Optimal Power Control for Multi-User Relay Networks over Fading Channels

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***Abstract*—In energy-constrained wireless networks such as sensor networks, it is important to maximize power efficiency by minimizing power consumption for a given quality of service such as the data rate; it is equally important to evenly or fairly distribute power consumption to all nodes to maximize the network life. In this paper, we develop optimal power control methods to balance the tradeoff between energy efficiency and fairness for wireless cooperative networks where several relays assist the communication of multiple source-destination pairs. Our optimal power control policy is derived in a quasi-closed form by solving a convex optimization problem with a properly chosen cost-function. We further develop novel stochastic optimization algorithms to dynamically learn the statistics of the wireless channels on-the-fly to approach the optimal power control policy. Moreover, our power control algorithm can be implemented distributedly at relays requiring each relay to know its local channel state information. Simulation results demonstrate the merits of our power control methods.**

***Index Terms*—Energy-efficient design, wireless relay networks, convex optimization, stochastic approximation.**

I. Introduction

COOPERATIVE communication has recently been introduced to enhance the performance of wireless networks, since it can mitigate the effect of path loss in wireless links and achieve cooperative diversity to counter the detrimental effect of signal fading inherent to wireless channels, thereby significantly improving the network coverage and capacity [1]. Several resource allocation schemes have been developed to maximize the throughput of cooperative networks [2], [3], [4], [5]. Although energy-efficient resource allocation has received growing interest for noncooperative wireless networking [7], [8], [9], [10], it was considered for decode-and-forward based cooperative networks only in a few recent works [11], [12].

In this paper we investigate energy-efficient resource allocation for multi-user wireless cooperative networks, where a set of relays assist communications between multiple source and destination pairs using an amplify-and-forward (AF) strategy. Since the achieved rate for a source and destination pair is a concave function of the power consumed by relays [2], we formulate a convex optimization problem for finding the optimal power control policy at each relay over fading channels. Through minimization of a class of β-fair cost functions proposed in our recent work [6], we derive the optimal power control policy in a quasi-closed form. This policy can achieve an important tradeoff between total power consumed by all relays (energy efficiency) and the power consumed by each individual relay (fairness). Specifically, using a β-fair cost function with $\beta = 0$, we minimize the sum-power of all relays, which yields most energy-efficient (even if unfair) power allocation. The power control policy becomes fairer with a larger β. As $\beta \to \infty$, it approaches the min-max allocation that is sometimes deemed the fairest for multi-hop networking [13].

Moreover, whereas most of the existing alternatives were developed based on static optimization solutions without considering dynamics of random channels [1]–[4], our optimal power allocation takes into account the time-varying nature of fading channels without *a priori* knowledge of the cumulative distribution function (cdf) of the channels. Specifically, relying on stochastic optimization tools, we develop stochastic power control schemes which can learn the underlying channel distribution to approach the optimal strategy on-the-fly.

The rest of this paper is organized as follows. The network model is described in Section II. An optimization problem is formulated and solved to obtain optimal power control schemes in Section III. Numerical results are provided in Section IV to demonstrate the merits of the proposed schemes, followed by conclusions in Section V.

II. Network Model

Suppose that N relay nodes R_i, $i = 1, \ldots, N$, are available to assist the data transmissions from M sources S_j, $j = 1, \cdots, M$, to their corresponding destination nodes D_j, $j = 1, \ldots, M$, as depicted in Fig. 1. Orthogonal transmissions are assumed among different users for simultaneous communications of all source-destination pairs. With bandwidth equally divided into as many bands as the number of users, transmissions for each source-destination pair are carried out in an allocated frequency band. The relays cooperate with each source in a time multiplexing manner [3], as will be specified later.

The signals transmitted from the sources to relays and from relays to destinations are subject to random fading. Let $h_{S_{ij}}$ denote the fading channel coefficient for the link from source S_j to relay R_i, and $h_{R_{ij}}$ denote the channel coefficient from relay R_i to destination D_j, $\forall\ i = 1, \ldots, N,\ j = 1, \ldots, M$;

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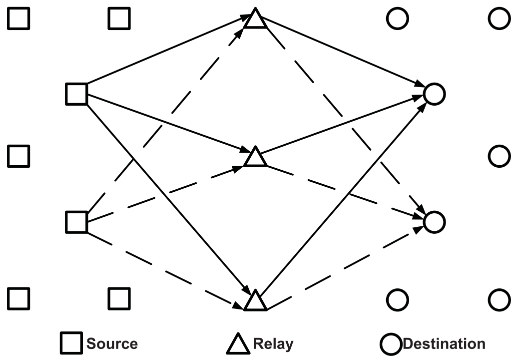

Fig. 1. A multi-user relay wireless network where three relays assist two source-destination pairs.

and let $\boldsymbol{h} := \{h_{S_{ij}}, h_{R_{ij}}, \ \forall ij\}$. We assume a block fading channel model, i.e., $\boldsymbol{h}$ remains invariant over a time slot whose duration is less than the coherent time of the channel, but is allowed to vary across successive slots according to a stationary and ergodic random process.

Denote P_{S_j} as the transmit power of S_j, $P_{R_{ij}}$ as the transmit power of relay R_i allocated for assisting S_j, and N_{R_i} and N_{D_j} are the variance of the additive white Gaussian noises (AWGN) at R_i and D_j, respectively. We assume that orthogonal transmissions are employed for AF cooperative networks to forward signals to the destination. Namely, each slot is equally divided into $N+1$ intervals. The source S_j broadcasts its signal to all the relays at the first interval, whereas each relay forward the signal to the destination D_j per interval in orders. The optimal decoder at a destination D_j effectively combines signals from all relays with a maximum-ratio combiner and the resulting signal-to-noise ratio (SNR) at D_j is given by[1]

$$\gamma_j = \sum_{i=1}^{N} \frac{P_{R_{ij}}}{\alpha_{R_{ij}} P_{R_{ij}} + \beta_{R_{ij}}} \tag{1}$$

where

$$\alpha_{R_{ij}} = \frac{\frac{N_{R_i}}{N+1}}{|h_{S_{ij}}|^2 P_{S_j}}, \quad \beta_{R_{ij}} = \frac{\frac{N_{D_j}}{N+1}\frac{N_{R_i}}{N+1}}{|h_{S_{ij}}|^2 |h_{R_{ij}}|^2 P_{S_j}} + \frac{\frac{N_{D_j}}{N+1}}{|h_{R_{ij}}|^2}. \tag{2}$$

The data rate for the source-destination pair $S_j \to D_j$ is then (in the unit of nats/sec/Hz):

$$r_j = \frac{1}{N+1}\log(1+\gamma_j). \tag{3}$$

It can be shown that r_j is a concave function of $P_{R_{ij}}$ [2, Lemma 1]. This concavity will help formulate a convex optimization for the problem under consideration in the sequel.

The system model in Fig. 1 are applicable to both cellular and ad hoc wireless networks where a number of relays are employed to assist the data transmissions from (multiple) sources to (multiple) destinations. Although a similar model is adopted in [2], [3], it only considered static wireless channels among sources, relays, and destinations. In contrast, we here consider more realistic time-varying fading channels and will develop optimal channel-adaptive power control schemes.

[1]We ignore the direct link from S_j to D_j for simplicity; incorporation of the SNR contribution from this link is straightforward.

III. Relay Power Optimization

Suppose that each source node S_j transmits with fixed power P_{S_j}, $j = 1, \ldots, M$. On the other hand, it is assumed that the relay nodes R_i, $i = 1, \cdots, N$, have full knowledge of current channel state information (CSI) $\boldsymbol{h}$, and they implement a power control policy $\boldsymbol{P}_R(\cdot)$ as a mapping from the channel state space to $\mathbb{R}_+^{NM}$, which allocates transmit power $\boldsymbol{P}_R(\boldsymbol{h}) := \{P_{R_{ij}}(\boldsymbol{h}), \ \forall ij\}$ to N relays that assist M sources. Although the ensuing derivations of our power control policy assume each relay knows $\boldsymbol{h}$, we will also show in Section III.D that our power control policy can be implemented in a distributed fashion which requires each relay to know its own local CSI.

For a given power control policy $\boldsymbol{P}_R(\cdot)$, let $\boldsymbol{P}_j(\boldsymbol{h}) := \{P_{R_{ij}}(\boldsymbol{h}), \ \forall i\}$ collect the power values for relays R_i, $i = 1, \cdots, N$, to assist source S_j at fading state $\boldsymbol{h}$. Then the maximum transmission rate over the link $S_j \to D_j$ for a given $\boldsymbol{h}$ is [cf. (3)]

$$r_j(\boldsymbol{P}_j(\boldsymbol{h})) = \frac{1}{N+1}\log(1+\gamma_j(\boldsymbol{P}_j(\boldsymbol{h}))) \tag{4}$$

where SNR $\gamma_j(\boldsymbol{P}_j(\boldsymbol{h}))$ is given by (1). Notice that since P_{S_j} is assumed to be fixed, $\alpha_{R_{ij}}$ and $\beta_{R_{ij}}$ in (2) are constants independent of the power control policy for a given $\boldsymbol{h}$.

Let $\bar{\boldsymbol{P}} := [\bar{P}_1, ..., \bar{P}_N]^T$ denote the vector of average power consumed by relay R_i, $i = 1, \ldots, N$. Our goal is to find an optimal joint power control policy $\boldsymbol{P}_R(\boldsymbol{h})$ for all relays by minimizing a properly chosen cost function of $\bar{\boldsymbol{P}}$, while ensuring a sufficiently large data rate for each source-destination pair. Let $\bar{\boldsymbol{a}} = [\bar{a}_1, ..., \bar{a}_M]^T$, where $\bar{a}_j$ is the minimum average data rate for the jth source-destination pair. Then, we formulate the following optimization problem for power control at all relays:

$$\begin{aligned} &\min_{\bar{\boldsymbol{P}} \geq \boldsymbol{0},\ \boldsymbol{P}_R(\boldsymbol{h}) \geq \boldsymbol{0}} \ \sum_{i=1}^{N} V(\bar{P}_i) \\ &\text{subject to} \ \ \bar{P}_i \geq \mathbb{E}_{\boldsymbol{h}}\Big[\sum_{j=1}^{M} P_{R_{ij}}(\boldsymbol{h})\Big], \ \forall i \\ &\qquad\qquad \mathbb{E}_{\boldsymbol{h}}\left[r_j(\boldsymbol{P}_j(\boldsymbol{h}))\right] \geq \bar{a}_j, \ \forall j \end{aligned} \tag{5}$$

where $V(\cdot)$ is a cost function that will be specified shortly, $\mathbb{E}_{\boldsymbol{h}}$ denotes the expectation over all fading realizations, and the notation $\boldsymbol{x} \geq 0$ for a vector $\boldsymbol{x} = [x_1, \cdots, x_N]^T$ stands for $x_i \geq 0$, $\forall i$. Since the major motivation for the design of energy-efficient networks is to maximize the network lifetime, it is important to fairly distribute power consumption among relay nodes. Hence a desirable cost function should facilitate an essential tradeoff between energy efficiency and fairness. To this end, we employ a class of β-fair cost functions defined as

$$V(\bar{P}_i) := V_\beta(\bar{P}_i) = (1+\beta)^{-1}\bar{P}_i^{1+\beta}. \tag{6}$$

This class of cost functions parameterized by a $\beta \geq 0$ was developed in our recent work [6]. It is clear that with $\beta = 0$, we have $V_\beta(\bar{P}_i) = \bar{P}_i$ and thus (5) is an average sum-power minimization problem which seeks most energy-efficient (even if unfair) power allocation at all relay nodes. In addition, it is shown in [6] that using a V_β with a larger β can result in a fairer power control scheme, and that as $\beta \to \infty$, the solution to (5) approaches the min-max power allocation that is sometimes deemed the fairest in multi-hop networking.

It is important to mention that $V_\beta(\bar{P}_i)$, $\forall\beta$, is a convex and increasing function of $\bar{P}_i$. It is also known from [2, Lemma 1] that $r_j(\boldsymbol{P}_j(\boldsymbol{h}))$ is a concave function of $\boldsymbol{P}_j(\boldsymbol{h})$. It then readily follows that (5) is a convex optimization problem, for which efficient solvers can be derived by drawing from rich convex programming tools [14], [15]. Notice that with the given fixed source powers, the optimization problem (5) may not be feasible if very large minimum rates $\bar{a}_j$ are requested by the sources. In this case, an admission control scheme should be invoked to render (5) feasible by dropping some source-destination links [21].

A. Lagrange Dual-based Approach

We next solve (5) using a Lagrangian dual based approach. Let $\boldsymbol{\lambda} := [\lambda_1, ..., \lambda_N]^T$ and $\boldsymbol{\mu} := [\mu_1, ..., \mu_M]^T$ collect the Lagrange multipliers associated with the constraints $\bar{P}_i \geq \mathbb{E}_{\boldsymbol{h}}[\sum_j P_{R_{ij}}(\boldsymbol{h})]$, $\forall i$, and $\mathbb{E}_h[r_j(\boldsymbol{P}_j(\boldsymbol{h}))] \geq \bar{a}_j$, $\forall j$, respectively. Then with the convenient notations $\boldsymbol{X} := \{\bar{\boldsymbol{P}},\ \boldsymbol{P}_R(\boldsymbol{h}), \forall \boldsymbol{h}\}$ and $\boldsymbol{\Lambda} := \{\boldsymbol{\lambda},\ \boldsymbol{\mu}\}$, the Lagrangian function of (5) is

$$\begin{aligned} L(\boldsymbol{X}, \boldsymbol{\Lambda}) &= \sum_{i=1}^{N} V_\beta(\bar{P}_i) + \sum_{i=1}^{N} \lambda_i \Big(\mathbb{E}_{\boldsymbol{h}} \Big[\sum_{j=1}^{M} P_{R_{ij}}(\boldsymbol{h}) \Big] - \bar{P}_i \Big) \\ &\quad + \sum_{j=1}^{M} \mu_j \left(\bar{a}_j - \mathbb{E}_{\boldsymbol{h}} \left[r_j(\boldsymbol{P}_j(\boldsymbol{h})) \right] \right) \\ &= \sum_{j=1}^{M} \mu_j \bar{a}_j + \sum_{i=1}^{N} \big(V_\beta(\bar{P}_i) - \lambda_i \bar{P}_i \big) + \\ &\quad \sum_{j=1}^{M} \mathbb{E}_{\boldsymbol{h}} \Big[\sum_{i=1}^{N} [\lambda_i P_{R_{ij}}(\boldsymbol{h})] - \mu_j r_j(\boldsymbol{P}_j(\boldsymbol{h})) \Big] \end{aligned} \tag{7}$$

The dual function is then given by

$$D(\boldsymbol{\Lambda}) = \min_{\boldsymbol{X} \geq \boldsymbol{0}} \ L(\boldsymbol{X}, \boldsymbol{\Lambda}) \tag{8}$$

and the dual problem of (5) is

$$\max_{\boldsymbol{\Lambda} \geq 0} \ D(\boldsymbol{\Lambda}) \tag{9}$$

Due to the convexity of (5), there is no duality gap between primal (5) and its dual problem (9). Therefore, the solution of (5) could be obtained by solving (9) [15].

To find the optimal $\boldsymbol{X}^*(\boldsymbol{\Lambda})$ that minimizes $L(\boldsymbol{X}, \boldsymbol{\Lambda})$ in (8), we need to solve two decoupled subproblems [cf. (7)]:

$$\min_{\bar{\boldsymbol{P}} \geq \boldsymbol{0}} \ \sum_{i=1}^{N} [V_\beta(\bar{P}_i) - \lambda_i \bar{P}_i] \tag{10}$$

$$\min_{\boldsymbol{P}_{\boldsymbol{R}}(\boldsymbol{h}) \geq \boldsymbol{0}} \ \sum_{j=1}^{M} \mathbb{E}_{\boldsymbol{h}} \Big[\sum_{i=1}^{N} [\lambda_i P_{R_{ij}}(\boldsymbol{h})] - \mu_j r_j(\boldsymbol{P}_j(\boldsymbol{h})) \Big] \tag{11}$$

It is clear that solving (10) is equivalent to solving:

$$\min_{\bar{P}_i \geq 0} \ \left[V_\beta(\bar{P}_i) - \lambda_i \bar{P}_i \right], \ \ \forall i \tag{12}$$

Using (6), we can find the solution to (12) as follows:

$$\bar{P}_i^*(\boldsymbol{\Lambda}) = {V'_\beta}^{-1}(\lambda_i) = (\lambda_i)^{1/\beta} \tag{13}$$

where ${V'_\beta}^{-1}$ denotes the (well-defined) inverse function of the first derivative of V_β.

The minimization in (11) can also be decoupled across different source-destination pairs and fading state $\boldsymbol{h}$. Hence (11) is equivalent to the following:

$$\min_{\boldsymbol{P}_j(\boldsymbol{h}) \geq \boldsymbol{0}} \ \Big[\sum_{i=1}^{N} [\lambda_i P_{R_{ij}}(\boldsymbol{h})] - \mu_j r_j(\boldsymbol{P}_j(\boldsymbol{h})) \Big], \quad \forall j, \forall \boldsymbol{h} \tag{14}$$

Recall that $r_j(\boldsymbol{P}_j(\boldsymbol{h}))$ is a concave function of $\boldsymbol{P}_j(\boldsymbol{h})$. Hence (14) is a convex optimization problem and its optimal solution

$$\boldsymbol{P}_j^*(\boldsymbol{h}; \boldsymbol{\Lambda}) := \arg \min_{\boldsymbol{P}_{R_j}(\boldsymbol{h}) \geq \boldsymbol{0}} \ \sum_{i=1}^{N} [\lambda_i P_{R_{ij}}(\boldsymbol{h})] - \mu_j r_j(\boldsymbol{P}_j(\boldsymbol{h})) \tag{15}$$

can be obtained via efficient algorithms. In fact, relying on the optimality condition for (14), we can derive the optimal $\boldsymbol{P}_j^*(\boldsymbol{h}; \boldsymbol{\Lambda})$ in a quasi-closed form, as will be elaborated in the subsequent subsection III-B.

Let us write the constraints in (5) in a compact form: $\boldsymbol{g}(\boldsymbol{X}) \leq \boldsymbol{0}$. Then for a given $\boldsymbol{\Lambda}$, with $\boldsymbol{X}^*(\boldsymbol{\Lambda}) := \{\bar{\boldsymbol{P}}^*(\boldsymbol{\Lambda}),\ \boldsymbol{P}_R^*(\boldsymbol{h}; \boldsymbol{\Lambda}), \forall \boldsymbol{h}\}$ available from (13) and (15), it can be shown that $\boldsymbol{g}(\boldsymbol{X}^*(\boldsymbol{\Lambda}))$ is a (sub-)gradient of the dual function $D(\boldsymbol{\Lambda})$ [14]. Therefore, the dual problem (9) can be solved through the following (sub-)gradient iteration

$$\boldsymbol{\Lambda}[n+1] = [\boldsymbol{\Lambda}[n] + s \cdot \boldsymbol{g}(\boldsymbol{X}^*(\boldsymbol{\Lambda}[n]))]^+ . \tag{16}$$

Specifically, we have

$$\begin{aligned} \lambda_i[n+1] &= \Big[\lambda_i[n] + s \cdot \Big(\mathbb{E}_{\boldsymbol{h}} \Big[\sum_{j=1}^{M} P^*_{R_{ij}}(\boldsymbol{h}; \boldsymbol{\Lambda}[n]) \Big] - \bar{P}_i^*(\boldsymbol{\Lambda}[n]) \Big) \Big]^+ \\ \mu_j[n+1] &= \Big[\mu_j[n] + s \cdot (\bar{a}_j - \mathbb{E}_{\boldsymbol{h}}[r_j^*(\boldsymbol{P}_j^*(\boldsymbol{h}; \boldsymbol{\Lambda}[n]))]) \Big]^+ \end{aligned} \tag{17}$$

where s is a small positive stepsize, n is the iteration index, and $[x]^+ := \max(0, x)$. Convergence of the gradient iteration (17) to the optimal Lagrange multipliers $\boldsymbol{\Lambda}^* := \{\boldsymbol{\lambda}^*,\ \boldsymbol{\mu}^*\}$ for (9) is guaranteed from any initial $\boldsymbol{\Lambda}(0) \geq \boldsymbol{0}$ [14].

Once the optimal $\boldsymbol{\Lambda}^*$ is obtained for (9), the zero duality gap between the primal (5) and the dual (9) implies that $\boldsymbol{X}(\boldsymbol{\Lambda}^*) = \{\bar{\boldsymbol{P}}^*(\boldsymbol{\Lambda}^*),\ \boldsymbol{P}_R^*(\boldsymbol{h}; \boldsymbol{\Lambda}^*), \forall \boldsymbol{h}\}$ also yields the optimal solution to (5). This then completes our quest for the optimal power control policy $\boldsymbol{P}_R^*(\boldsymbol{h}; \boldsymbol{\Lambda}^*)$ at all relays.

B. Optimal Power Control in Quasi-Closed Form

In this section we derive a quasi-closed form solution for the optimal power control defined in (15). Recall that the problem (14) is a convex optimization problem. Define

$$f(\boldsymbol{P}_j(\boldsymbol{h})) := \sum_{i=1}^{N} [\lambda_i P_{R_{ij}}(\boldsymbol{h})] - \mu_j r_j(\boldsymbol{P}_j(\boldsymbol{h})).$$

Suppose that for optimal $\boldsymbol{P}_j^*(\boldsymbol{h}; \boldsymbol{\Lambda})$, we have $P^*_{R_{ij}}(\boldsymbol{h}; \boldsymbol{\Lambda}) > 0$, $\forall i$. Then in this case we should have the gradient vector $\nabla f(\boldsymbol{P}_j(\boldsymbol{h}))$ equal to zero at the optimum; i.e., $\forall\ i$,

$$f'_i(\boldsymbol{P}_j^*(\boldsymbol{h})) := \left. \frac{\partial f(\boldsymbol{P}_j(\boldsymbol{h}))}{\partial P_{R_{ij}}(\boldsymbol{h})} \right|_{P_{R_{ij}}(\boldsymbol{h}) = P^*_{R_{ij}}(\boldsymbol{h})} = 0. \tag{18}$$

Using (3) and (18), we get (with $c := 1/(N+1)$)

$$\lambda_i = \frac{c\mu_j}{1+\sum_{n=1}^{N}\frac{P^*_{R_{nj}}(\boldsymbol{h};\boldsymbol{\Lambda})}{\alpha_{R_{nj}}P^*_{R_{nj}}(\boldsymbol{h};\boldsymbol{\Lambda})+\beta_{R_{nj}}}}\frac{\beta_{R_{ij}}}{(\alpha_{R_{ij}}P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})+\beta_{R_{ij}})^2}. \tag{19}$$

Defining convenient notations $a_i := \alpha_{R_{ij}}$, $b_i := \beta_{R_{ij}}$, and $x_i := P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})$, $\forall i$, we can then rewrite (19) as

$$\begin{cases}\lambda_1\left(1+\sum_{n=1}^{N}\frac{x_n}{a_nx_n+b_n}\right)=\frac{cu_jb_1}{(a_1x_1+b_1)^2}, & \text{(i)}\\ \lambda_i\left(1+\sum_{n=1}^{N}\frac{x_n}{a_nx_n+b_n}\right)=\frac{cu_jb_i}{(a_ix_i+b_i)^2}, \quad i>1. & \text{(ii)}\end{cases} \tag{20}$$

Since it is assumed that $P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})>0$, we have $(a_ix_i+b_i)>0$, $\forall i$. It then follows from (20) that

$$\sqrt{\frac{\text{(ii)}}{\text{(i)}}}=\sqrt{\frac{\lambda_i}{\lambda_1}}=\sqrt{\frac{b_i}{b_1}}\frac{a_1x_1+b_1}{a_ix_i+b_i} \tag{21}$$

Thus we can express x_i in terms of x_1 as

$$x_i=\frac{\sqrt{\frac{\lambda_1b_i}{\lambda_ib_1}}(a_1x_1+b_1)-b_i}{a_i}, \quad i>1. \tag{22}$$

Substituting (22) into (i) of (20), we obtain a standard quadratic equation with one unknown variable:

$$C_2x_1^2+C_1x_1+C_0=0 \tag{23}$$

where

$$\begin{aligned}C_2&=a_1^2+\sum_{i=1}^{N}\frac{a_1^2}{a_i},\\ C_1&=2a_1b_1+\sum_{i=1}^{N}\left(\frac{2a_1b_1}{a_i}-\frac{a_1b_i}{a_i}\sqrt{\frac{\lambda_ib_1}{\lambda_1b_i}}\right),\\ C_0&=b_1^2-\frac{c\mu_jb_1}{\lambda_1}+\sum_{i=1}^{N}\left(\frac{b_1^2}{a_i}-\frac{b_1b_i}{a_i}\frac{\lambda_ib_1}{\lambda_1b_i}\right).\end{aligned} \tag{24}$$

The solution x_1 for the equation (23) can be easily obtained in a closed-form: $x_1=(-C_1\pm\sqrt{C_1^2-4C_2C_0})/(2C_2)$. Subsequently, x_i, $i>1$, can be obtained from (22).

Recall that we assumed $x_i=P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})>0$, $\forall i$, in the foregoing derivation. However, it is not guaranteed that all the x_i obtained by solving (23) and (22) are positive. If this is true, (18) does not hold and the derivation from (19) to (24) becomes invalid.

In fact, without the assumption $P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})>0$, $\forall i$, the sufficient and necessary optimality condition for the optimal solution of convex optimization problem (14) is [14]:

$$[\nabla f(\boldsymbol{P}^*_j(\boldsymbol{h};\boldsymbol{\Lambda}))]^T(\boldsymbol{P}-\boldsymbol{P}^*_j(\boldsymbol{h};\boldsymbol{\Lambda}))\geq\boldsymbol{0} \tag{25}$$

for all $\boldsymbol{P}:=[P_1,\ldots,P_N]^T\geq\boldsymbol{0}$. Based on the complementary slackness condition for the optimal primal and dual variables, we prove in Appendix A that (25) implies the following:

$$\begin{cases}\text{if } P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})>0, & \text{then } f_i'(\boldsymbol{P}^*_j(\boldsymbol{h};\boldsymbol{\Lambda}))=0;\\ \text{if } f_i'(\boldsymbol{P}^*_j(\boldsymbol{h};\boldsymbol{\Lambda}))>0, & \text{then } P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})=0.\end{cases} \tag{26}$$

Notice that when $P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})=0$, we have either $f_i'(\boldsymbol{P}^*_j(\boldsymbol{h};\boldsymbol{\Lambda}))=0$ or $f_i'(\boldsymbol{P}^*_j(\boldsymbol{h};\boldsymbol{\Lambda}))>0$.

Based on (26), we can divided the whole index set $\mathcal{I}:=\{1,\ldots,N\}$ into two mutual exclusive subsets $\mathcal{I}_1$ and $\mathcal{I}_2$ (i.e., $\mathcal{I}_1\cap\mathcal{I}_2=\emptyset$ and $\mathcal{I}_1\cup\mathcal{I}_2=\mathcal{I}$), such that $P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})=0$, $\forall i\in\mathcal{I}_1$, whereas $P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})>0$, $\forall i\in\mathcal{I}_2$. An intuitive way to find the optimal power control $\boldsymbol{P}^*_j(\boldsymbol{h};\boldsymbol{\Lambda})$ is based on the exhaustive search over all the possible combinations of $\mathcal{I}_1$ and $\mathcal{I}_2$, as described in the following.

For a $(\mathcal{I}_1,\ \mathcal{I}_2)$ combination, we set $P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})=0$, $\forall i\in\mathcal{I}_1$, and $\forall i\in\mathcal{I}_2$, we solve the equations

$$\lambda_i = \frac{c\mu_j}{1+\sum_{n\in\mathcal{I}_2}\frac{P^*_{R_{nj}}(\boldsymbol{h};\boldsymbol{\Lambda})}{\alpha_{R_{nj}}P^*_{R_{nj}}(\boldsymbol{h};\boldsymbol{\Lambda})+\beta_{R_{nj}}}}\frac{\beta_{R_{ij}}}{(\alpha_{R_{ij}}P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})+\beta_{R_{ij}})^2}. \tag{27}$$

Based on (27), we can follow the derivations from (20) to (24) with the index set $\mathcal{I}$ replaced by $\mathcal{I}_2$ to obtain $P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})$, $\forall i\in\mathcal{I}_2$. If the resultant $P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})>0$, $\forall i\in\mathcal{I}_2$, and $f_i'(\boldsymbol{P}^*_j(\boldsymbol{h};\boldsymbol{\Lambda}))\geq 0$, $\forall i\in\mathcal{I}_1$, i.e., the optimality condition is satisfied, then the search is over and the optimal power control $\boldsymbol{P}^*_j(\boldsymbol{h};\boldsymbol{\Lambda})$ is obtained. Otherwise, we move to another $(\mathcal{I}_1,\ \mathcal{I}_2)$ combination to check whether a valid $\boldsymbol{P}^*_j(\boldsymbol{h};\boldsymbol{\Lambda})$ can be obtained. Although this method is technically sound, in the worst case we may need to search over all the 2^N possible $(\mathcal{I}_1,\ \mathcal{I}_2)$ combinations. Therefore, the computational complexity may be too high to afford unless the number of relays N is very small.

However, we can get a low-complexity solution by turning the exhaustive search into a "directional" search. To this end, we need the following lemma proved in Appendix B:

Lemma 1: *If $\lambda_i - c\mu_j/\beta_{R_{ij}}\geq 0$, then we must have $i\in\mathcal{I}_1$, i.e., $P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})=0$.*

Regarding λ_i as the unit power price and μ_j as the unit rate value, optimization in (14) is to minimize the net-cost (power cost minus rate reward) of data transmission for the jth source-destination pair. The partial first derivative $f_i'(\boldsymbol{P}_j(\boldsymbol{h}))$ thus stands for the "marginal net-cost gain" with respect to $P_{R_{ij}}(\boldsymbol{h})$, and the value $\lambda_i - c\mu_j/\beta_{R_{ij}}$ is the marginal net-cost gain $f_i'(\boldsymbol{0})$ evaluated at $\boldsymbol{P}^*_j(\boldsymbol{h})=\boldsymbol{0}$. It is easy to show that $f_i'(\boldsymbol{P}_j(\boldsymbol{h}))>\lambda_i-c\mu_j/\beta_{R_{ij}}$ for any $P_{R_{ij}}(\boldsymbol{h})>0$; hence marginal net-cost gain $f_i'(\boldsymbol{P}_j(\boldsymbol{h}))>0$ for $P_{R_{ij}}(\boldsymbol{h})>0$ if $\lambda_i-c\mu_j/\beta_{R_{ij}}\geq 0$. In this case, using any positive relay power $P_{R_{ij}}(\boldsymbol{h})$ increase the net-cost; so the optimal power for relay i that minimize the net-cost must be $P^*_{R_{ij}}(\boldsymbol{h};\boldsymbol{\Lambda})=0$. It is then clear that $\lambda_i-c\mu_j/\beta_{R_{ij}}$ serves an important channel quality indicator for power control decision. Only when the related channel gains $|h_{R_{ij}}|^2$ and $|h_{S_{ij}}|^2$ are large enough such that $\beta_{R_{ij}}$ is small [cf. (2)] and $\lambda_i-c\mu_j/\beta_{R_{ij}}$ becomes negative, it is worth for the relay i considering forwarding its received signal to destination j with a positive power.

If we define an index set $\mathcal{S}_1:=\{i\mid\lambda_i-c\mu_j/\beta_{R_{ij}}\geq 0\}$, Lemma 1 implies that $\mathcal{S}_1\subseteq\mathcal{I}_1$. Then the search space is reduced from $\mathcal{I}$ to $\mathcal{S}_2:=\mathcal{I}-\mathcal{S}_1$. To avoid exhaustive search over $\mathcal{S}_2$, we further prove the following lemma in Appendix C:

Lemma 2: *Suppose $\lambda_i\beta_{R_{ij}}\geq\lambda_k\beta_{R_{kj}}$. Then if $i\in\mathcal{I}_2$, we must also have $k\in\mathcal{I}_2$.*

The value of $\lambda_i\beta_{R_{ij}}$ is determined by the product of the ith relay's power price λ_i and $\beta_{R_{ij}}$ that is inversely proportional to the channel gains $|h_{R_{ij}}|^2$ and $|h_{S_{ij}}|^2$. It follows from

Lemma 2 that this value can be seen as the priority for i to be selected into the set $\mathcal{I}_2$. The smaller $\lambda_i\beta_{R_{ij}}$, the higher priority for relay i is selected to forward its received signal with a positive power; and a relay node cannot be selected unless all relay nodes with higher priority have been selected. As such, we can sort $\lambda_i\beta_{R_{ij}}$, $\forall i \in \mathcal{S}_2$, in a decreasing order to obtain a permutation of indices in $\mathcal{S}_2$: $\boldsymbol{\pi} := \{\pi(1), \ldots, \pi(|\mathcal{S}_2|)\}$ with $\lambda_{\pi(i)}\beta_{R_{\pi(i)j}} \geq \lambda_{\pi(i+1)}\beta_{R_{\pi(i+1)j}}$, $\forall i = 1, \ldots, |\mathcal{S}_2| - 1$. If we have a $\pi(i) \in \mathcal{I}_2$, then Lemma 2 implies that $\pi(k) \in \mathcal{I}_2$, $\forall k > i$. Hence the index set $\mathcal{I}_2$ can only be one of the following $|\mathcal{S}_2|$ subsets of $\mathcal{S}_2$: $\{\pi(1), \ldots, \pi(|\mathcal{S}_2|)\}$, $\{\pi(2), \ldots, \pi(|\mathcal{S}_2|)\}$, $\ldots$, $\{\pi(|\mathcal{S}_2|)\}$.

Using the "directional" search strategy implied by Lemmas 1 and 2, we have the following efficient algorithm to obtain the optimal power control $\boldsymbol{P}_j^*(\boldsymbol{h}; \boldsymbol{\Lambda})$:

Algorithm 1: *Search for Optimal Power Control:*

1. Calculate $\lambda_i - c\mu_j/\beta_{R_{ij}}$, $\forall i$. Put all the indices of $\mathcal{S}_1 = \{i \mid \lambda_i - c\mu_j/\beta_{R_{ij}} \geq 0\}$ into the set $\mathcal{I}_1$; i.e., set $P_{R_{ij}}^*(\boldsymbol{h}; \boldsymbol{\Lambda}) = 0$, $\forall i \in \mathcal{S}_1$.

2. Let $\mathcal{S}_2 = \mathcal{I} - \mathcal{S}_1$, and sort all the $|\mathcal{S}_2|$ indices of $\mathcal{S}_2$ in the decreasing order of $\lambda_i\beta_{R_{ij}}$ to obtain the permutation $\boldsymbol{\pi}$.

3. Initialize $n = 1$, and perform the following: Let $\mathcal{I}_1 = \mathcal{S}_1 \cup \{\pi(1), \ldots, \pi(n-1)\}$,[2] and $\mathcal{I}_2 = \{\pi(n), \ldots, \pi(|\mathcal{S}_2|)\}$; i.e., further set $P_{R_{\pi(k)j}}^*(\boldsymbol{h}; \boldsymbol{\Lambda}) = 0$ for $k = 1, \ldots, n-1$, and pose the equations (27) for $k = n, \ldots, |\mathcal{S}_2|$. After obtaining the closed-form solutions $P_{R_{\pi(k)j}}^*(\boldsymbol{h}; \boldsymbol{\Lambda})$, $k = n, \ldots, |\mathcal{S}_2|$, for (27) through (20)–(24), check the optimality condition (25). Output the resultant $\boldsymbol{P}_j^*(\boldsymbol{h}; \boldsymbol{\Lambda})$ if the condition is satisfied; otherwise let $n = n + 1$ and perform this step again.

Some comments are in order:

- The optimal $\boldsymbol{P}_j^*(\boldsymbol{h}; \boldsymbol{\Lambda})$ satisfying (25) for the convex optimization problem (14) always exists, and Algorithm 1 will find $\boldsymbol{P}_j^*(\boldsymbol{h}; \boldsymbol{\Lambda})$ in step 3.
- When solving the equations (27), we can define $x_1 = P_{R_{\pi(n)j}}^*(\boldsymbol{h}; \boldsymbol{\Lambda})$, and express all other variables x_i in terms of this x_1. Since $\lambda_{\pi(n)}\beta_{R_{\pi(n)j}} \geq \lambda_{\pi(k)}\beta_{R_{\pi(k)j}}$, $\forall k = n+1, \ldots, |\mathcal{S}_2|$, (22) implies that we have $x_i > 0$, $\forall i$, if $x_1 > 0$. Hence we can simply check whether the solution $x_1 = (-C_1 + \sqrt{C_1^2 - 4C_2C_0})/(2C_2)$ is positive to find out whether (25) is satisfied.
- The maximum number of iterations in Algorithm 1 is $|\mathcal{S}_2| \leq N$. In each iteration, the possible solution for $\boldsymbol{P}_j^*(\boldsymbol{h}; \boldsymbol{\Lambda})$ can be easily obtained in a closed-form. Hence, even in the worst case, the computational complexity of Algorithm 1 is in the order of $\mathcal{O}(N)$.

In summary, we have the following proposition:

Proposition 1: *For a given $\boldsymbol{\Lambda}$ and $\boldsymbol{h}$, the optimal $\boldsymbol{P}_j^*(\boldsymbol{h}; \boldsymbol{\Lambda})$ for (14) can be obtained in closed-form through Algorithm 1 with the worst case complexity of $\mathcal{O}(N)$.*

Notice that since (14) is a convex optimization problem, its solution can also be obtained through available convex programming software packages. For instance, a similar problem in [2][3] is solved using the Matlab CVX toolbox. In CVX package (as well as other software packages), classic algorithms such as (iterative) interior point methods are employed to solve general convex programs [16]. In contrast, we rely on the optimality condition to directly derive the optimal solution for the problem at hand in a quasi-closed form. Our solution can reveal the specific structure of the optimal power control policy, which cannot be provided by standard software packages, thereby providing more insights for relay power optimization. Moreover, our algorithm could be much more efficient in terms of computational complexity. Whereas the proposed algorithm has a worst case complexity of $\mathcal{O}(N)$, the interior point methods typically have a complexity higher than $\mathcal{O}(N^3)$ [15]. This is also corroborated by our simulations, which indicate that the cpu time for Algorithm 1 to obtain $\boldsymbol{P}_j^*(\boldsymbol{h}; \boldsymbol{\Lambda})$ for a three-relay network is less than 0.005% (5×10^{-5}) of that required by the CVX program.

C. Stochastic Power Control

With $\boldsymbol{P}_j^*(\boldsymbol{h}; \boldsymbol{\Lambda})$ available from Algorithm 1 and $\bar{\boldsymbol{P}}^*(\boldsymbol{\Lambda})$ given in (13), the gradient iteration (16) can find the optimal solution for (5) which yields an energy-efficient and fair resource allocation policy. To implement (16), we need the knowledge of fading channel cdf to evaluate the two expected values in (17). While channel cdf is required for theoretic studies in e.g., [8], practical mobile applications may need power control schemes that can operate without the knowledge of channel cdf, but approaching the optimal strategy by "learning" channel statistics on-the-fly.

It turns out that this can be achieved through a stochastic optimization paradigm [17], [18], [19]. Using this approach, we develop a *stochastic gradient* iteration based on (16) to solve (5) without knowing the channel cdf *a priori*. Specifically, we drop $\mathbb{E}_{\boldsymbol{h}}$ from (17) and then get a stochastic version of (17) based on per slot fading realization $\boldsymbol{h}[n]$ as follows:

$$\hat{\lambda}_i[n+1] = \left[\hat{\lambda}_i[n] + s \cdot \left(\sum_{j=1}^{M} P_{R_{ij}}^*(\boldsymbol{h}[n]; \hat{\boldsymbol{\Lambda}}[n]) - \bar{P}_i^*(\hat{\boldsymbol{\Lambda}}[n])\right)\right]^+$$
$$\hat{\mu}_j[n+1] = \left[\hat{\mu}_j[n] + s \cdot (\bar{a}_j - r_j(\boldsymbol{P}_j^*(\boldsymbol{h}[n]; \hat{\boldsymbol{\Lambda}}[n])))\right]^+ \tag{28}$$

where the hat notation with λ_i and μ_j is used to stress the fact that these iterations are stochastic estimates of those in (17), based on *instantaneous* (instead of average) power and rates. Notice that here n stands for both iteration and slot indices; in other words, each iterate of (28) will be run per slot. As with (16), we can also re-write (28) in a compact form:

$$\hat{\boldsymbol{\Lambda}}[n+1] = \left[\hat{\boldsymbol{\Lambda}}[n] + s \cdot \hat{\boldsymbol{g}}\left(\boldsymbol{h}[n]; \boldsymbol{X}^*(\hat{\boldsymbol{\Lambda}}[n])\right)\right]^+ \tag{29}$$

where $\hat{\boldsymbol{g}}\left(\boldsymbol{h}[n]; \boldsymbol{X}^*(\hat{\boldsymbol{\Lambda}}[n])\right)$ is the stochastic gradient depending on current fading realization $\boldsymbol{h}[n]$. Since a stationary and ergodic random fading process is assumed, we have $\mathbb{E}_{\boldsymbol{h}}[\hat{\boldsymbol{g}}(\boldsymbol{h}; \boldsymbol{X}^*(\boldsymbol{\Lambda}))] = \boldsymbol{g}(\boldsymbol{X}^*(\boldsymbol{\Lambda}))$, i.e., stochastic gradient $\hat{\boldsymbol{g}}$ is a random realization (or an unbiased estimate based on a single fading realization) of the "ensemble" gradient $\boldsymbol{g}$. Therefore,

[2]For $n = 1$, the set $\{\pi(1), \ldots, \pi(n-1)\}$ is an empty set.

[3]The problem in [2] is: $\max_{\boldsymbol{P}_j \geq \boldsymbol{0}} \left[w_j r_j(\boldsymbol{P}_j) - \sum_{i=1}^{N}[\lambda_i P_{R_{ij}}]\right]$.

(29) and (16) form a pair of *primary and averaged systems* [20].

In fact, the proposed stochastic gradient iteration (29) belongs to the same class as the well-known least-mean-square (LMS) algorithm in adaptive signal processing. As with the LMS algorithm, convergence of such a stochastic iteration can be established by the stochastic locking theorem, which justifies that the trajectory of the primary system (29) is always "locked" or stays close to that of the averaged system (16) in probability under some regularity conditions (primarily stochastic Lipschitz conditions for system perturbations), if a sufficiently small stepsize s is used [20]. It can be verified that these regularity conditions are satisfied for the primary and averaged systems of the wireless setup here, provided that the random fading channel has a continuous cdf. Indeed, we have the following lemma (proof of the lemma mimics the counterparts in our recent work [21], [22] and is omitted for conciseness):[4]

Lemma 3: *For ergodic fading channels with continuous cdf, if the primary system (16) and its averaged system (29) are initialized with $\hat{\mathbf{\Lambda}}[0] \equiv \mathbf{\Lambda}[0]$, then it holds over any time interval T that*

$$\max_{1 \le n \le T/s} \|\hat{\mathbf{\Lambda}}[n] - \mathbf{\Lambda}[n]\| \le c_T(s) \quad \text{w.p. } 1 \ , \qquad (30)$$

with the constant $c_T(s) \to 0$ as the stepsize $s \to 0$.

Since convergence of iteration (16) to $\mathbf{\Lambda}^*$ is guaranteed, the trajectory locking stated in Lemma 3 then implies that iteration (29) also converges to the optimal $\mathbf{\Lambda}^*$ in probability as stepsize $s \to 0$; i.e., $\lim_{n\to\infty} \Pr(|\hat{\mathbf{\Lambda}}[n] - \mathbf{\Lambda}^*| \ge \epsilon) = 0$ for any $\epsilon > 0$. Hence the resulting stochastic power control scheme $\boldsymbol{P}_R^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$ approaches the globally optimal solution of (5) on-the-fly. Depending on the stepsize s, there is actually a tradeoff emerging between convergence speed and optimality in stochastic optimization algorithms. This is a well-known tradeoff especially in the adaptive signals and systems literature [17], [20], [21]. As in any stochastic approximation scheme, the Lagrange multipliers in (16) only converge to or hover within a small neighborhood with size proportional to the stepsize s around optimal values; hence, one needs a small s to come "closer" to optimality, but the smaller the s is chosen, the slower convergence speed is experienced.

Convergence of stochastic iteration (29) to $\mathbf{\Lambda}^*$ in probability as $s \to 0$ can also be established through the fluid limit and/or Lyapunov drift techniques, when the traffic and fading process can be modeled as a finite-state Markov process. Under the latter condition, the proof can be readily derived along the similar lines to those in [17], [18], [19].

Relying on (29), we can then put forth a simple stochastic power control algorithm:

Algorithm 2: *On-line Stochastic Gradient Iterations:*
initialize with any $\hat{\mathbf{\Lambda}}[0]$;
repeat on-line: with $\hat{\mathbf{\Lambda}}[n]$ and $\boldsymbol{h}[n]$ available per slot n, perform the power control $\boldsymbol{P}_R^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$ at the relay nodes, and update $\hat{\mathbf{\Lambda}}[n+1]$ using (29).

---

[4]The lemma is indeed a variant of [20, Theorem 9.1] with our notations.

In Algorithm 2, we need to obtain the power control strategy $\boldsymbol{P}_R^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$ for the given $\hat{\mathbf{\Lambda}}[n]$ and $\boldsymbol{h}[n]$ per slot. Recall that the optimal $\boldsymbol{P}_j^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$, $\forall j = 1, \ldots, M$, can be calculated in a quasi-closed form using Algorithm 1 with the worst case complexity $\mathcal{O}(N)$. Hence obtaining $\boldsymbol{P}_R^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$ per iteration has a low complexity of $\mathcal{O}(MN)$ in the worst case. Using $\boldsymbol{P}_R^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$ and $\bar{\boldsymbol{P}}^*(\hat{\mathbf{\Lambda}}[n])$ from (13), $\hat{\mathbf{\Lambda}}[n+1]$ can be updated and then used for determining the relay-power control in the next slot. To further appreciate the significance of Algorithm 2, we stress the following result implied by Lemma 3 and convergence of (16).

Proposition 2: *For ergodic fading channels with continuous cdf, the power control $\boldsymbol{P}_R^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$ in Algorithm 2 approaches the globally optimal solution of (5) in probability.*

Although Proposition 2 states that convergence (and the asymptotic optimality) of Algorithm 2 is guaranteed when fading channels are stationary and ergodic, it is worth mentioning that due to its "stochastic learning" capability, the proposed stochastic iteration can also track non-stationary channels (induced by e.g., node mobility) by "re-learning" time-varying channel statistics and converge to a new optimum solution under the new condition. In summary, Algorithm 2 provides a desirable low-complexity approach to solving the optimization problem (5) to obtain a fair and energy-efficient power control strategy at all relay nodes.

D. Distributed Implementation

To apply our stochastic power control scheme to practical relay networks, we can implement Algorithm 2 in a distributed manner. Recall that the power control policy $\boldsymbol{P}_R^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$ needs to be found for a given $\hat{\mathbf{\Lambda}}[n]$ and $\boldsymbol{h}[n]$ in time slot n. This amounts to finding $\boldsymbol{P}_j^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$, $\forall j$. It is not difficult to see that for each j, solving (14) to get $\boldsymbol{P}_j^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$ only requires knowing the channel coefficients associated with the link $S_j \to D_j$, i.e., $h_{S_{ij}}$ and $h_{R_{ij}}$, $\forall i$. Since D_j can estimate $h_{R_{ij}}$, $\forall i$, and receive estimated $h_{S_{ij}}$, $\forall i$, from relays, it can run Algorithm 1 to obtain $\boldsymbol{P}_j^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$, which is then fed back and implemented at relays. Using $\boldsymbol{P}_j^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$, $\forall j$, from all destination nodes, each relay i can update $\hat{\lambda}_i[n+1]$, which is in turn fed forward to D_j, $\forall j$. With $\boldsymbol{P}_j^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$ available, D_j can also calculate the value of $r_j(\boldsymbol{P}_j^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])))$, and subsequently update $\hat{\mu}_j[n+1]$. Then, each D_j has all necessary information to solve (14) and make a power control decision for the next slot. Notice that each destination D_j can determine its SNR in (1) and calculate the data rate from (3). This data rate can be fed back to the source so that the source S_j can choose a proper coding scheme.

In summary, the stochastic power control can be implemented distributedly as follows:

- Initialize the Lagrange multipliers $\hat{\lambda}_i[0]$ and $\hat{\mu}_j[0]$ arbitrarily at each relay node R_i and destination node D_j, respectively.
- Per slot n, each D_j collects channel coefficients $\{h_{S_{ij}}, h_{R_{ij}}, \ \forall i\}$ and employs Algorithm 1 to obtain $\boldsymbol{P}_j^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$; it also update $\hat{\mu}_j[n+1]$ and feed back $P_{R_{ij}}^*(\boldsymbol{h}[n]; \hat{\mathbf{\Lambda}}[n])$ to R_i, $\forall i$.

- Per slot n, each R_i implements power control policy $\{P^*_{R_{ij}}(\boldsymbol{h}[n]; \hat{\boldsymbol{\Lambda}}[n]),\ \forall j\}$, update and then feed forward $\hat{\lambda}_i[n+1]$ to all destination nodes.

The distributed scheme here is in fact devised following the optimization decomposition paradigm [23]. As such, the whole problem is decomposed into subproblems solved at each D_j with local CSI, while $\hat{\lambda}_i[n+1]$, $\forall i$, are exchanged between R_i and D_j to coordinate global optimization.

IV. Numerical Results

We considered a wireless relay network with ten sources, ten destinations and three relays, operating with a system bandwidth $B = 100$ KHz and a time-slot duration $T_s = 1$ ms. The relays are located at coordinates (3,2), (5,2) and (7,2), whiles the source and destination nodes are deployed at the lines {(1,0) – (10,0)} and {(1,3) – (10,3)}, respectively. The average SNR for the link between nodes i and j is given by $\bar{\gamma}_{ij} = \frac{\bar{\gamma}}{((x_i-x_j)^2+(y_i-y_j)^2)^{n/2}}$, where the reference SNR $\bar{\gamma} = 10$ dBW, and the path loss exponent n was chosen to be 3.6 in simulations. The fading processes for different links were assumed to be independent and were generated from a Rayleigh distribution with the corresponding variance $\bar{\gamma}_{ij}$. The variances of AWGN N_{R_i} and N_{D_j} at relay and destination were assumed to be 4. The transmit power P_{S_j} for all source nodes was equal to 1 Watt. Using a stepsize $s = 0.005$, we ran the proposed stochastic power control algorithm with a data rate $\bar{a}_j = 35$ Kbps, $\forall j$, and several different values for β in the β-fair cost function in (5). Figure 2 (top) shows the average sum-power of three relay nodes, while Fig. 2 (bottom) depicts the average power consumed by each individual relay. It is observed that when $\beta = 0$, we indeed minimize the average sum-power in (5) and get the most efficient power allocation. However, this was achieved in an unfair manner as shown in Fig. 2 (bottom) where the third relay consumes much more power than the first relay. With a larger β, it is seen that more total power is consumed but fairness improves. For instance, when $\beta = 16$, all links consume almost the same average power, but about 5% more total power is consumed than the case with $\beta = 0$. These results demonstrate that different β-fair cost functions can nicely trade off power efficiency with fairness.

To gauge the performance of the proposed schemes, we consider two baseline heuristic schemes. For both schemes, the data rates $\bar{a}_j = 35$ Kbps are maintained per slot; i.e., an instantaneous (instead of average) rate constraint is enforced. In the first scheme relays use same transmit-powers to forward the signals for a $S_j \rightarrow D_j$ link but different powers are allowed across different source-destination links; whereas in the second scheme relays must use same transmit-powers for all source-destination links. Since instantaneous data rates are enforced, the heuristic schemes ignore the temporal diversity that the random fading brings. Hence much more powers need to be spent. Note that the second heuristic scheme ignores also part of the spatial diversity by enforcing same transmit-powers for all links, which incurs additional power consumptions. Table 1 shows the comparison of the individual average relay powers and average sum-power consumptions for the heuristic schemes and the proposed optimal scheme with $\beta = 16$.

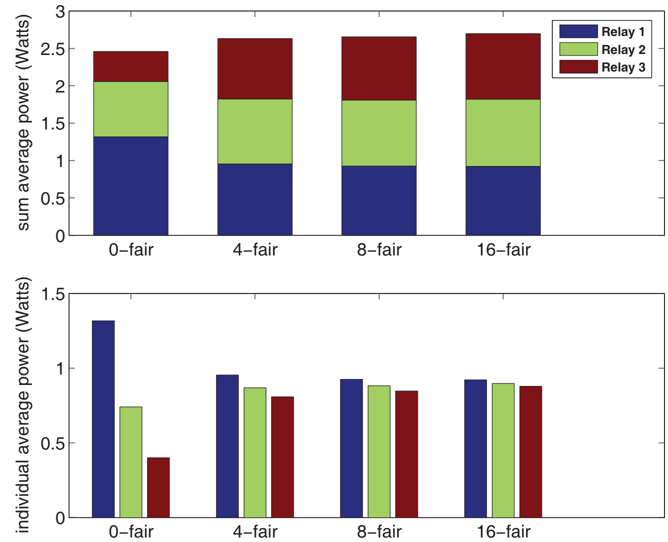

Fig. 2. Average sum-powers of all relays (top) and the powers consumed by individual relays (bottom).

TABLE I
COMPARISON OF RELAY POWER CONSUMPTIONS IN WATTS. (HEU1' AND HEU2' CORRESPOND TO HEU1 AND HEU2 ALLOWING 1% OUTAGE.)

	optimum	heu1	heu2	heu1'	heu2'
sum power	2.70	667.47	2777.13	81.90	213.54
relay 1 power	0.92	222.49	925.71	27.30	71.18
relay 2 power	0.90	222.49	925.71	27.30	71.18
relay 3 power	0.88	222.49	925.71	27.30	71.18

Our simulations show that compared to the proposed optimal scheme, about 247 times and 1028 times total powers are required for the two heuristic schemes, respectively. This is partly due to the very large power consumptions for these two schemes when a deep channel fading occurs. To make the heuristic schemes more efficient, we consider allowing 1% outage such that the relays defer transmissions upon deep fades. Although the heuristic schemes enforce the fairness in power consumptions among relays, it is seen that these two schemes allowing 1% outage still require more than 30 times and 78 times total powers than the derived optimal schemes. This clearly shows that the proposed energy-efficient schemes can result in significant power savings.

The fading cdf was assumed unknown in all simulations, and the proposed stochastic scheme was supposed to learn this knowledge on-the-fly to approach the optimal power control policy. To verify this, Fig. 3 depicts the evolutions of Lagrange multipliers $\hat{\lambda}_i$, $i = 1, 2, 3$, and $\hat{\mu}_j$, $j = 1, 2, 3, 4$,[5] in (28) for $\beta = 2$. Stochastic convergence of the iterations is clearly observed in Fig. 3, since the Lagrange multipliers converge to or hover within a small neighborhood (with a size proportional to stepsize s) around their corresponding optimal values $\lambda^*_1 = 0.9811$, $\lambda^*_2 = 0.7053$, $\lambda^*_3 = 0.5626$, $\mu^*_1 = 2.7228$, $\mu^*_2 = 1.6518$, $\mu^*_3 = 1.1425$, and $\mu^*_4 = 0.9143$.

It is also worth mentioning that our quasi-closed form solution for $\boldsymbol{P}^*_j(\boldsymbol{h}; \boldsymbol{\Lambda})$ can save significant cpu time than the

[5]The evolutions of $\hat{\mu}_j$, $j = 5, \ldots, 10$, are omitted for conciseness.

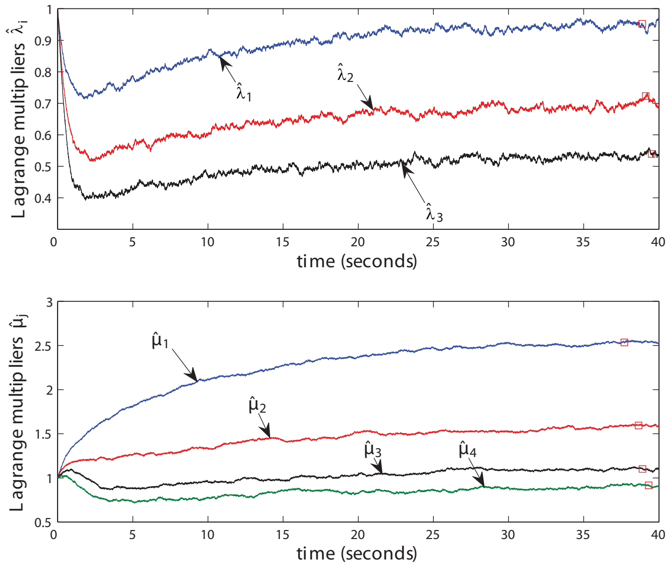

Fig. 3. Evolutions of Lagrange multipliers (the optimal values are indicated by square markers).

Matlab CVX toolbox version 1.0 [16]. Using Matlab 7.4.0 in Microsoft Windows XP with Intel Core 2 Duo CPU (T7500 2.20Hz) and 3GB of RAM , our simulations indicated that the average cpu time that Algorithm 1 took to obtain $\boldsymbol{P}_j^*(\boldsymbol{h};\boldsymbol{\Lambda})$ per iteration was 3.13×10^{-5} second, whereas the average cpu time for the same task with the CVX program was 1.25 seconds. In other words, our algorithm can save 99.995% cpu time compared to the CVX program.

V. Conclusions

We formulated a convex optimization problem for optimal power control in cooperative wireless networks over fading channels. The objective function was such chosen that the energy efficiency and fairness can be addressed jointly. Using a Lagrangian dual-based approach, we solved the optimization problem and derived the optimal power control policy in a quasi-closed form. We further developed a novel stochastic power method that can learn the statistics of the fading channels and adaptively approach the optimal strategy on-the-fly. Moreover, we showed that our power control method can be implemented in a distributed fashion at each relay, which only requires each relay to known its own local channel state information. Our simulations demonstrated that our power control policy indeed can achieve the balance between energy efficiency and fairness.

Appendix

A. Proof of (26) via Complementary Slackness Condition

Using the definition of $f(\boldsymbol{P}_j(\boldsymbol{h}))$, we rewrite (14) as

$$\min_{\boldsymbol{P}_j(\boldsymbol{h})} f(\boldsymbol{P}_j(\boldsymbol{h})) \quad \text{s. to} \quad -P_{R_{ij}}(\boldsymbol{h}) \le 0, \quad \forall i \tag{31}$$

With $\boldsymbol{\nu} := [\nu_1, \ldots, \nu_N]^T$ collecting the Lagrange multipliers for $-P_{R_{ij}}(\boldsymbol{h}) \le 0$, $\forall i$, the Lagrangian of (31) is:

$$L(\boldsymbol{P}_j(\boldsymbol{h}), \boldsymbol{\nu}) = f(\boldsymbol{P}_j(\boldsymbol{h})) - \sum_{i=1}^{N} \nu_i P_{R_{ij}}(\boldsymbol{h}). \tag{32}$$

From the complementary slackness condition for the optimal primal $\boldsymbol{P}_{R_j}^*(\boldsymbol{h})$ and dual variables $\boldsymbol{\nu}^*$ [15], we have:

$$\begin{cases} \nu_i^* > 0 \;\Rightarrow\; P_{R_{ij}}^*(\boldsymbol{h}) = 0, \\ P_{R_{ij}}^*(\boldsymbol{h}) > 0 \;\Rightarrow\; \nu_i^* = 0. \end{cases} \tag{33}$$

According to the Karush-Kuhn-Tucker (KKT) optimality condition, we also have

$$\left.\frac{\partial L(\boldsymbol{P}_j(\boldsymbol{h}), \nu^*)}{\partial P_{R_{ij}}(\boldsymbol{h})}\right|_{P_{R_{ij}}(\boldsymbol{h}) = P_{R_{ij}}^*(\boldsymbol{h})} = f_i'(\boldsymbol{P}_j^*(\boldsymbol{h})) - \nu^* = 0. \tag{34}$$

Namely, $f_i'(\boldsymbol{P}_j^*(\boldsymbol{h})) = \nu^*$. This together with (33) then readily implies (26).

B. Proof of Lemma 1

Suppose $\lambda_i - c\mu_j/\beta_{R_{ij}} \ge 0$. Using again the convenient notations $a_i := \alpha_{R_{ij}}$, $b_i := \beta_{R_{ij}}$, and $x_i := P_{R_{ij}}^*(\boldsymbol{h};\boldsymbol{\Lambda})$, $\forall i$, we can write

$$\begin{aligned} f_i'(\boldsymbol{P}_j^*(\boldsymbol{h};\boldsymbol{\Lambda})) &= \lambda_i - \frac{c\mu_j}{1 + \sum_{n=1}^{N} \frac{x_n}{a_n x_n + b_n}} \frac{b_i}{(a_i x_i + b_i)^2} \\ &\ge \lambda_i - c\mu_j \frac{b_i}{(a_i x_i + b_i)^2} \\ &\ge \lambda_i - \frac{c\mu_j}{b_i} \ge 0 \end{aligned} \tag{35}$$

where the first inequality is due to $\sum_{n=1}^{N} \frac{x_n}{a_n x_n + b_n} \ge 0$, and the second inequality is due to $\frac{b_i}{(a_i x_i + b_i)^2} \le \frac{1}{b_i}$ for $x_i \ge 0$.

We next show that we cannot have $x_i > 0$. In fact, if $x_i > 0$, it is easy to see that the first two inequality in (35) should be strictly satisfied; i.e., we must have $f_i'(\boldsymbol{P}_j^*(\boldsymbol{h};\boldsymbol{\Lambda})) > 0$. However, if the latter is true, it follows from (26) that we must have $x_i := P_{R_{ij}}^*(\boldsymbol{h};\boldsymbol{\Lambda}) = 0$. This leads to a contradiction. Hence, if $\lambda_i - c\mu_j/\beta_{R_{ij}} \ge 0$ for a certain i, we must have $P_{R_{ij}}^*(\boldsymbol{h};\boldsymbol{\Lambda}) = 0$, i.e., $i \in \mathcal{I}_1$.

C. Proof of Lemma 2

Use the convenient notations $a_i := \alpha_{R_{ij}}$, $b_i := \beta_{R_{ij}}$, and $x_i := P_{R_{ij}}^*(\boldsymbol{h};\boldsymbol{\Lambda})$, $\forall i$. Suppose that $\lambda_i b_i \ge \lambda_k b_k$, and $i \in \mathcal{I}_2$, i.e., $x_i > 0$. Then we have

$$\begin{aligned} 0 &= f_i'(\boldsymbol{P}_j^*(\boldsymbol{h};\boldsymbol{\Lambda})) \\ &= \lambda_i - \frac{c\mu_j}{1 + \sum_{n \in \mathcal{I}_2} \frac{x_n}{a_n x_n + b_n}} \frac{b_i}{(a_i x_i + b_i)^2} \\ &> \lambda_i - \frac{c\mu_j}{1 + \sum_{n \in \mathcal{I}_2} \frac{x_n}{a_n x_n + b_n}} \frac{1}{b_i} \end{aligned}$$

where $f_i'(\boldsymbol{P}_j^*(\boldsymbol{h};\boldsymbol{\Lambda})) = 0$ follows from (26) and the inequality is due to $\frac{b_i}{(a_i x_i + b_i)^2} < \frac{1}{b_i}$ for $x_i > 0$. This then implies

$$\lambda_i b_i < \frac{c\mu_j}{1 + \sum_{n \in \mathcal{I}_2} \frac{x_n}{a_n x_n + b_n}}. \tag{36}$$

Now suppose we have $k \notin \mathcal{I}_2$, i.e., $x_k = 0$. Clearly, in this case we have

$$f_k'(\boldsymbol{P}_j^*(\boldsymbol{h};\boldsymbol{\Lambda})) = \lambda_k - \frac{c\mu_j}{1 + \sum_{n \in \mathcal{I}_2} \frac{x_n}{a_n x_n + b_n}} \frac{1}{b_k} \ge 0$$

Consequently,

$$\lambda_k b_k \geq \frac{c\mu_j}{1 + \sum_{n \in \mathcal{I}_2} \frac{x_n}{a_n x_n + b_n}} > \lambda_i b_i$$

where the second inequality follows from (36). But this clearly contradicts with $\lambda_i b_i \geq \lambda_k b_k$. Therefore, we must have $k \in \mathcal{I}_2$, i.e., $x_k > 0$ if $i \in \mathcal{I}_2$, under the condition $\lambda_i b_i \geq \lambda_k b_k$.

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