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Fair Energy-Efficient Network Design for Multihop Communications

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***Abstract*—We consider the energy-efficient network resource allocation that minimizes a cost function of average user powers for multi-hop wireless networks. A class of fair cost functions is derived to balance the tradeoff between efficiency and fairness in energy-efficient designs. Based on such cost functions, optimal routing, scheduling and power control schemes are developed. Relying on stochastic optimization tools, we further develop stochastic network resource allocation schemes which are capable of dynamically learning the traffic and channel statistics, and converging to the optimal strategy on-the-fly.**

Keywords: Fair energy efficiency, network resource allocation, stochastic optimization.

I. Introduction

In wireless networks, mobile devices are typically powered by tiny batteries that cannot be recharged in a convenient way. For this reason, energy efficiency has emerged as a critical issue and energy-efficient network designs have been extensively pursued in [1], [2]. A major motivation for energy-efficient design is to maximize the network operating lifetime. To this end, fair energy consumptions among user nodes are also important. This was partially addressed in [3], [4], [5], where network designs were put forth to maximize the length of time until the first node drains out its battery.

Fairness is defined in different ways in the literature. Recently the so-called α-fairness has received growing attention, and has proven success in development of the network utility maximization (NUM) paradigm [6]. It was shown that these α-fair utility functions $U_\alpha(\cdot)$ originated in [7] can nicely balance network throughput and fairness; but simple transformation of α-fair utility functions *cannot* yield a desirable cost function for fair energy-efficient network design considered here. Inspired by the design principles of $U_\alpha(\cdot)$, we propose that a new class of functions $V_\beta(\cdot) = (1+\beta)^{-1}(\cdot)^{1+\beta}$ may serve as what we call the "β-fair" cost functions. It is shown that minimizing a β-fair cost function of average powers can balance energy efficiency and fairness. Using such a cost function with $\beta = 0$, we have a sum-power minimization problem which seeks the most energy-efficient (even if unfair) resource allocation, as in [1], [2]. With a larger β, the resultant resource allocation becomes fairer; and with $\beta \to \infty$, the solution approaches the min-max allocation which is sometimes deemed the fairest in multi-hop networking [8], and corresponds to the solutions in [3], [4], [5].

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Based on the β-fair cost functions, we then explore energy-efficient routing, scheduling and power control for multi-hop wireless networks. For such networks, we establish that the optimal routing and scheduling follow a (queue) backpressure and maximum weight matching principle originated in [9], [10]. To illustrate the related power control, we consider a small or medium-scaled network for which time-division multiple-access (TDMA) is a simple and efficient scheme to eliminate interference [2]. It is proved that in this case the optimal power control can be derived through a greedy water-filling approach. When the network traffic statistics and fading channel cumulative distribution function (cdf) are known *a priori*, we show that the optimal routing, scheduling and power control scheme can be obtained using fast-convergent gradient iterations. In addition, stochastic approximation is employed to develop on-line routing, scheduling and power control algorithm which can dynamically learn the underlying traffic and channel statistics and converge to the optimal benchmark.

The rest of the paper is organized as follows. Section II specifies the network model and the problem of interest. Section III describes the β-fair cost functions. Sections IV derives the optimal routing, scheduling and power control scheme for TDMA multi-hop wireless networks. Section V develops the stochastic network resource allocation capable of dynamically learning the traffic and channel statistics to converge to the optimal strategy. Numerical results are provided in Section VI, followed by conclusions.

II. Network Model and Problem Formulation

Consider a multi-hop wireless network comprising J nodes $\{N_i\}_{i=1}^J$. Node N_i wishes to deliver packets for data flows indexed by k. For every flow k, packet arrives at node N_i according to an ergodic process with average rate $\bar{a}_i^k$. In this network, nodes rely on multi-hop transmissions to deliver packets to the intended destinations. To this end, N_i can select an average rate $\bar{r}_{ij}^k$ for transmitting the kth flow's packets to its neighboring node N_j. Let $\mathcal{N}_i$ denote the set of neighbors of N_i. To support the arrival rate $\bar{a}_i^k$, a flow conservation equation should be satisfied such that the total input rate $\bar{a}_i^k + \sum_{j\in\mathcal{N}_i} \bar{r}_{ji}^k$ is not greater than the scheduled total output rate $\sum_{j\in\mathcal{N}_i} \bar{r}_{ij}^k$, i.e., $\bar{a}_i^k \le \sum_{j\in\mathcal{N}_i}(\bar{r}_{ij}^k - \bar{r}_{ji}^k)$. The flow conservation equation implies that the average packet arrival rates lie within the "network capacity" region. As the capacity region of wireless network grows with the power consumptions, the energy-efficient design here is to obtain the minimum power cost that

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ensures the latter condition.

For illustration, we assume a simple small- or medium-scale network, for which TDMA is a simple and efficient scheme to eliminate interference [2]. In this TDMA network, data packet transmissions over links $N_i \to N_j$ are slot based. Without loss of generality (w.l.o.g.), we suppose that the available bandwidth is $B = 1$ and the additive white Gaussian noise at every receiver has unit variance. All the wireless links $N_i \to N_j$ are subject to random fading. Let $\boldsymbol{\gamma} := \{\gamma_{ij},\ \forall ij\}$ collect all the fading channel gains and thus the normalized receive signal-to-noise ratios (SNRs), which remain invariant over coherent time slots but can vary across successive slots according to a stationary and ergodic random process with a continuous joint cdf. At the outset, we allow each slot to be shared by links over non-overlapping time factions τ_{ij}, $\forall ij$. Supposing w.l.o.g. that each slot has unit duration, these non-negative fractions τ_{ij} must clearly satisfy $\sum_{ij} \tau_{ij} \le 1$. Given τ_{ij} and transmit-power p_{ij}, the maximum instantaneous rate (Shannon capacity) of the link ij is

$$c_{ij}(\tau_{ij}, p_{ij}) = \begin{cases} \tau_{ij} \log_2(1 + \gamma_{ij} p_{ij}/\tau_{ij}), & \tau_{ij} > 0, \\ 0, & \tau_{ij} = 0. \end{cases} \tag{1}$$

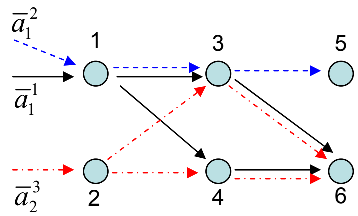
Note that since the link ij is only active during τ_{ij} faction of time, the transmit-power for its active period is p_{ij}/τ_{ij} for $\tau_{ij} > 0$. It will be also shown in the sequel that although we allow time-sharing among links at the outset, the optimal policy schedules almost surely a single link for transmission per slot. This could then facilitate the distributed implementation of the proposed scheme.

Consider now the average rates $\bar{r}_{ij}^k$ of all flows k traversing the link $N_i \to N_j$, for which the ergodic information capacity $\mathbb{E}_{\boldsymbol{\gamma}}\left[c_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}))\right]$ is determined by (1) for the given time and power allocation policy $\{\tau_{ij}(\boldsymbol{\gamma}),\ p_{ij}(\boldsymbol{\gamma}),\ \forall ij,\ \forall \boldsymbol{\gamma}\}$. Since the information capacity dictates the achievable rate limit, clearly the average rates $\bar{r}_{ij}^k$ must satisfy: $\sum_k \bar{r}_{ij}^k \le \mathbb{E}_{\boldsymbol{\gamma}}\left[c_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}))\right]$. Upon selecting a cost function $V(\bar{p}_i)$ for the average power $\bar{p}_i \ge \mathbb{E}_{\boldsymbol{\gamma}}[\sum_j p_{ij}(\boldsymbol{\gamma})]$ spent by node N_i, our energy-efficient design is then to minimize the total power cost by solving the following problem:

$$\begin{aligned} \min_{\boldsymbol{X} \in \mathcal{B}} \ & \sum_i V(\bar{p}_i) \\ \text{s. to} \ & \bar{p}_i \ge \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{j \in \mathcal{N}_i} p_{ij}(\boldsymbol{\gamma})\right], \quad \forall i \\ & \bar{a}_i^k \le \sum_{j \in \mathcal{N}_i} (\bar{r}_{ij}^k - \bar{r}_{ji}^k), \quad \forall i, k \\ & \sum_k \bar{r}_{ij}^k \le \mathbb{E}_{\boldsymbol{\gamma}}\left[c_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}))\right], \quad \forall i, j \end{aligned} \tag{2}$$

where $\boldsymbol{X} := \{\bar{p}_i,\ \bar{r}_{ij}^k,\ \tau_{ij}(\boldsymbol{\gamma}),\ p_{ij}(\boldsymbol{\gamma}),\ \forall i, j, k\}$ collects the optimization variables, and the set

$$\begin{aligned} \mathcal{B} := \big\{ & \bar{p}_i,\ \bar{r}_{ij}^k,\ \tau_{ij}(\boldsymbol{\gamma}),\ p_{ij}(\boldsymbol{\gamma}) \mid \bar{p}_i \ge 0,\ \bar{r}_{ij}^k \ge 0, \\ & \tau_{ij}(\boldsymbol{\gamma}) \ge 0,\ p_{ij}(\boldsymbol{\gamma}) \ge 0,\ \textstyle\sum_{ij} \tau_{ij}(\boldsymbol{\gamma}) \le 1 \big\} \end{aligned}$$



$$\begin{aligned} & \min \textstyle\sum_{i=1}^4 V(\bar{p}_i) \\ \text{s. to} \ & \bar{p}_1 \ge \mathrm{E}_\gamma[p_{13}(\boldsymbol{\gamma}) + p_{14}(\boldsymbol{\gamma})], \quad \bar{p}_2 \ge \mathrm{E}_\gamma[p_{23}(\boldsymbol{\gamma}) + p_{24}(\boldsymbol{\gamma})], \\ & \bar{p}_3 \ge \mathrm{E}_\gamma[p_{35}(\boldsymbol{\gamma}) + p_{36}(\boldsymbol{\gamma})], \quad \bar{p}_4 \ge \mathrm{E}_\gamma[p_{46}(\boldsymbol{\gamma})], \\ & \bar{a}_1^1 \le \bar{r}_{13}^1 + \bar{r}_{14}^1, \quad 0 \le \bar{r}_{36}^1 - \bar{r}_{13}^1, \quad 0 \le \bar{r}_{46}^1 - \bar{r}_{14}^1, \\ & \bar{a}_1^2 \le \bar{r}_{13}^2, \qquad 0 \le \bar{r}_{35}^2 - \bar{r}_{13}^2, \\ & \bar{a}_2^3 \le \bar{r}_{23}^3 + \bar{r}_{24}^3, \quad 0 \le \bar{r}_{36}^3 - \bar{r}_{23}^3, \quad 0 \le \bar{r}_{46}^3 - \bar{r}_{34}^3, \\ & \bar{r}_{13}^1 + \bar{r}_{13}^2 \le \mathrm{E}_\gamma[c_{13}(\boldsymbol{\tau}(\boldsymbol{\gamma}), \mathbf{p}(\boldsymbol{\gamma}))], \quad \bar{r}_{14}^1 \le \mathrm{E}_\gamma[c_{14}(\boldsymbol{\tau}(\boldsymbol{\gamma}), \mathbf{p}(\boldsymbol{\gamma}))], \\ & \bar{r}_{23}^3 \le \mathrm{E}_\gamma[c_{23}(\boldsymbol{\tau}(\boldsymbol{\gamma}), \mathbf{p}(\boldsymbol{\gamma}))], \quad \bar{r}_{24}^3 \le \mathrm{E}_\gamma[c_{24}(\boldsymbol{\tau}(\boldsymbol{\gamma}), \mathbf{p}(\boldsymbol{\gamma}))], \\ & \bar{r}_{35}^2 \le \mathrm{E}_\gamma[c_{35}(\boldsymbol{\tau}(\boldsymbol{\gamma}), \mathbf{p}(\boldsymbol{\gamma}))], \quad \bar{r}_{36}^1 + \bar{r}_{36}^3 \le \mathrm{E}_\gamma[c_{36}(\boldsymbol{\tau}(\boldsymbol{\gamma}), \mathbf{p}(\boldsymbol{\gamma}))], \\ & \bar{r}_{46}^1 + \bar{r}_{46}^3 \le \mathrm{E}_\gamma[c_{46}(\boldsymbol{\tau}(\boldsymbol{\gamma}), \mathbf{p}(\boldsymbol{\gamma}))] \end{aligned}$$

Fig. 1. A network with six nodes and three active data flows.

denotes the feasible set for $\boldsymbol{X}$.

To illustrate, consider a network with six nodes and three active data flows in Fig. 1. For this example, the specific form of problem (2) is also presented in Fig. 1. Notice that the vector $\boldsymbol{X}$ does not need to contain all the possible $\bar{p}_i$, $\bar{r}_{ij}^k$, $\tau_{ij}(\boldsymbol{\gamma})$ and $p_{ij}(\boldsymbol{\gamma})$, but needs only the ones that appear for the given topology and/or routing table.

Considering a simple TDMA network facilitates development of the optimal power control scheme. However, it is worth mentioning that the proposed approach applies to general network setting. For a large multi-hop network, TDMA becomes inefficient and frequency reuse must be taken into account. For such a network, the general principle of the proposed routing, scheduling and power control scheme applies. However, the optimal power control policy is difficult to derive since interference among transmissions over wireless links need to be considered.

III. Fair Energy Efficiency

In our formulation (2), selection of cost function V will play an important role. To entail efficient solvers, we would like (2) to be a convex optimization problem, for which a necessary condition is that $V(\bar{p}_i)$ is a *convex* function of $\bar{p}_i$. This function must be also an *increasing* function since a higher power expenditure naturally incurs a higher cost. A major motivation for energy-efficient network design is to maximize the network lifetime, for which it is important to fairly distribute power consumptions among the nodes. Hence the desirable cost functions, if possible, should also facilitate a *fair* energy-efficient design.

The notion of fairness characterizes how competing user nodes/links share (usually limited) resources. In the literature fairness has been defined in different ways. Recently, the so-called α-fairness has received growing attention, and has proven success in development of the well-known NUM paradigm. Specifically, it was shown that a maximizer of the so-called α-fair utility functions satisfies the definition of α-fair allocation. Here α-fair utility function refers to a family of functions parameterized by $\alpha \geq 0$ [7]:

$$U_\alpha(\cdot) = \begin{cases} (\cdot)^{1-\alpha}/(1-\alpha), & \text{for } \alpha \neq 1, \\ \log(\cdot), & \text{for } \alpha = 1. \end{cases} \quad (3)$$

A feasible allocation $\boldsymbol{x} := [x_1, \ldots, x_J]^T$ is called α-fair if, for any other feasible allocation $\boldsymbol{y} := [y_1, \ldots, y_J]^T$, we have $\sum_{j=1}^{J} (y_j - x_j)/(x_j)^\alpha \leq 0$. The notion of α-fairness includes max-min fairness (with $\alpha \to \infty$) [8], proportional fairness (with $\alpha = 1$) [11], and throughput maximization (with $\alpha = 0$) as special cases. The larger α means more fairness. The α-fair utility function $U_\alpha(x)$ in (3) is a *concave and increasing* function. Using $U_\alpha(x)$ to quantify the benefit of user rates, we can formulate a convex utility maximization problem to nicely balance overall throughput and fairness [7]. However, simple transformation of these α-fair utility functions *cannot* yield the desirable cost functions[1].

For the latter purpose, we follow the design principles of $U_\alpha(x)$ to propose a new class of *β-fair cost* functions parameterized by $\beta \geq 0$:

$$V_\beta(\cdot) = (\cdot)^{1+\beta}/(1+\beta). \quad (4)$$

Different from $U_\alpha(\cdot)$, $V_\beta(\cdot)$ is a *convex* (instead of concave) and increasing function. To define the β-fairness, consider minimizing the total cost for e.g., a power allocation $\boldsymbol{p} := [p_1, \ldots, p_J]^T$:

$$\min_{\boldsymbol{p}} \sum_{j=1}^{J} V_\beta(p_j), \quad \text{s. to} \quad \boldsymbol{p} \in \mathcal{P} \quad (5)$$

where $\mathcal{P}$ denotes the convex, closed and compact feasible set for $\boldsymbol{p}$, e.g, the one specified by the constraints in (2). Since $V_\beta(p_j)$ is a convex function of p_j, the problem (5) is a convex optimization problem. With the first derivative $V'_\beta(p) = p^\beta$, the optimal vector $\boldsymbol{p}^*$ for (5) must hold for any other feasible vector $\boldsymbol{p}$ that [12]:

$$\sum_{j=1}^{J} (p_j^*)^\beta (p_j - p_j^*) \geq 0, \quad \forall \boldsymbol{p} \in \mathcal{P}. \quad (6)$$

Based on the latter, we can then define the β-fairness as:

Defintion 1: *A feasible vector $\boldsymbol{p}^*$ is β-fair if it satisfies (6).*

It is clear that with $\beta = 0$ (i.e., $V_0(p_j) = p_j$), the problem (5) becomes the sum-power minimization problem which seeks the most energy-efficient (even if unfair) resource allocation, as in [1], [2]. When $\beta > 0$, the "marginal cost gain" for $V_\beta(p_j)$ is $V'_\beta(p_j) = (p_j)^\beta$, which increases as p_j increases. Therefore, in order to minimize the total cost, the node having spent less power will be more likely scheduled, so that the β-fairness is achieved.

Since the marginal cost gain $V'_\beta(p_j) = (p_j)^\beta$ increases as β increases, it implies that with a larger β, the resultant resource allocation for (5) becomes fairer. As β becomes arbitrarily large, we show in Appendix A that:

Proposition 1: *The β-fair vector approaches the min-max fair vector as $\beta \to \infty$.*

Proposition 1 establishes that as $\beta \to \infty$, the solution for (5) approaches the min-max allocation, which is sometimes deemed the fairest in multi-hop networking [8], and correspond to the solutions in [3], [4], [5].

In summary, the class of convex and increasing β-fair functions parameterized by $\beta \geq 0$ can balance overall costs with fairness. Hence, they can form a foundation for fair energy-efficient network designs pursued in this paper.

IV. Optimal Routing, Scheduling and Power Control

Using $V_\beta(\cdot)$ as the cost function, we next investigate the optimal routing, scheduling and power control scheme for the problem of interest (2).

For a TDMA channel, it has been established in our recent work [13] that $c_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}))$ is a jointly concave function in $\tau_{ij}(\boldsymbol{\gamma})$ and $p_{ij}(\boldsymbol{\gamma})$. Together with the fact that $V_\beta(\bar{p}_i)$ is a convex function of $\bar{p}_i$, it readily follows that (2) is a convex optimization problem which can be efficiently solved using a Lagrange dual based approach. Let $\boldsymbol{\lambda} := \{\lambda_i, \ \forall i\}$ collect the Lagrange multipliers associated with the constraints $\bar{p}_i \geq \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_j p_{ij}(\boldsymbol{\gamma})\right]$, and $\boldsymbol{\mu} := \{\mu_i^k, \ \forall i, k\}$ collect the multipliers for $\bar{a}_i^k \leq \sum_j (\bar{r}_{ij}^k - \bar{r}_{ji}^k)$. Then with the convenient notation $\boldsymbol{\Lambda} := \{\boldsymbol{\lambda}, \boldsymbol{\mu}\}$, the (partial) Lagrangian function of (2) is

$$\begin{aligned} L(\boldsymbol{X}, \boldsymbol{\Lambda}) &= \sum_i V_\beta(\bar{p}_i) + \sum_i \lambda_i \left(\mathbb{E}_{\boldsymbol{\gamma}} \left[\sum_j p_{ij}(\boldsymbol{\gamma}) \right] - \bar{p}_i \right) \\ &\quad + \sum_{ik} \mu_i^k \left(\bar{a}_i^k - \sum_j (\bar{r}_{ij}^k - \bar{r}_{ji}^k) \right) \\ &= \sum_{ik} \mu_i^k \bar{a}_i^k + \sum_i \left(V_\beta(\bar{p}_i) - \lambda_i \bar{p}_i \right) \\ &\quad + \sum_{ij} \left(\lambda_i \mathbb{E}_{\boldsymbol{\gamma}} \left[p_{ij}(\boldsymbol{\gamma}) \right] - \sum_k (\mu_i^k - \mu_j^k) \bar{r}_{ij}^k \right). \end{aligned} \quad (7)$$

The dual function is then given by

$$\begin{aligned} D(\boldsymbol{\Lambda}) = \min_{\boldsymbol{X} \in \mathcal{B}} \ & L(\boldsymbol{X}, \boldsymbol{\Lambda}) \\ \text{s. to} \ & \sum_k \bar{r}_{ij}^k \leq \mathbb{E}_{\boldsymbol{\gamma}} \left[c_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma})) \right], \ \forall i, j \end{aligned} \quad (8)$$

and the dual problem of (2) is

$$\max_{\boldsymbol{\Lambda} \geq 0} \ D(\boldsymbol{\Lambda}). \quad (9)$$

[1] For instance, $-U_\alpha(x)$ is a convex but *decreasing* function, and is thus *not* a valid cost function. To see it, consider (2) with $V(\bar{p}_i) = -U_\alpha(\bar{p}_i) = -\bar{p}_i$ for $\alpha = 0$. Then here we have an objective $\min \sum_i (-\bar{p}_i) \equiv \max \sum_i \bar{p}_i$, which is not meaningful.

Due to the convexity of (2), there is no duality gap between (2) and its dual problem (9). Therefore, the solution of (2) can be obtained by solving (9) [12].

To this end, we need to first solve the problem in (8). For a given $\mathbf{\Lambda}$, it is clear from (7) that the optimal $\bar{p}_i(\mathbf{\Lambda})$ can be obtained by solving $\forall i$,

$$\min_{\bar{p}_i \geq 0} \left(V_\beta(\bar{p}_i) - \lambda_i \bar{p}_i\right). \tag{10}$$

From the definition of V_β, the solution to (10) is

$$\bar{p}_i(\mathbf{\Lambda}) = (\lambda_i)^{1/\beta}. \tag{11}$$

To find the optimal $\bar{r}_{ij}^k(\mathbf{\Lambda})$, $\tau_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda})$ and $p_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda})$ for (8), we need to solve [cf. (7)]

$$\begin{aligned} \min_{\bar{r}_{ij}^k,\, \tau_{ij}(\boldsymbol{\gamma}),\, p_{ij}(\boldsymbol{\gamma})} \quad & \sum_{ij} \left(\lambda_i \mathbb{E}_{\boldsymbol{\gamma}}\left[p_{ij}(\boldsymbol{\gamma})\right] - \sum_k (\mu_i^k - \mu_j^k)\bar{r}_{ij}^k\right) \\ \text{s. to} \quad & \sum_k \bar{r}_{ij}^k \leq \mathbb{E}_{\boldsymbol{\gamma}}\left[c_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}))\right], \quad \forall i,j \end{aligned} \tag{12}$$

Following the (queue) backpressure and maximum weight matching principle in [9], [10], it can be shown that the optimal scheduling for (12) is to pick a flow $k_{ij}(\mathbf{\Lambda})$ with the *largest differential backlog* $\mu_i^k - \mu_j^k$ to be routed at link ij; i.e.,

$$k_{ij}(\mathbf{\Lambda}) = \arg\max_k \, (\mu_i^k - \mu_j^k). \tag{13}$$

With such a schedule, it is also clear the optimal time and power allocation for (12) should ensure

$$\sum_k \bar{r}_{ij}^k(\mathbf{\Lambda}) = \bar{r}_{ij}^{k_{ij}(\mathbf{\Lambda})}(\mathbf{\Lambda}) = \mathbb{E}_{\boldsymbol{\gamma}}\left[c_{ij}(\tau_{ij}(\boldsymbol{\gamma}\ \mathbf{\Lambda}), p_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda}))\right]$$

Upon denoting $w_{ij}(\mathbf{\Lambda}) := \mu_i^{k_{ij}(\mathbf{\Lambda})} - \mu_j^{k_{ij}(\mathbf{\Lambda})}$, it then follows that the optimal allocation $\tau_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda})$ and $p_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda})$ can be found by solving [cf. (12)]

$$\min_{\tau_{ij}(\boldsymbol{\gamma}),\, p_{ij}(\boldsymbol{\gamma})} \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{ij}\left[\lambda_i p_{ij}(\boldsymbol{\gamma}) - w_{ij}(\mathbf{\Lambda}) c_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}))\right]\right]. \tag{14}$$

The next lemma proved in Appendix B shows that the optimal time and power allocation for (14) follows a greedy policy ($[x]^+ := \max\{0,x\}$):

Lemma 1: *For each fading realization $\boldsymbol{\gamma}$, consider per link ij the power allocation*

$$\tilde{p}_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda}) = \left[\frac{w_{ij}(\mathbf{\Lambda})}{\lambda_i \ln 2} - \frac{1}{\gamma_{ij}}\right]^+ \tag{15}$$

and subsequently what we term the link quality indicator

$$\phi_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda}) = \lambda_i \tilde{p}_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda}) - w_{ij}(\mathbf{\Lambda}) \log_2(1 + \gamma_{ij}\tilde{p}_l(\boldsymbol{\gamma};\mathbf{\Lambda})). \tag{16}$$

Then for ergodic fading channels with continuous cdf, the almost surely unique solution for (14) yields the optimal time and power allocation per $\boldsymbol{\gamma}$ as

$$\begin{cases} \tau_{l(\boldsymbol{\gamma};\mathbf{\Lambda})}(\boldsymbol{\gamma};\mathbf{\Lambda}) = 1, \;\; p_{l(\boldsymbol{\gamma};\mathbf{\Lambda})}(\boldsymbol{\gamma};\mathbf{\Lambda}) = \tilde{p}_{l(\boldsymbol{\gamma};\mathbf{\Lambda})}(\boldsymbol{\gamma};\mathbf{\Lambda}), \\ \tau_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda}) = p_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda}) = 0, \quad \forall ij \neq l(\boldsymbol{\gamma};\mathbf{\Lambda}) \end{cases} \tag{17}$$

where $l(\boldsymbol{\gamma};\mathbf{\Lambda}) = \arg\min_{ij}\ \phi_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda})$.

Basically, Lemma 1 asserts that a “winner-takes-all” assignment per fading state $\boldsymbol{\gamma}$ along with a water-filling power allocation across $\boldsymbol{\gamma}$ realizations constitutes with probability one (w.p. 1) the optimal solution of (14), provided that the distribution function of the random fading channel is continuous. Regarding λ_i as a power price and $w_{ij}(\mathbf{\Lambda})$ as rate reward value, $\phi_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda})$ in (16) can be interpreted as a *net cost* (power cost minus rate reward) for link ij over $\boldsymbol{\gamma}$. The optimal resource allocation for (14) should minimize the total net cost across users per $\boldsymbol{\gamma}$. As illustrated in Fig. 2, this amounts to a *greedy water-filling* solution, where power and time allocations are decoupled.

In the first step, transmit-power $\tilde{p}_{ij} := p_{ij}/\tau_{ij}$ during the active time fraction $\tau_{ij} > 0$, is allocated per user across $\boldsymbol{\gamma}$ following a water-filling principle as in (15). The link quality indicators $\phi_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda})$ in (16) then represent the smallest potential of net cost when allocating the entire coherent time slot to link ij for the fading realization $\boldsymbol{\gamma}$. In the second step, the entire time slot is greedily assigned to the “winner” link $l(\boldsymbol{\gamma};\mathbf{\Lambda})$ for minimum net cost per $\boldsymbol{\gamma}$.

Note that although the time slots were allowed to be time shared by multiple links at the outset, the almost surely optimum solution here dictates no sharing. Hence the optimal joint time and power allocation reduces to a decoupled link scheduling and power control policy. Based on the latter, the optimal average rates $\bar{r}_{ij}^k(\mathbf{\Lambda})$ for (12) are given by

$$\bar{r}_{ij}^k(\mathbf{\Lambda}) = \mathbb{E}_{\boldsymbol{\gamma}}\left[r_{ij}^k(\boldsymbol{\gamma};\mathbf{\Lambda})\right] \tag{18}$$

where

$$r_{ij}^k(\boldsymbol{\gamma};\mathbf{\Lambda}) = \begin{cases} c_{ij}(\tau_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda}), p_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda})), & \text{if } k = k_{ij}(\mathbf{\Lambda}), \\ 0, & \text{otherwise.} \end{cases} \tag{19}$$

Using the vector $\boldsymbol{X}(\mathbf{\Lambda}) := \{\bar{p}_i(\mathbf{\Lambda}),\ \bar{r}_{ij}^k(\mathbf{\Lambda}),\ \tau_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda}),\ p_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda}),\ \forall i,j,k\}$ with its entries given by (11), (18) and (17), the (sub-)gradient[2] of the dual function $D(\mathbf{\Lambda})$ with respect to $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ can be evaluated as [16]:

$$\begin{aligned} D_{\boldsymbol{\lambda}}(\mathbf{\Lambda}) &= \left\{\mathbb{E}_{\boldsymbol{\gamma}}\left[\sum\nolimits_j p_{ij}(\boldsymbol{\gamma};\mathbf{\Lambda})\right] - \bar{p}_i(\mathbf{\Lambda}),\ \forall i\right\}, \\ D_{\boldsymbol{\mu}}(\mathbf{\Lambda}) &= \left\{\bar{a}_i^k - \sum\nolimits_j (\bar{r}_{ij}^k(\mathbf{\Lambda}) - \bar{r}_{ji}^k(\mathbf{\Lambda})),\ \forall i,k\right\}. \end{aligned} \tag{20}$$

Relying on (20), we can then solve the dual problem (9) using the following gradient iterations (indexed by t):

$$\begin{aligned} \boldsymbol{\lambda}(t+1) &= \left[\boldsymbol{\lambda}(t) + s \cdot D_{\boldsymbol{\lambda}}(\mathbf{\Lambda}(t))\right]^+, \\ \boldsymbol{\mu}(t+1) &= \left[\boldsymbol{\mu}(t) + s \cdot D_{\boldsymbol{\mu}}(\mathbf{\Lambda}(t))\right]^+ \end{aligned} \tag{21}$$

where s is a small stepsize. Convergence of gradient iteration (21) to optimal $\mathbf{\Lambda}^* := \{\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*\}$ is guaranteed from any initial $\mathbf{\Lambda}(0)$, and convergence rate is linear (geometric) under conditions [16].

Having obtained the optimal $\mathbf{\Lambda}^*$ for (9), the zero duality gap between the primal (2) and dual (9) then implies that replacing

[2]Note that since here the sub-gradient is unique for any given $\mathbf{\Lambda}$ due to the almost surely uniqueness of optimal time and power allocation, this sub-gradient is indeed the gradient of $D(\mathbf{\Lambda})$.

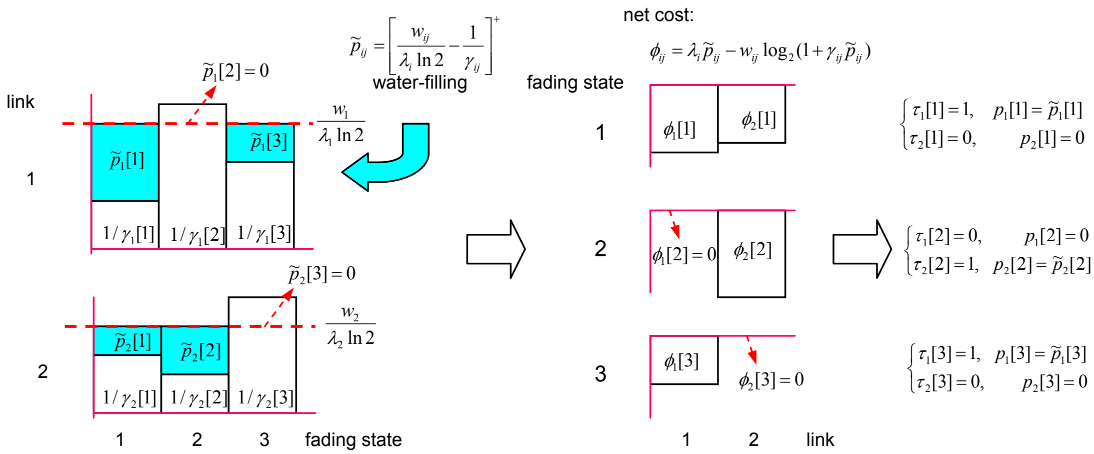


Fig. 2. The greedy water-filling approach.

$\mathbf{\Lambda}$ with $\mathbf{\Lambda}^*$ in (11), (18) and (17) provides the optimal solution, namely, $\boldsymbol{X}(\mathbf{\Lambda}^*)$ for (2).

It is clear that the optimal power control for (2) is a greedy one as dictated by Lemma 1. This greedy scheme can effectively exploit the capacity of random wireless channels for energy efficiency since it "water-fills" the available power resources across $\boldsymbol{\gamma}$ realizations, with higher power (and rate) assigned to higher quality links. This ensures the optimum utilization of the temporal diversity that random fading brings. In addition, the water-filling power allocation together with the "winner-takes-all" time-slot assignment specified in (17) captures also the multi-user diversity through intelligently scheduling a user-link with a "best" channel enabling the smallest net cost per slot. Notice that the term "winner-takes-all" should not be misunderstood. Although one winner (link) is chosen per coherent time slot, the optimally scheduled winner (as well as its assigned power) varies across fading realizations. In this spirit, all the available fading diversity and the "ergodic" energy efficiency of the intended wireless channels are obtained by the proposed power control scheme.

In fact, the "winner-takes-all" policy here subsumes three cases: (i) no transmission when all links experience deep fades, i.e., $\gamma_{ij} \leq (\lambda_i^* \ln 2)/w_{ij}(\mathbf{\Lambda}^*)$, $\forall ij$;[3] (ii) transmission over the single winner-link if $\phi_{ij}^*(\boldsymbol{\gamma};\mathbf{\Lambda}^*)$ admits a unique minimizer; and (iii) transmission over a randomly chosen winner-link if $\phi_{ij}^*(\boldsymbol{\gamma};\mathbf{\Lambda}^*)$ has multiple minima. Continuity of channel cdf ensures that having multiple winner-links, i.e., case (iii), is an even of Lebesgue measure zero. Thus, the allocation $(\tau_{ij}^*(\boldsymbol{\gamma};\mathbf{\Lambda}^*), p_{ij}^*(\boldsymbol{\gamma};\mathbf{\Lambda}^*))$ is almost surely unique; and hence, it is almost surely optimal for e.g., Rayleigh, Rice and Nakagami channel models with continuous fading coefficients.

V. Stochastic Network Resource Allocation

The gradient iteration (21) is efficient to find the optimal routing, scheduling and power control for (2). But a key knowledge we need in (21) is that of the average traffic arrival rates and joint fading channel cdf; only then can we evaluate the expected values encountered. Assumption of this knowledge is reasonable for theoretic studies in e.g., [13]. However, practical mobile applications motivate schemes that can operate even without such a knowledge, while approaching the optimal strategy by "learning" the required statistics on-the-fly. Interestingly, this can be achieved by a *stochastic gradient* iteration developed based on (21) to solve (2) without the channel and traffic statistics *a priori*. To this end, we consider simply dropping $\mathbb{E}_{\boldsymbol{\gamma}}$ from (21), and replacing average packet arrival rates with instantaneous arrival rates, to devise the on-line iteration per slot n as:

$$\begin{aligned} \hat{\boldsymbol{\lambda}}[n+1] &= \left[\hat{\boldsymbol{\lambda}}[n] + s \cdot \hat{D}_{\boldsymbol{\lambda}}(\boldsymbol{\gamma}[n], \hat{\mathbf{\Lambda}}[n])\right]^+, \\ \hat{\boldsymbol{\mu}}[n+1] &= \left[\hat{\boldsymbol{\mu}}[n] + s \cdot \hat{D}_{\boldsymbol{\mu}}(\boldsymbol{\gamma}[n], \hat{\mathbf{\Lambda}}[n])\right]^+ \end{aligned} \tag{22}$$

where hats are to stress that these iterates are stochastic estimates of those in (21), based on the "stochastic" gradients

$$\begin{aligned} \hat{D}_{\boldsymbol{\lambda}}(\boldsymbol{\gamma}[n], \hat{\mathbf{\Lambda}}[n]) &= \left\{\sum\nolimits_j p_{ij}(\boldsymbol{\gamma}[n]; \hat{\mathbf{\Lambda}}[n]) - \bar{p}_i(\hat{\mathbf{\Lambda}}[n]),\ \forall i\right\}, \\ \hat{D}_{\boldsymbol{\mu}}(\boldsymbol{\gamma}[n], \hat{\mathbf{\Lambda}}[n]) &= \Big\{a_i^k[n] - \sum\nolimits_j (r_{ij}^k(\boldsymbol{\gamma}[n], \hat{\mathbf{\Lambda}}[n]) \\ &\qquad - r_{ji}^k(\boldsymbol{\gamma}[n], \hat{\mathbf{\Lambda}}[n])),\ \forall i,k\Big\}. \end{aligned} \tag{23}$$

The stochastic gradients in (23) are based on the *instantaneous* (instead of average) packet arrivals $a_i^k[n]$, powers $p_{ij}(\boldsymbol{\gamma}[n]; \hat{\mathbf{\Lambda}}[n])$ and rates $r_{ij}^k(\boldsymbol{\gamma}[n], \hat{\mathbf{\Lambda}}[n])$. Hence, the stochastic iteration (22) bypasses the need for traffic and channel statistics. In addition, it is easy to see that these stochastic gradients are unbiased (random) estimates of the gradients in (20), provided that the packet arrival and channel fading processes are jointly ergodic. Therefore, (22) and (21) consists of a pair of so-called *primary and averaged systems* [17].

In fact, the proposed stochastic gradient iteration (22) belongs to the same class as the well-known least-mean-square (LMS) algorithm in adaptive signal processing [17]. As with the LMS algorithm, convergence of such a stochastic iteration

[3] In this case $\tilde{p}_{ij}^*(\boldsymbol{\gamma};\mathbf{\Lambda}^*) = 0$, $\forall ij$, implying all links defer.

can be established by a stochastic locking theorem, which justifies that the trajectory of the primary system (22) is always "locked", i.e., stays close to, that of its averaged system (21) in probability under some regularity conditions (primarily stochastic Lipschitz conditions for system perturbations), if a sufficiently small stepsize s is used [17]. It can be confirmed that these regulation conditions are satisfied for the primary and averaged systems of the wireless setup here, provided that the random fading channel has continuous cdf. The proof mimics the counterparts in our recent works [14], [15], and is omitted for conciseness. Since convergence of iteration (21) to $\mathbf{\Lambda}^*$ is guaranteed, the trajectory locking then implies that iteration (22) converges also to the optimal $\mathbf{\Lambda}^*$, and thus dynamic routing, scheduling and power control scheme $\{r_{ij}^k(\boldsymbol{\gamma}[n], \hat{\mathbf{\Lambda}}[n]),\ \tau_{ij}(\boldsymbol{\gamma}[n]; \hat{\mathbf{\Lambda}}[n]),\ p_{ij}(\boldsymbol{\gamma}[n]; \hat{\mathbf{\Lambda}}[n]),\ \forall i,j,k\}$ approaches the globally optimal one for (2) on-the-fly.

Convergence of the stochastic iteration (22) to $\mathbf{\Lambda}^*$ in probability as $s \to 0$, can be also established through the fluid limit and/or Lyapunov drift techniques, when the traffic and fading process confines to a finite-state Markov process. The proof can be readily derived following the similar lines as those in [18], [19], [20]. Enhancing the latter, our result here accommodates also the general wireless channels with continuous fading coefficients.

Summarizing, we can establish the following result.

Proposition 2: *For stationary and ergodic packet arrivals and fading channels, the stochastic gradient iterates in (22) converge to $\mathbf{\Lambda}^*$ in probability, from any initial $\hat{\mathbf{\Lambda}}[0]$ as the stepsize $s \to 0$; and, consequently, dynamic routing, scheduling and power control scheme $\{r_{ij}^k(\boldsymbol{\gamma}[n], \hat{\mathbf{\Lambda}}[n]),\ \tau_{ij}(\boldsymbol{\gamma}[n]; \hat{\mathbf{\Lambda}}[n]),\ p_{ij}(\boldsymbol{\gamma}[n]; \hat{\mathbf{\Lambda}}[n]),\ \forall i,j,k\}$ converges to the globally optimal one for (2).*

From the iterations in (22), it can be found that the stochastic gradient drift $a_i^k[n] - \sum_j (r_{ij}^k[n] - r_{ji}^k[n])$ for Lagrange multiplier $\hat{\mu}_i^k[n]$ is also the difference between incoming and outgoing traffic for flow k at node i per slot n. Therefore, $\hat{\mu}_i^k[n]$ in fact represents the queue backlog $q_i^k[n]$ scaled by the stepsize s, i.e., $\hat{\mu}_i^k[n] \equiv sq_i^k[n]$, and $\hat{\mu}_i^k - \hat{\mu}_i^k$ represents the scaled differential queue backlog. Hence, the dynamic routing and scheduling scheme with (22) needs to simply schedule a flow $k_{ij}(\hat{\mathbf{\Lambda}})$ with largest differential *queue backlog* to be routed at link ij. This is the "queue backpressure routing" principle in [9], [10], which is shown to be also part of the stochastic energy-optimization algorithm here.

Solving the optimization problem (2) on-line, iteration (22) is a desirable stochastic approach to obtaining the energy-efficient and fair routing, scheduling and power control scheme. Convergence (and near-optimality) of (22) is confirmed by Proposition 2 when fading channels are stationary and ergodic. It is worth mentioning that due to its "stochastic learning" capability, the proposed stochastic iteration can also track even non-stationary channels (induced by e.g., mobility), if dynamic changes of the channel statistics are relatively slow with respect to the convergence speed of the proposed scheme. In the latter case, iteration (22) will be capable of "re-learning" the varying channels and converging to the new optimum. This makes the proposed scheme more attractive for practical mobile applications.

So far the proposed algorithm is assumed to be performed by a central controller. However, the distributed implementation of the proposed scheme can be devised through the optimization decomposition paradigm [6]. Using the Lagrange dual method, the optimal resource allocation for e.g., (2), can be obtained by iteratively solving $\boldsymbol{X}^*(\mathbf{\Lambda})$ for the dual function, and based on the latter, accordingly updating the dual variables using stochastic gradient iterations (22) online. From (7), finding $\boldsymbol{X}^*(\mathbf{\Lambda})$ corresponds to solving some decoupled subproblems in (10) and (12). It is clear that the optimization over $\bar{p}_i$ in (10) can be performed per node N_i, and the flow scheduling decision in (13) can be performed per link ij. In addition, the optimization over time and power allocation $(\boldsymbol{\tau}(\cdot), \boldsymbol{p}(\cdot))$ in (14) is carried out per fading realization $\boldsymbol{\gamma}$ and the proposed stochastic resource allocation scheme is expected to facilitate the distributed operations because it can operate without the knowledge of the (possibly jointly-coupled) fading distribution of sensor links. In addition, the simple greedy strategy adopted in the optimal resource allocation facilitates its distributed operations as well. Detailed development of the distributed scheme is omitted due to limited space.

VI. Numerical Results

In this section, we present numerical tests of the proposed stochastic schemes for TDMA networks with system bandwidth $B = 100$ KHz and slot duration $T_s = 1$ ms.

Consider first a simple single-hop (a special case of multi-hop) network with $L = 4$ active wireless links. The fading processes for the links are independent and γ_l, $l = 1, \ldots, 4$, are generated from a Rayleigh distribution with variance $\bar{\gamma}_l$. The average normalized SNR for the links are assumed to be $\bar{\gamma}_1 = 8$, $\bar{\gamma}_2 = 6$, $\bar{\gamma}_3 = 4$, and $\bar{\gamma}_4 = 2$ dBW. All the links need to maintain a constant packer arrival rate $a_l = 100$ Kbps, $\forall l$. Using a stepsize $s = 0.002$, we run (22) when different β-fair cost functions are adopted. Fig. 3 (top) shows the resultant average sum-power, whereas Fig. 3 (bottom) shows the individual average power consumptions by links 1–4 for β=0, 4, 8, and 16. When β=0, we indeed minimize the sum of average powers, and thus in this case (22) yields the most energy-efficient design. However, this is achieved in an unfair manner, as witnessed by Fig. 3 (bottom) where link 4 consumes much more power than link 1. With a larger β, it is shown that more total power needs to be spent but the fairness improves. For instance, when β=16, all links consume almost same average powers but compared with the β=0 case, 27% more total power is spent. For comparison, Fig. 3 also includes the resultant power consumptions for a suboptimal fixed-access scheme, where equal time factions (i.e., $\tau_l(\boldsymbol{\gamma}) = 1/4$,, $\forall l$, $\forall \boldsymbol{\gamma}$) are assigned to the four links per slot, and then each link implements water-filling based power allocation (across fading realizations) to adapt its transmit-power per assigned time fraction. Since such a fixed-access scheme ignores the multi-user diversity, it is seen that more than 3 times total power is required than the derived optimal resource allocation

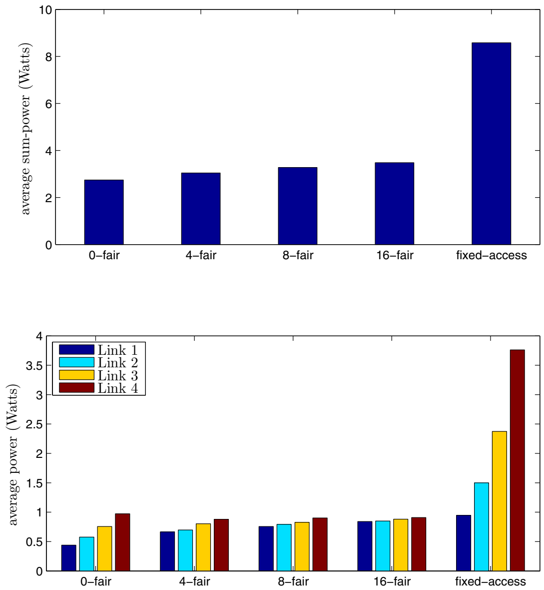


Fig. 3. Resultant average sum and individual powers for the single-hop case.

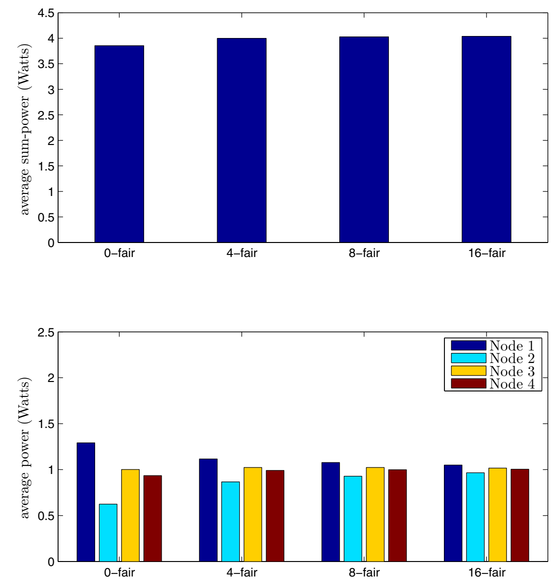


Fig. 4. Resultant average sum and individual powers for the multi-hop case.

schemes. In addition, fairness is also overlooked. Fig. 3 clearly demonstrates that the proposed energy-efficient schemes can result in large power savings, whereas different β-fair cost functions can trade off overall energy efficiency and fairness.

We next consider the multi-hop wireless network with six nodes and three active data flows in Fig. 1. The locations (x_i, y_i) for nodes 1–6 are (0,1), (0,0), (1,1), (1,0), (2,1) and (2,0), respectively. Supposing the reference normalized SNR $\bar{\gamma} = 8$ dBW, the average normalized SNR for a link ij is given by $\bar{\gamma}_{ij} = \frac{\bar{\gamma}}{((x_i-x_j)^2+(y_i-y_j)^2)^{n/2}}$ where $n = 3.6$ is the path loss component adopted in the simulations. The fading processes for the links are independent and are generated from a Rayleigh distribution with the corresponding variance $\bar{\gamma}_{ij}$. The arrival processes for the three flows are assumed to be Bernoulli distributed with given average rates $\bar{a}_i^k$ and probabilities $\pi_i^k \in (0,1)$ [21]. As a result, the arrival rate per time slot n is given by

$$a_i^k[n] = \begin{cases} 0, & \text{with probability } \pi_i^k, \\ \bar{a}_i^k/(1-\pi_i^k), & \text{with probability } 1-\pi_i^k. \end{cases} \quad (24)$$

The parameters for the arrival processes are set to: $\bar{a}_1^1 = \bar{a}_1^2 = \bar{a}_2^3 = 100$ Kbps, and $\pi_1^1 = 0.4$, $\pi_1^2 = 0.5$, $\pi_2^3 = 0.6$. Using a stepsize $s = 0.002$, we run (22) for different β-fair cost functions adopted in (2). Fig. 4 (top) shows the resultant average sum-power, whereas Fig. 4 (bottom) shows the individual average power consumptions by nodes 1–4 for β=0, 4, 8, and 16. When β=0, the most energy-efficient resource allocation is achieved, however, in an unfair manner. In this case, node 1 spends much more power than node 2. With a larger β, it is shown that slightly more total power needs to be spent but the fairness improves. When β=16, the nodes consume almost same average powers, with only 4.8% more total power spent than the β=0 case. This can certainly benefit the network lifetime.

The traffic and channel statistics are assumed unknown *a priori* in all simulations, and the proposed stochastic schemes are supposed to learn this knowledge on-the-fly and approach the optimal policy. To confirm this, Fig. 5 depicts the evolution of the Lagrange multipliers $\hat{\lambda}_i$ in (22) when $\beta = 4$ for the second test case. Convergence of the iterations is clear. Notice that here only a "stochastic" convergence is achieved. In other words, the Lagrange multipliers only converge to, or hover within, a small neighborhood (with a size proportional to stepsize s) around the optimal values.

VII. Conclusions

In this paper, we derived the optimal routing, scheduling and power control scheme that minimizes a β-fair cost function of average powers for energy-efficient transmissions in TDMA multi-hop wireless networks. It has been shown that the resultant scheme can nicely balance overall energy efficiency and fairness. Drawing from the stochastic optimization techniques, the proposed stochastic scheme is also capable of self-adaptively approaching the optimal strategy on-line, without requiring *a-priori* knowledge of the traffic, channel and/or mobility statistics.

Appendix

A. Proof of Proposition 1

The proof modifies from that for [7, Lemma 3]. To show the wanted min-max fairness, consider the problem (5) with

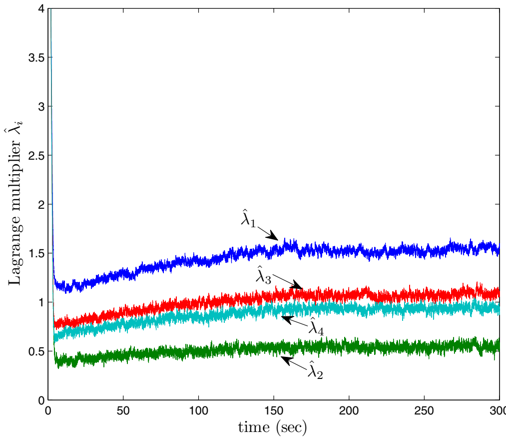


Fig. 5. Evolution of Lagrange multipliers $\hat{\lambda}_i$.

a feasible set $\mathcal{P} = \{\boldsymbol{p} \mid \boldsymbol{A}^T\boldsymbol{p} \geq \boldsymbol{v}, \;\; 0 \leq \boldsymbol{p} \leq p_{\max}\}$; namely, we consider the following problem (P):

$$\min_{\boldsymbol{p}} \sum_{j=1}^{J} V_\beta(p_j), \quad \text{s. to} \quad \boldsymbol{A}^T\boldsymbol{p} \geq \boldsymbol{v}, \;\; 0 \leq \boldsymbol{p} \leq p_{\max}. \quad (25)$$

In wireless networks, linear constraints $\boldsymbol{A}^T\boldsymbol{p} \geq \boldsymbol{v}$ can be contributed by the quality-of-service (QoS) requirements; i.e., the (sum-) power for a certain links need to be greater than a value to support the prescribed data-flow rates. Considering these linear constraints facilitate studying the "bottleneck" problem relevant to the min-max fairness. The bounds $0 \leq \boldsymbol{p} \leq p_{\max}$ are natural for physical networks.

Let $\boldsymbol{p}^{(\beta)}$ denote the optimal solution of (P) for a sequence of $\beta \to \infty$. Clearly $\{\boldsymbol{p}^{(\beta)}\}$ is a sequence in a compact set $\mathcal{P}$, and thus there exists a subsequence β_k, $k = 1, \ldots, \infty$, of β sequence such that $\boldsymbol{p}^{(\beta_k)}$ converges to some $\vec{\boldsymbol{p}} \in \mathcal{P}$ as $k \to \infty$.

We next prove that $\vec{\boldsymbol{p}}$ must be the min-max vector by showing a contradiction otherwise. Assume that $\vec{\boldsymbol{p}}$ is not a min-max vector, then there exists a node i whose power $\vec{p}_i$ can be decreased with increasing other nodes' $\vec{p}_j$ that are less than $\vec{p}_i$. Let $\mathcal{L}_1$ be the set of "saturated" constraints involving node i, i.e., $(\boldsymbol{A}^T\boldsymbol{p})_l = v_l$, $\forall l \in \mathcal{L}_1$; and let $\mathcal{L}_2$ be the set of other constraints involving node i. Notice that $\mathcal{L}_1$ cannot be an empty set, since otherwise $\vec{\boldsymbol{p}}$ will not be an optimal solution for (P) because we can further decrease $\vec{p}_i$ and consequently the total power cost within the feasible set $\mathcal{P}$. For each $l \in \mathcal{L}_1$ involving other nodes, there must also exist a node $u(l)$ whose power $\vec{p}_{u(l)}$ is less than $\vec{p}_i$, i.e., $\vec{p}_{u(l)} < \vec{p}_i \leq p_{\max}$. Otherwise, $\vec{p}_i$ cannot be decreased with increasing other $\vec{p}_j < \vec{p}_i$.

Now define a small constant δ by[4]

$$\delta := \frac{1}{5} \min \left\{ \min_{l \in \mathcal{L}_1} \{\vec{p}_i - \vec{p}_{u(l)}\}, \; \min_{l \in \mathcal{L}_2} \{(\boldsymbol{A}^T\boldsymbol{p})_l - v_l\} \right\}. \quad (26)$$

[4]In (26), minimum of an empty set, if any, is defined to be a large number, say ∞, instead of 0.

From the convergence of $\boldsymbol{p}^{(\beta_k)}$ to $\vec{\boldsymbol{p}}$, we can find a k_0 such that for $k \geq k_0$, it holds for all j,

$$\vec{p}_j - \delta \leq p_j^{(\beta_k)} \leq \vec{p}_j + \delta. \quad (27)$$

Define a sequence of vector $\boldsymbol{q}^{(\beta_k)}$ with:

$$q_j^{(\beta_k)} = \begin{cases} p_j^{(\beta_k)} - \delta, & \text{if } j = i, \\ p_j^{(\beta_k)} + \delta, & \text{if } j = u(l), \text{ for } l \in \mathcal{L}_1, \\ p_j^{(\beta_k)}, & \text{otherwise.} \end{cases} \quad (28)$$

It is easy to show that $\boldsymbol{A}^T\boldsymbol{q}^{(\beta_k)} \geq \boldsymbol{v}$ and $0 \leq \boldsymbol{q}^{(\beta_k)} \leq p_{\max}$ for $k \geq k_0$ since we choose a small enough δ in (26). This implies that $\boldsymbol{q}^{(\beta_k)}$ is a feasible vector for (P).

Consider now the difference

$$\begin{aligned} 0 \;\; &\geq \;\; D^{(\beta_k)} \\ &= \;\; \sum_j \left[V_{\beta_k}(p_j^{(\beta_k)}) - V_{\beta_k}(q_j^{(\beta_k)}) \right] \\ &= \;\; \left[V_{\beta_k}(p_i^{(\beta_k)}) - V_{\beta_k}(p_i^{(\beta_k)} - \delta) \right] + \\ &\quad\;\; \sum_{l \in \mathcal{L}_1} \left[V_{\beta_k}(p_{u(l)}^{(\beta_k)}) - V_{\beta_k}(p_{u(l)}^{(\beta_k)} + \delta) \right] \end{aligned} \quad (29)$$

where $D^{(\beta_k)} \leq 0$ follows from the optimality of $\boldsymbol{p}^{(\beta_k)}$ and feasibility of $\boldsymbol{q}^{(\beta_k)}$.

From the mean value theorem, there exists numbers $m_i^{(\beta_k)}$ such that

$$\begin{cases} p_i^{(\beta_k)} - \delta \leq m_i^{(\beta_k)} \leq p_i^{(\beta_k)}, \\ V_{\beta_k}(p_i^{(\beta_k)}) - V_{\beta_k}(p_i^{(\beta_k)} - \delta) = V'_{\beta_k}(m_i^{(\beta_k)})\delta. \end{cases} \quad (30)$$

Combing with (27), we also have

$$m_i^{(\beta_k)} \geq p_i^{(\beta_k)} - \delta \geq \vec{p}_i - 2\delta.$$

Similarly, there exists some numbers $m_{u(l)}^{(\beta_k)}$ such that

$$\begin{cases} p_{u(l)}^{(\beta_k)} \leq m_i^{(\beta_k)} \leq p_{u(l)}^{(\beta_k)} + \delta, \\ V_{\beta_k}(p_{u(l)}^{(\beta_k)}) - V_{\beta_k}(p_{u(l)}^{(\beta_k)} + \delta) = -V'_{\beta_k}(m_{u(l)}^{(\beta_k)})\delta; \end{cases} \quad (31)$$

and combing with (27) and (26), we have

$$m_{u(l)}^{(\beta_k)} \leq p_{u(l)}^{(\beta_k)} + \delta \leq \vec{p}_{u(l)} + 2\delta \leq \vec{p}_i - 5\delta + 2\delta = \vec{p}_i - 3\delta.$$

Therefore, we have from (29)

$$\begin{aligned} D^{(\beta_k)} \;\; &= \;\; \delta \left[V'_{\beta_k}(m_i^{(\beta_k)}) - \sum_{l \in \mathcal{L}_1} V'_{\beta_k}(m_{u(l)}^{(\beta_k)}) \right] \\ &\geq \;\; \delta \left[V'_{\beta_k}(\vec{p}_i - 2\delta) - G V'_{\beta_k}(\vec{p}_i - 3\delta) \right] \\ &= \;\; \delta(\vec{p}_i - 2\delta)^{\beta_k} \left[1 - G \left(\frac{\vec{p}_i - 3\delta}{\vec{p}_i - 2\delta} \right)^{\beta_k} \right] \end{aligned} \quad (32)$$

where G is the cardinality of $\mathcal{L}_1$, and the inequality follows from the convexity of V_{β_k} and the bounds on $m_i^{(\beta_k)}$ and $m_{u(l)}^{(\beta_k)}$. But since $((\vec{p}_i - 3\delta)/(\vec{p}_i - 2\delta))^{\beta_k} \to 0$ as $\beta_k \to \infty$, and $(\vec{p}_i - 2\delta)^{\beta_k} > 0$, we must have $D^{(\beta_k)} > 0$ for some k large enough. This is clearly a contradiction with $D^{(\beta_k)} \leq 0$. Therefore, $\vec{\boldsymbol{p}}$ must be the min-max vector and the proof is complete.

B. Proof of Lemma 1

Let us define convenient notation

$$\phi_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}); \boldsymbol{\Lambda}) := \lambda_i p_{ij}(\boldsymbol{\gamma}) - w_{ij}(\boldsymbol{\Lambda}) c_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma})). \quad (33)$$

To prove the lemma, we will need the following two claims.

Claim 1: *For any $\boldsymbol{\Lambda} \geq 0$, it holds that $\tau_{ij}(\boldsymbol{\gamma})\phi_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) \leq \phi_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}); \boldsymbol{\Lambda})$, $\forall \tau_{ij}(\boldsymbol{\gamma}) \geq 0$, $\forall p_{ij}(\boldsymbol{\gamma}) \geq 0$.*

Proof: Consider the following two cases:

1) If $\tau_{ij}(\boldsymbol{\gamma}) > 0$, substituting (1) into (33) yields

$$\phi_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}); \boldsymbol{\Lambda}) = \lambda_i p_{ij}(\boldsymbol{\gamma}) - w_{ij}(\boldsymbol{\Lambda})\tau_{ij}(\boldsymbol{\gamma}) \log_2\left(1 + \gamma_{ij}\frac{p_{ij}(\boldsymbol{\gamma})}{\tau_{ij}(\boldsymbol{\gamma})}\right).$$

Upon defining $\tilde{p} := p/\tau$, the latter can be rewritten as

$$\phi_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}); \boldsymbol{\Lambda}) = \tau_{ij}(\boldsymbol{\gamma})\tilde{\phi}_{ij}(\tilde{p}_{ij}(\boldsymbol{\gamma}); \boldsymbol{\Lambda})$$

where

$$\tilde{\phi}_{ij}(\tilde{p}_{ij}(\boldsymbol{\gamma}); \boldsymbol{\Lambda}) := \lambda_i \tilde{p}_{ij}(\boldsymbol{\gamma}) - w_{ij}(\boldsymbol{\Lambda}) \log_2(1 + \gamma_{ij}\tilde{p}_{ij}(\boldsymbol{\gamma})).$$

Since $\tilde{\phi}_{ij}(\tilde{p}_{ij}(\boldsymbol{\gamma}); \boldsymbol{\Lambda})$ is a convex function of $\tilde{p}_{ij}(\boldsymbol{\gamma}) \geq 0$, the optimal $\tilde{p}_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ minimizing $\tilde{\phi}_{ij}(\tilde{p}_{ij}(\boldsymbol{\gamma}); \boldsymbol{\Lambda})$ is given by a water-filling formula as in (15). Substituting the latter into $\tilde{\phi}_{ij}(\tilde{p}_{ij}(\boldsymbol{\gamma}); \boldsymbol{\Lambda})$ yields the link quality indicator $\phi_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ in (16). It thus holds that $\tau_{ij}(\boldsymbol{\gamma})\phi_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) \leq \phi_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}); \boldsymbol{\Lambda})$, $\forall p_{ij}(\boldsymbol{\gamma}) \geq 0$.

2) If $\tau_{ij}(\boldsymbol{\gamma}) = 0$, it clearly holds that $\tau_{ij}(\boldsymbol{\gamma})\phi_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) = 0$. On the other hand, (33) and (1) imply that $\phi_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}); \boldsymbol{\Lambda}) = \lambda_i p_{ij}(\boldsymbol{\gamma}) \geq 0$, $\forall p_{ij}(\boldsymbol{\gamma}) \geq 0$. Therefore, $\tau_{ij}(\boldsymbol{\gamma})\phi_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) \leq \phi_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}); \boldsymbol{\Lambda})$, $\forall p_{ij}(\boldsymbol{\gamma}) \geq 0$.

Cases 1) and 2) imply that $\tau_{ij}(\boldsymbol{\gamma})\phi_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) \leq \phi_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}); \boldsymbol{\Lambda})$, $\forall \tau_{ij}(\boldsymbol{\gamma}) \geq 0, \forall p_{ij}(\boldsymbol{\gamma}) \geq 0$. □

Claim 2: *For any $\boldsymbol{\Lambda} \geq 0$, it holds that $\phi_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) \leq 0$, $\forall \boldsymbol{\gamma}$.*

Proof: Substituting (15) into (16) and then differentiating (16) yields

$$\frac{\partial \phi_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda})}{\partial \gamma_{ij}} = \begin{cases} \frac{1}{\gamma_{ij}}\left(\frac{\lambda_i}{\gamma_{ij}} - \frac{w_{ij}(\boldsymbol{\Lambda})}{\ln 2}\right), & \text{if } \gamma_{ij} > \frac{\lambda_i \ln 2}{w_{ij}(\boldsymbol{\Lambda})} \\ 0, & \text{if } \gamma_{ij} \leq \frac{\lambda_i \ln 2}{w_{ij}(\boldsymbol{\Lambda})} \end{cases} \quad (34)$$

and thus $\frac{\partial \phi_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda})}{\partial \gamma_{ij}} < 0$, $\forall \gamma_{ij} > \lambda_i \ln 2/w_{ij}(\boldsymbol{\Lambda})$. Since $\phi_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ is a continuous function of γ_{ij} and $\phi_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) = 0$, $\forall \gamma_{ij} \leq \lambda_i \ln 2/w_{ij}(\boldsymbol{\Lambda})$, the claim readily follows. □

We are now ready to prove Lemma 1 based on Claims 1 and 2. With the winner index $l(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ defined in Lemma 1, it holds for each fading state $\boldsymbol{\gamma}$ that

$$\sum_{ij} \phi_{ij}(\tau_{ij}(\boldsymbol{\gamma}), p_{ij}(\boldsymbol{\gamma}); \boldsymbol{\Lambda}) \geq \sum_{ij} \tau_{ij}(\boldsymbol{\gamma})\phi_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) \geq \phi_{l(\boldsymbol{\gamma};\boldsymbol{\Lambda})}(\boldsymbol{\gamma}; \boldsymbol{\Lambda})\left(\sum_{ij} \tau_{ij}(\boldsymbol{\gamma})\right) \geq \phi_{l(\boldsymbol{\gamma};\boldsymbol{\Lambda})}(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$$

where the first inequality is due to Claim 1; the second inequality is due to the definition of $l(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) := \arg\min_{ij} \phi_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$; and the third one is due to the facts that $\phi_{l(\boldsymbol{\gamma};\boldsymbol{\Lambda})}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) \leq 0$ from Claim 2 and $\sum_{ij} \tau_{ij}(\boldsymbol{\gamma}) \leq 1$. Furthermore, the equality can be achieved using the allocation specified in (17), which is thus optimal for (14).

The almost sure uniqueness of $\{\tau_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}),\ p_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}),\ \forall ij\}$ is implied by the fact that there is almost surely a single winner $l(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ that minimizes $\phi_{ij}(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ per $\boldsymbol{\gamma}$, provided that the fading process has a continuous cdf. The rigorous proof for this mimics its counterpart in our recent work [15].

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