Improved Digital Signatures Based on Elliptic Curve Endomorphism Rings

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1. Introduction

2. Digital Signature Scheme
   - Elliptic Curve Background
   - GPS Signatures

3. Our Improvements
   - #1: Isogeny-to-Ideal
   - #2: Ideal-to-Isogeny
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- Topic of discussion: Galbraith-Petit-Silva digital signature scheme (AsiaCrypt 2017).

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- Implementations are not widely available because:
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  – signing would be too inefficient to be practical.
- Our work presents three major ways to improve efficiency, and implements the scheme in SAGE.

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$$\phi : E(\mathbb{F}_{23}) \to E'(\mathbb{F}_{23}),\ \deg(\phi) = 3$$

$$\phi(x, y) = \left(\frac{x^3 + 10x^2 + 16x + 10}{x^2 + 10x + 2}, \frac{(x^3 + 15x^2 + 15x + 14)y}{x^3 + 15x^2 + 6x + 10} \right)$$

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- Knowledge of an elliptic curve's endomorphism ring can be used as Trapdoor Information.

Hard and Easy Problems with Isogenies

Consider a supersingular $E(\mathbb{F}_{p^2})$, and a hash function H which outputs isogenies with domain E.

	End(E) known	End(E) unknown
Preimage resistant	✓	✓
2^{nd} Preimage resistant	X	✓
Collision resistant	X	✓

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- Given E, compute an arbitrary isogeny with domain E and smooth degree (e.g. 2^m).

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- Given E, $\mathrm{End}(E)$, and $\phi : E \to E'$, find $\mathrm{End}(E')$.
- The GPS signature scheme uses these discrepancies to make an Identification Protocol.

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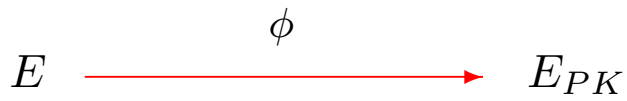

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$$E \xrightarrow{\phi} E_{PK}$$

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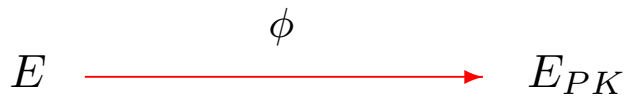

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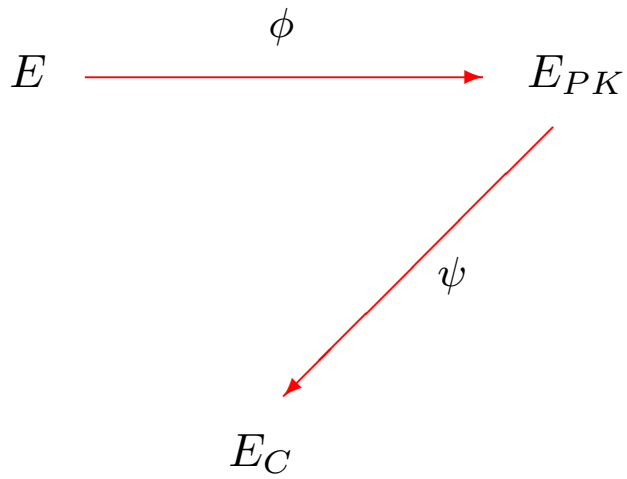

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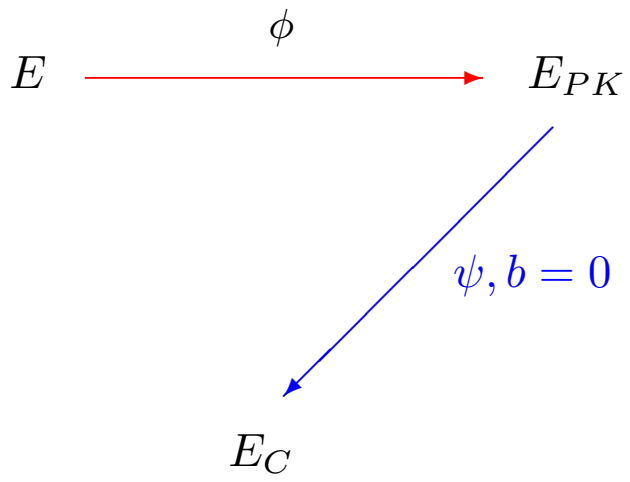

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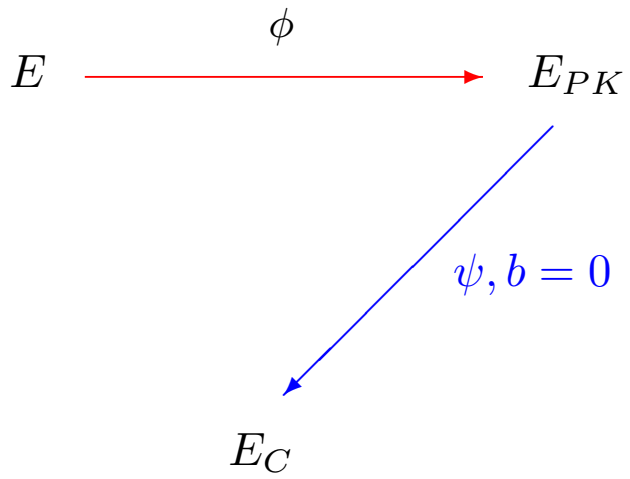

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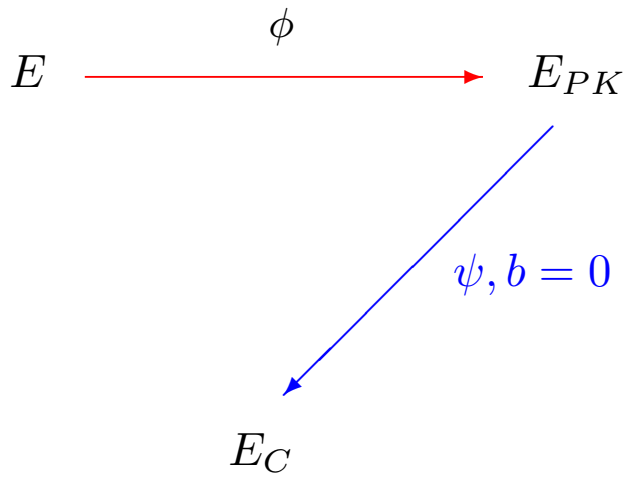

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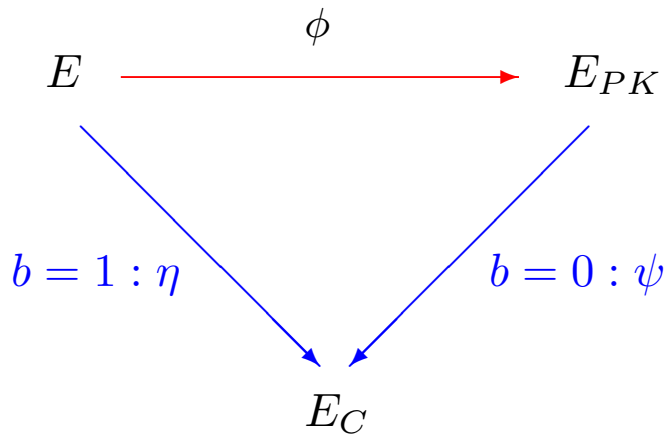

Warning: $\eta \neq \psi \circ \phi$

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- This decrease in cost from this modification is only marginal, but this computation is used widely throughout the signing algorithm.

- When $b = 1$, the output isogeny η will be a composition of isogenies, each with prime power degrees less than L.

$$\eta = \eta_k \circ \cdots \circ \eta_1.$$

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- When constructing the ideal I, it can be done for each ℓ^e individually and then combined.
- We improve the runtime of step (ii) (isogeny-to-ideal) from $poly(\ell^e)$ to $poly(\ell, e)$.

- Let $\langle Q \rangle = \ker \eta_i \subset E(\mathbb{F}_{p^n})$. To find the ideal I, we need a solution $\alpha \in \mathrm{End}(E)$ to

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- We instead propose finding random solutions to $\alpha_j(Q) = 0$ and $\deg \alpha_j \equiv 0 \mod \ell^j$ iteratively for $j = 1, \ldots, e$.

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- In the worst case, this would result in an extension of degree $\mathrm{LCM}\{\deg \eta_i\}_{i=1}^{k} \leq L^k$.

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- We provide formulas for the isomorphisms in terms of the j-invariant of E_i.
- This process bounds the necessary extension degree by $\max\{\deg \eta_i\}_{i=1}^{k} \leq L$.

- Our third major contribution is a proof of security for multiple parallel instances of the GSP signature scheme.

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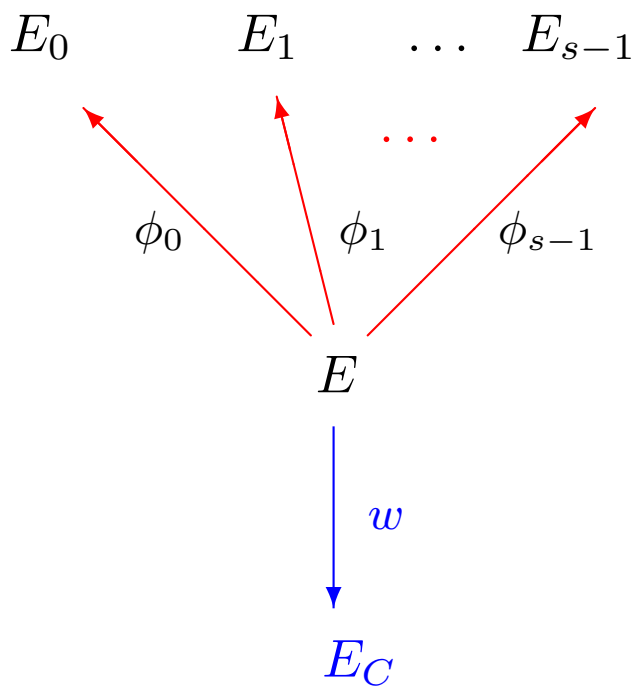

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- Allowing for a greater number of challenge bits decreases the number of rounds by a logarithmic factor.
- We provide a proof of Completeness, Soundness, and Zero-Knowledge.
- Output of the algorithm for step (iii) (ideal renormalization) in the original work has a particular form. This makes our ZK proof highly non-trivial, which is why this modification has not achieved before.

n	$\log_2 p$	Isogenies	(ii)	(iii)	(iv)
[3,10]	8.7	0.100	0.073	0.064	0.109
[3,20]	24.2	0.217	0.215	0.366	0.190
[3,43], [97]	61.1	1.000	1.356	0.883	0.492
[3,113]	155.4	6.356	9.442	6.989	2.297
[3,373], [587]	510.7	174.917	126.520	173.020	45.270

Table: Time (sec) per step

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- The runtime is still too inefficient, the bottleneck is computing the extension fields.
- Our code for the KLPT subroutine is fast.

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