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Trailing Edge Noise Prediction Based on Solutions to the Orr Sommerfeld Equation

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Trailing edge noise is important in many applications including low speed fan noise, wind turbines, airframe noise and helicopter broadband noise. It has been studied extensively and in recent years attention has been given to surface treatments and trailing edge modifications that reduce the radiated sound levels. In this paper we will discuss the fundamentals of trailing edge noise and how it is related to the turbulence in the boundary layer upstream of the trailing edge. Using the Orr Sommerfeld equation the radiated sound and the turbulence in the boundary layer upstream of the trailing edge will be shown to depend on the non-linear terms in the Lamb vector of the turbulence, or Lighthill's stress tensor, and to be directly related to the vorticity outside the viscous sublayer.

I. Introduction

Trailing edge noise was first modeled by Ffowcs Williams and Hall [1] based on Lighthill's Acoustic Analogy. They related the far field sound to the Lighthill's stress tensor based on a solution to the inhomogeneous acoustic wave equation using a Greens function that matched the boundary conditions on a semi-infinite flat plate. The far field sound intensity was shown to scale on the fifth power of the flow speed, and was louder that the dipole sources associated with the unsteady loading on the blade surface by a factor of $1/M$ where M is the flow Mach number. Howe [2] also developed a model for trailing edge noise based on his alternative acoustic analogy that related the far field sound to the Lamb vector of the turbulent flow near the edge, and a low frequency Greens function that approximated the Greens function for a sharp edge used by Ffowcs Williams and Hall[1]. The problem with both these approaches is that the turbulence in the vicinity of the edge, including the turbulence in the blade wake must be prescribed. The details of the wake turbulence are hard to model and this issue was addressed by Amiet [3,4] who developed a trailing edge noise model based on the blade surface pressure fluctuations specified upstream of the edge. It was assumed that the pressure perturbations were convected at a fixed phase speed past the trailing edge and that there was no pressure discontinuity across the blade wake. To match these boundary conditions Amiet used the Schwartzschlid solution to the wave equation in the presence of a sharp edge to determine the acoustic waves that propagate to the far field. The valuable part of Amiet's model is that it only requires the blade surface pressure spectrum as the input to the noise radiation model, and this is far easier to characterize, both and analytically and empirically, than the turbulence in the blade boundary layer and wake.

One important characteristic of broadband trailing edge noise is that the scattered sound on the surface of the blade must match the acoustic waves that propagate to the far field. This means that the wavelength of the surface pressure fluctuations at a given frequency must match the acoustic wavelength. At low Mach numbers the acoustic wavelength is of order $1/M$ times the wavelength of the turbulent flow. The edge scattering mechanism develops waves of this scale in the direction perpendicular to the edge, but not parallel to the edge. The consequence is that the far field sound only couples with certain components of the turbulence wave number spectrum [5]. If the analysis is carried out in the wave number domain with wave numbers k_1,k_2,k_3, where k_1 is in the flow direction, k_2 is normal to the surface, and k_3

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is parallel to the trailing edge, then only the $k_3=0$ component of the turbulence or surface pressure wave number spectra will couple with the acoustic field. This is represented by the spanwise integral of the turbulent velocity and pressure perturbations. This characteristic helps with the modeling of the turbulence, but also eliminates sources caused by some of the dominant coherent structures that are known to occur in turbulent boundary layers. For example, vortex streaks that are aligned with the mean flow often dominate the turbulent flow but are not necessarily strong contributors to the far field sound. This highlights one of the most difficult issues in relating turbulence or surface pressure measurements to the acoustic radiation. To obtain a one to one correlation between the surface pressure perturbations and the far field sound the surface pressure must be instantaneously integrated parallel to the trailing edge, and this is hard, if not impossible to achieve experimentally.

II. Trailing Edge Noise Prediction

Brooks, Marcolini and Pope [6] carried out a comprehensive series of experiments on a NACA 0012 airfoil as a function of Reynolds's number, Mach number, angle of attack, trailing edge thickness and boundary layer trips. They were able to scale their results and provide an empirical prediction procedure that applies to airfoils of different sizes and in different flow regimes. However, it is an empirical method and so is limited by the airfoil shape, and predicting what happens when the boundary layer characteristics are changed is not an option.

More general predictions are typically based on Amiet's method. This assumes that the far field sound is a function of the surface pressure wave number spectrum. In the original method [3] it was assumed that the surface pressure spectrum could be modeled by a uniformly convected pressure disturbance that has the same frequency spectrum as the pressure at a fixed point, and a spanwise integral length scale that is proportional to the disturbance convection speed, and inversely proportional to the frequency. This very simple approach is remarkably accurate as shown in Figure 15.5 in [5]. To extend this to boundary layers that are more complex than a canonical zero pressure gradient turbulent boundary layer over a flat plate more detailed modeling is required, and this has been the subject of on-going research by many different groups.

III. Predicting Surface Pressure Spectra

The prediction of surface pressure wave number spectra has been widely researched and there are both empirical and analytical models available. The most detailed empirical model for the pressure spectrum is given by Goody [7] who collapsed a wide range of experimental data based on inner and outer scaling variables. For the wave number spectrum Chase's model is considered the state of the art, (Blake[8]). However, these empirical models are for zero pressure gradient boundary layers over smooth walls, and do not apply to the more general case of a blade of finite thickness at an angle of attack to the upstream flow. Mean flow pressure gradients and cross flow cause the boundary layer to thicken as it approaches the trailing edge, and flow reversal and separation can occur. These effects are very different from the idealized case of a two dimensional flat plate boundary layer.

The first step in addressing the issue of a non-zero pressure gradient boundary layer is to reconsider Lighthill's acoustic analogy and use it to obtain a model for the surface pressure fluctuations that are caused by the turbulence in the boundary layer. In light of Amiet's model, the incompressible part of the flow is separated from the acoustic waves scattered by the trailing edge of the blade. The assumption is that only the incompressible part of the turbulent flow need be considered since the acoustic scattering is well defined by the Schartzschlid solution. In the near field, that is inside the flow, the acoustic waves are of small amplitude compared to the hydrodynamic fluctuations and the turbulence can be considered as being incompressible providing that the mean flow Mach number is small. Lighthill's wave equation then reduces to a Poisson equation for the pressure with the source terms specified by the double divergence of Lighthill's stress tensor T_{ij}. The Poisson equation is solved with a non-penetration boundary condition at the surface and the surface pressure related directly to the spatial distribution of T_{ij}. The source term is further split into two parts. The mean flow turbulence interaction term that is the product of the mean flow gradients and the turbulent velocity fluctuations, and the non-linear turbulence-turbulence interaction term which generates the turbulent shear stresses. Clearly, to solve the problem, details of both terms are needed, but the modeling of turbulence-turbulence interactions is challenging, and can only be successfully accomplished using an LES or DNS calculation ([9], [10]). However, Grasso et al [11], show that the turbulence-turbulence interaction term is only important at high frequencies, and the mean shear turbulence interaction dominates at low frequencies. Also, Blake [7] suggests that away from the convective part of the wave number spectrum the turbulence-turbulence interaction cannot be neglected.

It is often argued that the mean flow turbulence interaction term provides insight into the regions of the boundary layer that are most responsible for the surface pressure fluctuations [7,10]. This term depends on the normal component of the turbulent velocity u_2, the mean shear U', and the distance from the wall y_2. The surface pressure is given by the integral of $\rho_o u_2 U' \exp(-k_s y_2)(k_1/k_s)$ over y_2, where $k_s = k_1^2 + k_3^2$. This suggests that the source of the pressure fluctuations is located in regions of large mean shear U'. However, for some flows, such as a wall jet the highest turbulence intensity occurs at the location of maximum flow speed where the shear is a minimum. This is a concern that can lead to numerical inaccuracies, but, as shown by Gonzalez[12], this problem can be eliminated using integration by parts so the source term is $\rho_o(u_2\exp(-k_1 y_2))' U (k_1/k_s)$. This raises an important issue regarding the appropriate interpretation of the source term in Lighthill's analogy. From the analogy alone it is not clear whether the surface pressure fluctuations are caused by turbulence in regions of large mean shear or regions of the high mean flow speed. It follows that generalizations about how the mean shear affects the surface pressure cannot be made using the analogy alone.

IV. The Blake TNO Model

One approach to reducing trailing edge noise is to introduce upstream devices that modify the mean flow velocity profile. To determine the impact of such devices a general model is required that can be used for an arbitrary mean flow. The most commonly used approach (Blake[8], Grasso[11], Gonzalez[12]) is to use the Blake TNO model which uses the solution to Lighthill's equation to obtain the surface pressure spectrum in terms of the mean shear and a wave number spectrum model of the turbulence in the boundary layer. Typically, a von Karman turbulence spectrum is used with a variation of length scale and turbulence intensity as a function of distance from the wall. The mean flow and turbulence intensity can be obtained from RANS calculations for any surface flow, but the length scales have to be chosen based on empirical estimates. This can be an issue, for example, Grasso [11] shows calculations for a turbulent boundary layer over a flat plate that over estimate the spectral level at the peak frequency by 20dB when compared to experimental measurements. In contrast Gonzalez [12] applied the same approach to a wall jet flow and obtained good agreement across all flow speeds. In Gonzalez calculations the mean flow was considered to be the superposition of two uncorrelated components, the outer flow that scaled on the wall jet half height and maximum flow speed, and an inner region that scaled on the friction velocity and momentum thickness, and this two parameter model of the wall jet showed a remarkably good collapse with experimental measurements.

V. Vorticity Based Models

Lighthill's acoustic analogy was developed to show how a region of turbulent flow could generated acoustic waves that propagate to the acoustic far field. It provides a basis for the coupling of the turbulent velocity fluctuations to the acoustic perturbations. Within the turbulent flow the acoustic perturbations are negligible in low Mach number flows and so the turbulence can be considered as incompressible. The approaches described above all assume that Lighthill's analogy can be applied within the flow but some care is needed because Lighthill's wave equation is only a subset of the full Navier Stokes equations that are required to specify the fluid motion. The Navier Stokes equations are a set of five coupled partial differential equations, three of which describe momentum fluctuations, the fourth the continuity of the flow and the fifth gives the relationship between pressure and density. Using exact algebraic manipulations the continuity equation can be replaced by Lighthill's equation, and, at low Mach numbers, a linear relationship can be specified between pressure and density. In the incompressible limit the Navier Stokes equations reduce to four coupled partial differential equations that must be solved simultaneously. If the turbulent velocity fluctuations are known exactly, for example using DNS calculations then the pressure perturbations can be obtained using the Poissons equation discussed above[10]. However, if the velocity is not known precisely then the fully coupled equations must be solved to obtain the pressure fluctuations. One approach is to use the vorticity equation, which eliminates the pressure term. Further, for a parallel shear flow the vorticity equation can be reduced using Squires theorem to the Orr Sommerfeld (OS) equation in terms of a single variable. Landau [13] and Chase [14] showed how the surface pressure fluctuations can be found from the fourth order inhomogeneous form of the OS equation. Landau [13] used a modal approach but noted that this was numerically difficult to evaluate for high Reynolds number flows. Chase solved the inhomogeneous OS equation using the variation of parameters in the limit that the phase speed tended to infinity. The key result is that in order for the unsteady flow to match both the non penetration condition and the no slip condition at the wall all four independent solutions to the homogeneous OS equation must be included. In a recent study Glegg et al[15] have shown how the solution can be simplified by considering the flow near the wall as the sum of an irrotational flow and a thin layer of rotational flow that adds to give zero unsteady flow velocity at the wall based on the local solution to the OS equation. It was shown that the pressure perturbations on the surface can be related to the wall shear stress that exactly cancels the irrotational part of the near wall flow, and that, by using the Biot Savort law, the irrotational flow at the wall is directly related to the vorticity in the boundary layer outside the viscous sub layer. The Biot-Savort law is exact, and linear, so is not affected by turbulence-turbulence interactions, or the effect of mean shear on the transmission of pressure, which Jacobs and Durban [16] specify as shear sheltering. This does not mean that turbulence-turbulence interaction is not important because the unsteady part of the vorticity is dependent on these terms. The inviscid vorticity equation can be written such that the rate of change of vorticity $\partial\boldsymbol{\omega}/\partial t$ is equal to the curl of the Lamb vector $(\boldsymbol{\omega}+\boldsymbol{\omega}_o)$ x$(\mathbf{U}+\mathbf{u})$ (where $\boldsymbol{\omega}_o$ and $\mathbf{U}$ are the mean vorticity and flow speed). Hence the source term for the OS equation encompasses both the mean flow turbulence interaction $\boldsymbol{\omega}\mathrm{x}\mathbf{U}+\boldsymbol{\omega}_o\mathrm{x}\mathbf{u}$, the non linear effects of image vorticity due to the wall, and the turbulence-turbulence interaction $\boldsymbol{\omega}\mathrm{x}\mathbf{u}$. Alternatively Chase [14] shows that the source term in the OS equation depends on three components of the Lighthill stress tensor. However, the advantage of using the Biot-Savort law with the vorticity as the source term for pressure fluctuations is that the effective source of surface pressure fluctuations is defined by a single parameter that encompasses all the subtle interactions between the constituent non-linear and linear terms in Lighthill's stress tensor.

The results shown in [15] give the surface pressure as the integral of the source term $(\rho_o c/k_s)(k_1\omega_3-k_3\omega_1)\exp(-k_s y_2)$ as a function of y_2, where c is the phase speed of the vorticity, which, in a constant mean shear, is equal to the local flow speed U, and $k_s=(k_1^2+k_3^2)^{1/2}$. We note that both streamwise and spanwise vorticity contribute to the surface pressure, but for trailing edge noise, which depends on the $k_3=0$ component, only the spanwise vorticity is relevant. This highlights one of the difficulties with predicting trailing edge noise from surface pressures. In typical boundary layer flows the vorticity is dominated by vortex streaks in which the vorticity is aligned with the mean flow, so the ω_1 term dominates the surface pressure. However, these vortex steaks do not couple with the trailing edge to create sound waves and we must look to the much weaker spanwise component of the vorticity to obtain a source term for far field sound. This of course is almost impossible to measure accurately even using PIV technology. It is noteworthy that this conclusion is entirely consistent with Howe's theory of aerodynamic noise.

A modeling formula for a fully developed turbulent boundary is given in [15] and is compared to the empirical Goody model [7] for surface pressure spectra. Figure 1 shows a comparison between these two models as a function of Reynolds's number, and good agreement is found over the entire frequency range.

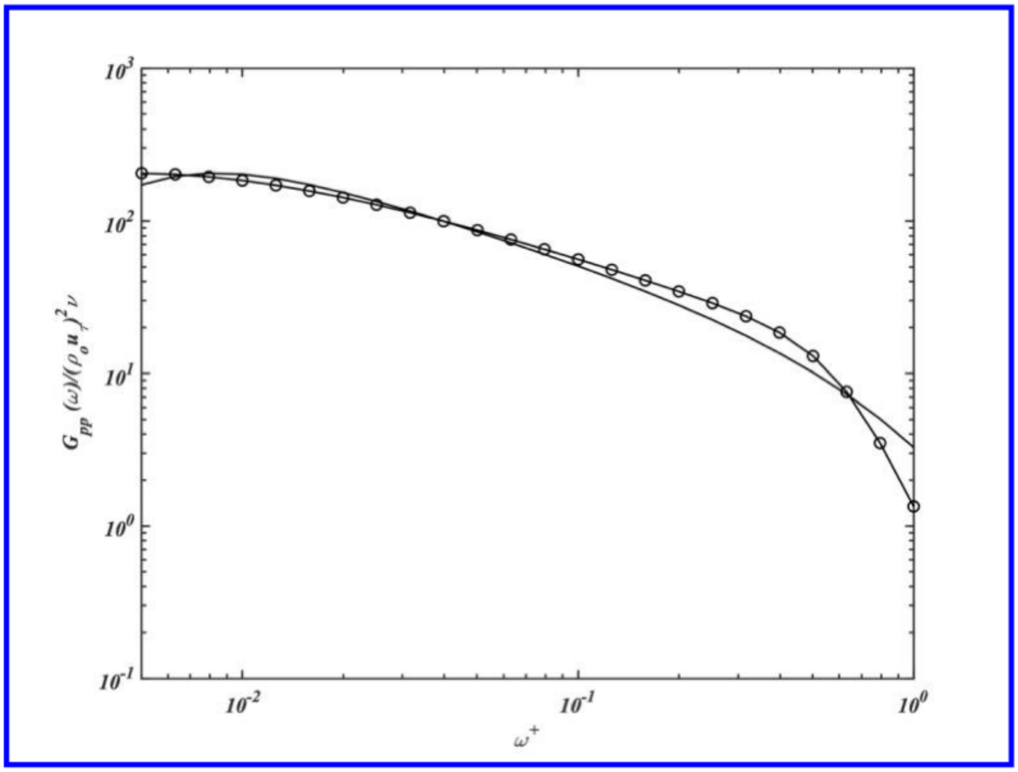

Figure 1: The predicted surface pressure spectrum from the vorticity model compared to the Goody model. Solid line with low frequency peak is prediction using the scaling with y_c=10 wall units, $U_c=15u_\tau$,,L=4000 wall units, and the circles are Goody's model.

VI. Modified Turbulent Boundary Layers

In their experiments on trailing edge noise Clark et al [17] modified the boundary flow near the trailing edge by using various types of finlets. Some of these finlet configurations were investigated using LES by Bodlin et al [18] who found that the primary features of the finlet flow were a strong streamwise vortex and a blockage effect that lifted the mean flow away from the trailing edge.

The mechanisms for trailing edge noise reduction suggested by these results are most easily interpreted using the vortex theory described above. Firstly, the strong streamwise vorticity is a feature of a junction flow in which the spanwise vorticity is distorted and stretched so it's core aligns with the mean flow. Kelvins theorem shows that the magnitude of the circulation associated with the spanwise vorticity does not change, but it's length scale is dramatically increased in the flow direction, and this reduces the high frequency content of the surface pressure spectrum. In addition to the stretching that takes place the modified mean flow lifts the vorticity away from the surface and this also reduces the high frequency content of the surface pressure spectrum based on the exponential factor $\exp(-\omega h/U)$ where h is the change in height above the surface caused by the lift up. For vorticity close to the surface, that typically has small scales, the stretching effect will be most important, and this is expected to scale on the finlet spacing. However, when the finlet spacing is so small that viscous blockage dominates the flow in the gaps then only the lift up mechanism will occur.

VII. Conclusions

This paper has discussed the modeling of the turbulent flow in a boundary layer as it relates to surface pressure fluctuations and trailing edge noise. It has been shown that the key feature of the turbulence that affects the radiated noise is the spanwise vorticity. Estimates of the turbulence wavenumber spectrum based on this model have been shown to agree well with experimental data of the surface pressure fluctuations.

The effects of modifying a turbulent boundary layer using surface mounted structures with the objective of reducing trailing edge noise has also been discussed. The hypothesis was made that the structures cause the vorticity in the flow to be stretched and elevated away from the surface. This can either increase or decrease the surface pressure spectrum on the surface, but in general the stretching of the vorticity reduces trailing edge noise which only depends on the vorticity component normal to the edge. Vortex stretching increases the length scale of the disturbance and this reduces the high frequency content of the spectrum. In addition, the blockage effects of finlet type structures near a trailing edge cause the mean flow to lift the vorticity away from the surface, and this gives additional reductions in trailing edge noise.

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