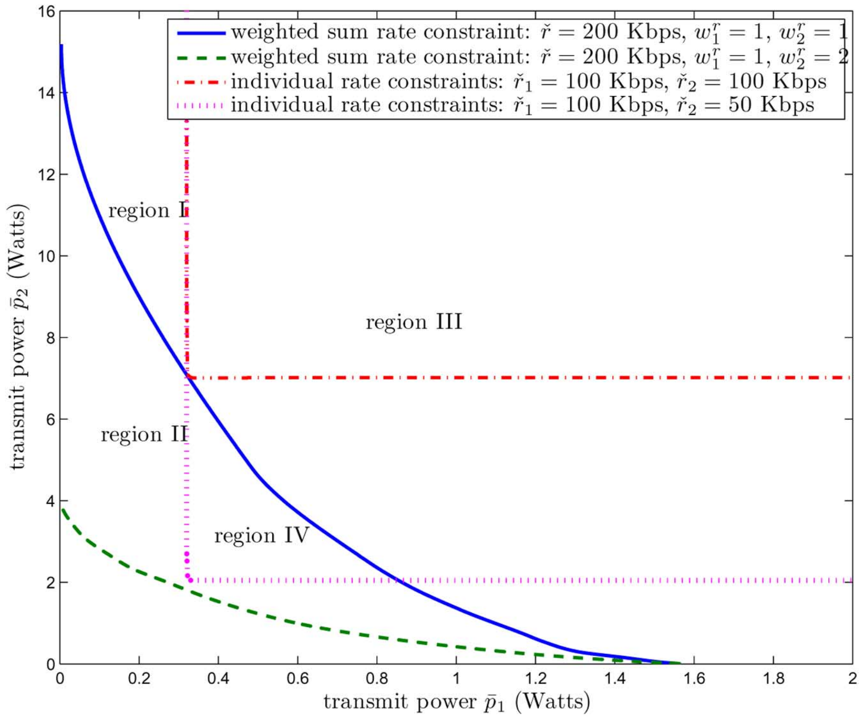

Fig. 6. Power regions for the finite rate set case when two users have 10–dB difference in average normalized SNRs: $\bar{h}_1/(N_0B) = 10$ dBW, and $\bar{h}_2/(N_0B) = 0$ dBW.

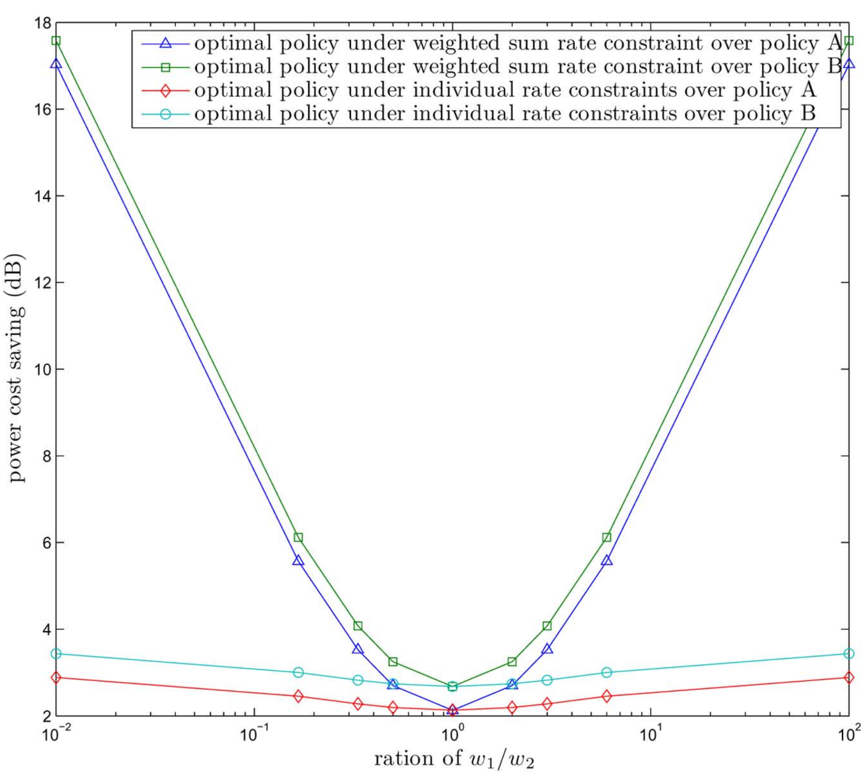

Fig. 7. Power savings for the infinite-size codebook case when two users have identical average normalized SNRs: $\bar{h}_1/(N_0B) = \bar{h}_2/(N_0B) = 0$ dBW. (Policy A: equal time allocation and separate water-filling; Policy B: equal time allocation among users and equal power per fading state for each user).

to achieve the same transmission rate with uncoded M-QAM relative to that using capacity-achieving codewords.

With $\bar{h}_1/(N_0B) = 10$ dBW and $\bar{h}_1/(N_0B) = 0$ dBW, we also test our power-efficient resource allocation under the same four sets of rate constraints i)–iv). The power regions for continuous rates and finite rates are plotted in Figs. 5 and 6, respectively. Since the first user has a considerably more reliable channel (i.e., higher average SNR) than user 2, the required average transmit-

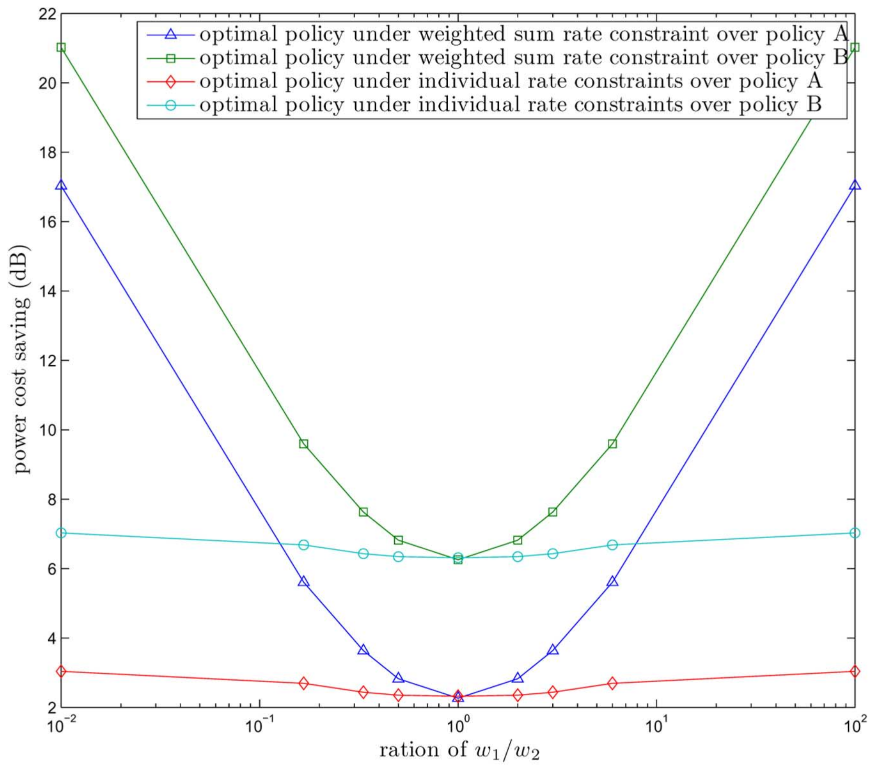

Fig. 8. Power savings for the finite rate set case when two users have identical average normalized SNRs: $\bar{h}_1/(N_0B) = \bar{h}_2/(N_0B) = 0$ dBW. (Policy A: equal time allocation and separate water-filling. Policy B: equal time allocation among users and equal power per fading state for each user).

power of user 1 is much lower than that of user 2 most of the time. Apart from this difference, the results are similar to those in Figs. 3 and 4.

We next compare this paper's optimal power-efficient resource allocation with two alternative suboptimal policies. Policy A assigns equal time fractions (i.e., $\tau_1(\boldsymbol{h}) = \tau_2(\boldsymbol{h}) = 1/2 \,\forall\, \boldsymbol{h}$) to the two users per block. Then each user terminal implements water-filling separately to adapt its transmission rate per assigned time fraction. In policy B, each user is assigned equal time fraction and transmits with equal power per block. Fig. 7 depicts the power savings of the optimal policies under two different sets of rate constraints i) and iii), over the policies A and B for the infinite-size codebook case when two users have identical average normalized SNRs. It is seen that when the ratio w_1/w_2 of two users' power prices is far away from 1, the optimal policies under a sum–rate constraint result in huge power savings (near 20 dB) over the other two suboptimal polices. However, in this case, the optimal policies under individual rate constraints exhibit a smaller advantage (around 3 dB) in power savings over the suboptimal policies. This is because with the sum average rate constraint, this paper's policies offer more flexible time and rate allocations From Fig. 7, we also observe that the separate water-filling of policy A only achieves small power savings (less than 1 dB) over the equal power strategy in policy B.

Fig. 8 depicts the same comparison with finite rates. The same trends are observed. However, in this case separate water-filling of policy A achieves considerable power savings (4 dB) over the equal-power policy B. Fig. 9 depicts similar power savings with continuous rates under two different sets of rate constraints ii) and iv), when two users have 10 dB in SNR difference. Note that the optimal policies under individual rate constraints achieve sizable power savings (near 9 dB), over the suboptimal policies.

VII. Concluding Remarks

Based on full transmit-CSI, we developed power-efficient resource allocation strategies for TDMA fading channels. For power minimization either under an average sum–rate constraint or under average individual rate constraints, the (almost surely) optimal allocation policies boil down to a low-complexity greedy water-filling approach. Analogous (in fact dual) approaches can be developed also for capacity-achieving resource allocation over time-division fading broadcast and multiple-access channels.

Interestingly, although arbitrary time sharing among users was allowed per time block at the outset, the optimal resource allocation turned out to comprise a winner-takes-all policy, which avoids the difficulty of implementing arbitrary time- sharing in practice, where time is usually divided with granularity of one time unit determined by the available bandwidth. Another interesting feature of the novel power-efficient resource allocation strategies is that the access point (which naturally has full CSI) is the one determining the time allocation and feeding it back to users. Then given the optimal scalar λ^* or vector Lagrange multiplier $\boldsymbol{\lambda}^*$, the terminals only need their own CSI to determine the optimum transmission rate. If uplink and downlink transmissions are operated in a time-division duplexing mode, the users can even obtain their own CSI without feedback from the access

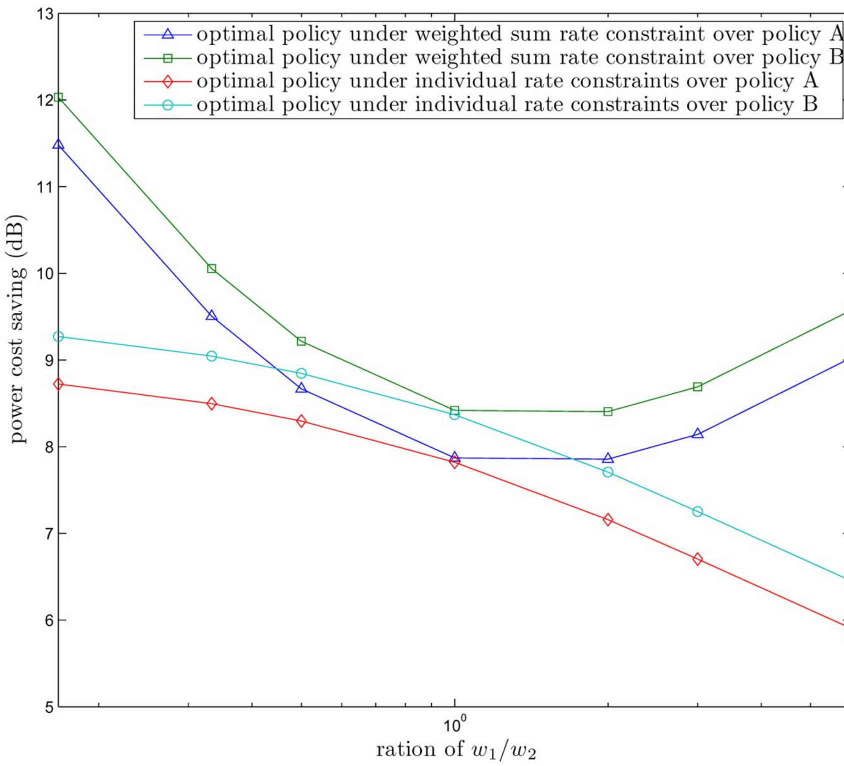

Fig. 9. Power savings for the infinite-size codebook case when two users have 10-dB difference in average normalized SNRs: $\bar{h}_1/(N_0B) = 10$ dBW, and $\bar{h}_2/(N_0B) = 0$ dBW. (Policy A: equal time allocation and separate water-filling. Policy B: equal time allocation among users and equal power per fading state for each user).

point since CSI at the transmitters can be acquired via training over the reciprocal uplink and downlink channels. Together with the fact that the access point needs only a few bits to indicate the time allocation (since the optimal policy is a winner-takes-all one), this feature is attractive from a practical implementation viewpoint.

As far as future research is concerned, it is interesting to study power minimization over fading channels with delay constraints and/or using quantized (instead of full) CSI. Delay-constrained power minimization can be possibly viewed as dual to the delay-limited capacity maximization in [10]. Results on quantized CSI with finite rate transceivers can be found in [22].[3]

APPENDIX

A. Proof of Lemma 1

To prove the convexity of $P_k(\tau, r)$ in (1), it suffices to show that for a convex combination $(\tau_c, r_c) = \theta(\tau_a, r_a) + (1-\theta)(\tau_b, r_b)$ with $\theta \in (0,1)$, we have $P_k(\tau_c, r_c) \leq \theta P_k(\tau_a, r_a) + (1-\theta)P_k(\tau_b, r_b)$. Consider the following three cases.

c1) $\tau_a > 0$ and $\tau_b > 0$: In this case, we have also $\tau_c > 0$ and $P_k(\tau, r) = (\tau/h_k)(2^{r/\tau} - 1)$. Because $(1/h_k)(2^r - 1)$ is a convex function of r, it follows that its *perspective* $(\tau/h_k)(2^{r/\tau} - 1)$ is also jointly convex in τ and r when $\tau > 0$ [15, Sec. 2.6]. Using the latter, it follows readily from (1) that $P_k(\tau_c, r_c) \leq \theta P_k(\tau_a, r_a) + (1-\theta)P_k(\tau_b, r_b)$.

c2 $\tau_a = 0$, $r_a > 0$, and $\tau_b > 0$: In this case, (1) yields $P_k(\tau_a, r_a) = \infty$; and thus

$$\theta P_k(\tau_a, r_a) + (1-\theta)P_k(\tau_b, r_b) = \infty \geq P_k(\tau_c, r_c).$$

c3) $\tau_a = 0, r_a = 0$ and $\tau_b > 0$: Since $(\tau_c, r_c) = ((1-\theta)\tau_b, (1-\theta)r_b)$ and $\tau_c > 0$, it follows that

$$\begin{aligned} P_k(\tau_c, r_c) &= (1-\theta)(\tau_b/h_k)(2^{r_b/\tau_b} - 1) \\ &= \theta P_k(\tau_a, r_a) + (1-\theta)P_k(\tau_b, r_b) \end{aligned}$$

because

$$P_k(\tau_a, r_a) = 0 \text{ and } P_k(\tau_b, r_b) = (\tau_b/h_k)(2^{r_b/\tau_b} - 1).$$

From c1)–c3), it clearly holds that $P_k(\tau_c, r_c) \leq \theta P_k(\tau_a, r_a) + (1-\theta)P_k(\tau_b, r_b)$ and thus P_k is convex.

To prove that $\mathcal{P}(\check{r})$ is convex, for two vectors $\bar{\boldsymbol{p}}_a$, $\bar{\boldsymbol{p}}_b \in \mathcal{P}$, we must have two allocation policies $(\boldsymbol{\tau}_a, \boldsymbol{r}_a)$, $(\boldsymbol{\tau}_b, \boldsymbol{r}_b) \in \mathcal{F}(\check{r})$ such that $\bar{p}_{a,k} \geq \mathbb{E}_{\boldsymbol{h}}[P_k(\boldsymbol{\tau}_a(\boldsymbol{h}), \boldsymbol{r}_a(\boldsymbol{h}))]$ and $\bar{p}_{b,k} \geq \mathbb{E}_{\boldsymbol{h}}[P_k(\boldsymbol{\tau}_b(\boldsymbol{h}), \boldsymbol{r}_b(\boldsymbol{h}))]$, $\forall\ k$. Now consider a convex combination $(\boldsymbol{\tau}_c, \boldsymbol{r}_c) := (\theta\boldsymbol{\tau}_a + (1-\theta)\boldsymbol{\tau}_b, \theta\boldsymbol{r}_a + (1-\theta)\boldsymbol{r}_b)$, where $\theta \in (0,1)$. It is easy to see that this new policy $(\boldsymbol{\tau}_c, \boldsymbol{r}_c) \in \mathcal{F}(\check{r})$.

[3]The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U. S. Government.

As $P_k(\tau, r)$ is a jointly convex function of $(\boldsymbol{\tau}, \boldsymbol{r})$, it holds that $P_k(\tau_{c,k}, r_{c,k}) \leq \theta P_k(\tau_{a,k}, r_{a,k}) + (1-\theta) P_k(\tau_{b,k}, r_{b,k})$. The latter implies that for a vector $\bar{\boldsymbol{p}}_c = \theta \bar{\boldsymbol{p}}_a + (1-\theta)\bar{\boldsymbol{p}}_b$, we have

$$\begin{aligned}\bar{p}_{c,k} &= \theta \bar{p}_{a,k} + (1-\theta)\bar{p}_{b,k} \\ &\geq \theta \mathbb{E}_{\boldsymbol{h}}\left[P_k(\tau_{a,k}(\boldsymbol{h}), r_{a,k}(\boldsymbol{h}))\right] \\ &\quad + (1-\theta)\mathbb{E}_{\boldsymbol{h}}\left[P_k(\tau_{b,k}(\boldsymbol{h}), r_{b,k}(\boldsymbol{h}))\right] \\ &\geq \mathbb{E}_{\boldsymbol{h}}\left[P_k(\tau_{c,k}(\boldsymbol{h}), r_{c,k}(\boldsymbol{h}))\right].\end{aligned}$$

Since $(\boldsymbol{\tau}_c, \boldsymbol{r}_c) \in \mathcal{F}(\check{r})$, we have any convex combination $\bar{\boldsymbol{p}}_c = \theta \bar{\boldsymbol{p}}_a + (1-\theta)\bar{\boldsymbol{p}}_b \in \mathcal{P}(\check{r})$ for any two vectors $\bar{\boldsymbol{p}}_a$, $\bar{\boldsymbol{p}}_b \in \mathcal{P}(\check{r})$. The convexity of $\mathcal{P}(\check{r})$ thus follows.

B. Proof of Lemma 2

To prove the lemma, we will need the following two properties.

Property 1: For any fixed λ, it holds that $\tau_k(\boldsymbol{h})\varphi_k^*(\lambda, \boldsymbol{h}) \leq \varphi_k(\lambda, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))$, $\forall\, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h})$.

Proof: Consider the following two cases:

c1) $\tau_k(\boldsymbol{h}) > 0$: From the definition of $P_k(\tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))$ in (1), the net-cost in (8) becomes $\varphi_k(\lambda, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))= (w_k \tau_k(\boldsymbol{h})/h_k)(2^{r_k(\boldsymbol{h})/\tau_k(\boldsymbol{h})} - 1) - \lambda r_k(\boldsymbol{h})$. Upon defining $\rho_k(\boldsymbol{h}) := r_k(\boldsymbol{h})/\tau_k(\boldsymbol{h})$, we have

$$\varphi_k(\lambda, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h})) = \tau_k(\boldsymbol{h})\varphi_k(\lambda, \rho_k(\boldsymbol{h}))$$

where

$$\varphi_k(\lambda, \rho_k(\boldsymbol{h})) := (w_k/h_k)(2^{\rho_k(\boldsymbol{h})} - 1) - \lambda \rho_k(\boldsymbol{h}).$$

It is easy to see that $\varphi_k(\lambda, \rho_k(\boldsymbol{h}))$ is a convex function for $\rho_k(\boldsymbol{h}) \geq 0$; and after equating its derivative w.r.t. $\rho_k(\boldsymbol{h})$ to zero, the optimal $\rho_k^*(\boldsymbol{h})$ minimizing $\varphi_k(\lambda, \rho_k(\boldsymbol{h}))$ is given by

$$\rho_k^*(\boldsymbol{h}) = \left[\log_2 \lambda - \log_2(w_k \ln 2/h_k)\right]^+.$$

Substituting $\rho_k^*(\boldsymbol{h})$ into $\varphi_k(\lambda, \rho_k(\boldsymbol{h}))$ yields the link quality indicator $\varphi_k^*(\lambda, \boldsymbol{h}) = \varphi_k(\lambda, \rho_k^*(\boldsymbol{h}))$. It then follows readily that $\tau_k(\boldsymbol{h})\varphi_k^*(\lambda, \boldsymbol{h}) \leq \varphi_k(\lambda, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))$.

c2) $\tau_k(\boldsymbol{h}) = 0$: In this case, we have $\tau_k(\boldsymbol{h})\varphi_k^*(\lambda, \boldsymbol{h}) = 0$. And for $\varphi_k(\lambda, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))$, it follows that [cf. (1)]: i) if $r_k(\boldsymbol{h}) = 0$, then $\varphi_k(\lambda, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h})) = 0$; and ii) if $r_k(\boldsymbol{h}) > 0$, then $\varphi_k(\lambda, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h})) = \infty$. But i) and ii) imply that $\tau_k(\boldsymbol{h})\varphi_k^*(\lambda, \boldsymbol{h}) \leq \varphi_k(\lambda, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h}))$.

The property clearly follows from c1) and c2). □

Property 2: For any fixed λ, it holds that $\varphi_k^*(\lambda, \boldsymbol{h}) \leq 0$, $\forall\, \boldsymbol{h}$.

Proof: By the definition of $\varphi_k^*(\lambda, \boldsymbol{h})$ in (10), it follows after differentiation that

$$\frac{\partial \varphi_k^*(\lambda, \boldsymbol{h})}{\partial h_k} = \begin{cases} \frac{1}{h_k}\left(\frac{w_k}{h_k} - \frac{\lambda}{\ln 2}\right), & h_k > \frac{w_k \ln 2}{\lambda} \\ 0, & h_k \leq \frac{w_k \ln 2}{\lambda} \end{cases} \tag{65}$$

and thus $\frac{\partial \varphi_k^*(\lambda, \boldsymbol{h})}{\partial h_k} < 0, \forall\, h_k > w_k \ln 2/\lambda$. Since $\varphi_k^*(\lambda, \boldsymbol{h})$ is a continuous function of h_k and $\varphi_k^*(\lambda, \boldsymbol{h}) = 0\, \forall\, h_k \leq w_k \ln 2/\lambda$, the property follows readily. □

We are now ready to prove Lemma 2 based on Properties 1 and 2. With the winner user index $k^*(\lambda, \boldsymbol{h})$ defined in (11), it holds that

$$\begin{aligned}\sum_{k=1}^{K} \varphi_k(\lambda, \tau_k(\boldsymbol{h}), r_k(\boldsymbol{h})) &\geq \sum_{k=1}^{K} \tau_k(\boldsymbol{h})\varphi_k^*(\lambda, \boldsymbol{h}) \\ &\geq \varphi_{k^*}^*(\lambda, \boldsymbol{h}) \left(\sum_{k=1}^{K} \tau_k(\boldsymbol{h})\right) \\ &\geq \varphi_{k^*}^*(\lambda, \boldsymbol{h})\end{aligned}$$

where the first inequality is due to Property 1, the second inequality is due to $\varphi_{k^*}^*(\lambda, \boldsymbol{h}) \leq \varphi_k^*(\lambda, \boldsymbol{h})\ \forall\, k$ from (11), and the last inequality holds because $\varphi_{k^*}^*(\lambda, \boldsymbol{h}) \leq 0$ from Property 2 and the time allocation constraint $\sum_{k=1}^{K} \tau_k(\boldsymbol{h}) \in [0, 1]$. Furthermore, the equality can be achieved using the allocation specified in (12); i.e., $\sum_{k=1}^{K} \varphi_k(\lambda, \tau_k^*(\boldsymbol{h}), r_k^*(\boldsymbol{h})) = \varphi_{k^*}^*(\lambda, \boldsymbol{h})$. It is then clear that (12) is optimal for (6), and the proof is complete.

C. Proof of Theorem 1

Since the primal problem (4) is a strictly feasible and convex optimization problem, it follows from the strong duality theorem [15, p. 226] that its solution coincides with the optimal $\boldsymbol{\tau}^*(\lambda^*, \boldsymbol{h})$ and $\boldsymbol{r}^*(\lambda^*, \boldsymbol{h})$ in Lemma 2, which solves (6) for $\lambda = \lambda^*$. Furthermore, the optimal λ^* should satisfy the complementary slackness condition [15, p. 243]: either $\lambda^* = 0$ and $\mathbb{E}_{\boldsymbol{h}}\left[r_{k^*}(\lambda^*, \boldsymbol{h})\right] > \check{r}$; or $\lambda^* > 0$ and $\mathbb{E}_{\boldsymbol{h}}\left[r_{k^*}(\lambda^*, \boldsymbol{h})\right] = \check{r}$. But if $\lambda^* = 0$, we have $r_{k^*}(\lambda^*, \boldsymbol{h}) = 0\ \forall\, \boldsymbol{h}$ from (12) and thus $\mathbb{E}_{\boldsymbol{h}}\left[r_{k^*}(\lambda^*, \boldsymbol{h})\right] = 0 < \check{r}$, which does not satisfy the average sum-rate constraint. Therefore, we must have $\lambda^* > 0$ and $\mathbb{E}_{\boldsymbol{h}}\left[r_{k^*}(\lambda^*, \boldsymbol{h})\right] = \check{r}$.

D. Proof of Lemma 3

With the winner user–mode pair $(k^*(\lambda, \boldsymbol{h}), m_{k^*}^*(\lambda, \boldsymbol{h}))$ defined in (27), we first note that

$$\varphi_{k^*, m_{k^*}^*}(\lambda, \boldsymbol{h}) \leq \varphi_{k,0}(\lambda, \boldsymbol{h}) = 0. \tag{66}$$

Subsequently, we have

$$\begin{aligned}&\sum_{k=1}^{K} \sum_{m=0}^{M_k} \tau_{k,m}(\boldsymbol{h})\varphi_{k,m}(\lambda, \boldsymbol{h}) \\ &\geq \varphi_{k^*, m_{k^*}^*}(\lambda, \boldsymbol{h}) \sum_{k=1}^{K} \sum_{m=0}^{M_k} \tau_{k,m}(\boldsymbol{h}) \\ &\geq \varphi_{k^*, m_{k^*}^*}(\lambda, \boldsymbol{h})\end{aligned}$$

where the first inequality is due to the definition of $(k^*(\lambda, \boldsymbol{h}), m_{k^*}^*(\lambda, \boldsymbol{h}))$, and the second inequality is due to (66) and $\sum_{k=1}^{K} \sum_{m=0}^{M_k} \tau_{k,m}(\boldsymbol{h}) \leq 1$. Furthermore, the equality is achieved using the allocation in (26), which thus minimizes $\sum_{k=1}^{K} \sum_{m=0}^{M_k} \tau_{k,m}(\boldsymbol{h})\varphi_{k,m}(\lambda, \boldsymbol{h})$ per $\boldsymbol{h}$ and is in turn optimal for (23).

$$
\begin{aligned}
\mathbb{E}_{\boldsymbol{h}}\left[r_k^*(\boldsymbol{\lambda}^*,\boldsymbol{h})\right] &= \mathbb{E}_{\boldsymbol{h}}\left[\mathbb{1}_{\{\varphi_i^*(\lambda_i^*,\boldsymbol{h})>\varphi_k^*(\lambda_k^*,\boldsymbol{h}),\ \forall\, i\}}\left[\log_2\lambda-\log_2(w_k\ln 2/h_k)\right]^+\right] \\
&= \int_{\frac{w_k\ln 2}{\lambda_k^*}}^{\infty}\log_2\left(\frac{\lambda_k^* h_k}{w_k\ln 2}\right)\prod_{i\neq k}F_i(\varphi_i^{-1}(\varphi_k(h_k)))dF_k(h_k) \qquad (67)\\
\bar{p}_k^* &= \mathbb{E}_{\boldsymbol{h}}\left[\frac{1}{h_k}(2^{[\log_2\lambda-\log_2(w_k\ln 2/h_k)]^+}-1)\mathbb{1}_{\{\varphi_i^*(\lambda_i^*,\boldsymbol{h})>\varphi_k^*(\lambda_k^*,\boldsymbol{h}),\ \forall i\}}\right] \\
&= \int_{\frac{w_k\ln 2}{\lambda_k^*}}^{\infty}\left(\frac{\lambda_k^*}{w_k\ln 2}-\frac{1}{h_k}\right)\prod_{i\neq k}F_i(\varphi_i^{-1}(\varphi_k(h_k)))dF_k(h_k). \qquad (68)
\end{aligned}
$$

E. Proof of Corollary 1

Let $\mathbb{1}_{\{\cdot\}}$ denote the indicator function ($\mathbb{1}_{\{x\}}=1$ if x is true and zero otherwise). Since $\frac{\partial\varphi_k(\lambda_k^*,\boldsymbol{h})}{\partial h_k}\leq 0$ from (65) in Appendix C, we have (67) and (68) (shown at the top of the page) [cf. Theorem 3]; and the corollary follows readily.

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