

# A Bit-Map-Assisted Dynamic Queue Protocol for Multiaccess Wireless Networks With Multiple Packet Reception

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**Abstract**—A novel network-assisted (signal processing based) medium access control (MAC) protocol known as the bit-map-assisted dynamic queue (BMDQ) is presented in this paper. The protocol is explicitly designed for a wireless slotted system with multiple packet reception (MPR) capability. In the proposed protocol, the traffic in the channel is viewed as a flow of transmission periods (TPs). Each TP has a bit-map (BM) slot at the beginning followed by a data transmission period (DP). The BM slot is reserved for user detection so that accurate knowledge of active user set (AUS) can be obtained. Then, given the knowledge of the AUS and the channel MPR matrix, the number of users that can access the channel simultaneously in each packet slot in the DP is chosen to maximize the conditional throughput of every packet slot. Compared with other conventional and network-assisted MAC protocols, the proposed BMDQ protocol yields better performance. Its maximum steady-state throughput is close to the channel MPR capacity, and it can achieve the same throughput with lower traffic load and smaller delay. Performance issues are investigated analytically and via simulations.

**Index Terms**—Medium access control protocols, multiple packet reception, packet radio access, random access, slotted DS-CDMA systems.

## I. INTRODUCTION

MULTIPLE access schemes allow multiple users to share a common channel. Random access methods provide each user a flexible way of gaining access to the channel whenever the user has information (packets) to be sent. A consequence of randomness of user access is that there is contention among the users for access to the channel, resulting in collisions of contending transmissions. The first MAC protocol for random access in wireless communications, called ALOHA, was proposed in the 1970s [1]. Since then, many other protocols, such as the tree algorithm and a class of adaptive schemes [1]–[4] have been developed, and better performance has been obtained. These conventional MAC protocols are based on a noiseless collision channel model where it is assumed that a packet can be successfully received if and only if there are no collisions. When there are concurrent transmissions, collision occurs, and the colliding packets have to be discarded and retransmitted later. Therefore, a goal

of the conventional MAC protocols is to avoid concurrent transmissions. Advances in multiaccess techniques and signal coding and processing techniques such as space-time coding, blind equalization [15], and multiuser reception [16]–[18], make correct reception of one or more packets in the presence of concurrent transmissions possible, thereby endowing the channel with the so-called multipacket reception (MPR) capability [14]. With the MPR capability, it is possible to improve the network throughput, thereby saving the limited wireless communication resources. As shown in [5]–[7], some conventional MAC protocols can be modified for the MPR channel. In these methods, the basic concept of conventional MAC protocols is still used where the unsuccessfully received packets are retransmitted later randomly according to some schemes. Unfortunately, such retransmission schemes can have adverse effects on the network throughput and delay.

A novel network-assisted collision resolution method known as network-assisted diversity multiple access (NDMA) was recently proposed by Tsatsanis *et al.* [8]. In the NDMA protocol [8], the channel traffic is viewed as a flow of epochs. In an epoch, instead of discarding the colliding packets, the slot with  $k$  users' collided packets is retransmitted  $k - 1$  times so that the individual packet can be recovered by solving a  $k \times k$  source separation problem. In [8], the authors require detection of active users, and this is accomplished by assigning orthogonal identification user codes. Moreover, the authors in [8] assume user synchronism and negligible multipath delay spread. In [9] and [26], approaches have been investigated to alleviate the requirements of orthogonal codes, user synchronism, and negligible multipath delay spread. In [8], [9], and [26], the channel MPR capability (if in existence) is not taken into consideration. In this paper, we also assume user synchronism and negligible multipath delay spread but take channel MPR capability into consideration. In our approach, each epoch (called transmission period) has a time-division multiple access (TDMA) slot in the beginning and this slot is used for user detection. Clearly, the contention-based approaches of [9] and [26] are potentially useful for user detection in our case as well, as TDMA approach is more wasteful in terms of bandwidth, and moreover, [9] and [26] allow user asynchronism and multipath delay spread. We leave this for future research. Finally, we note that an extension of [8] to include a receive antenna array is given in [27]; this provides some MPR capability. In this paper, we assume a single antenna at the receiver.

Unlike NDMA, Tong *et al.* [10]–[14] explicitly design protocols for MPR channels based on a model given by an MPR ma-

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trix. Among them, a dynamic queue protocol was proposed in [10] and [11] for general MPR channels. It can achieve a performance comparable with that of the approaches of [12]–[14] with much simpler implementation. Similar to the NDMA approach, in the dynamic queue protocol, the channel traffic is viewed as a flow of transmission periods (TPs). However, when collision occurs, instead of transmitting and retransmitting all colliding packets in all the slots, just an appropriate subset of users is allowed access to the channel in each slot. The size of the access set is chosen so that the multiaccess interference would not be “high,” and the channel MPR capability can be optimally exploited. With a transmission structure similar to that in the dynamic tree protocol [2], the dynamic queue protocol chooses the size of the access set so that the expected duration of TP is minimized, based on the channel MPR capability and the probability that a user has a packet to transmit in the TP. The implementation and the steady-state analysis in [10] has been made under the assumption that each user can hold at most one packet newly generated in the current TP (one-packet-buffer) and to be transmitted in the next TP. To extend [10] and [11] to the multiple-packet-buffer case, one may view each user with multiple packets as several virtual users each with one packet. In [10] and [11], one requires the knowledge of the probability  $p$  that a user generates a packet within one time slot, and  $p$  is assumed to be the same for all users. Clearly, when virtual users are created,  $p$  will not be the same for all virtual users derived from the same parent user, even if it holds for all real users. Thus, extension of [10] and [11] to the multiple-packet-buffer case, although conceptually straightforward, appears to be nontrivial.

The approach of [10] and [11] falls under the category of random-access MAC protocols [29, Sec. 4.3]. According to [29, Sec. 4.3], reservation-based protocols are a subset of random-access MAC protocols. In reservation-based protocols, the desire to transmit is broadcast before the actual transmission. In this paper, we propose a reservation-based MAC protocol known as the bit-map-assisted dynamic queue (BMDQ) protocol for general MPR channels and provide performance analysis for both the infinite-buffering case and the one-packet-buffering case. Unlike [10], with the proposed BMDQ protocol, the active user set (AUS) can be determined from the bit-map (reservation) slot; then, the principle of dynamic queue [10] is applied to construct the TP, however, with the knowledge of AUS. The proposed BMDQ protocol can achieve better network performance than that of the dynamic queue protocol of [10] with simpler implementation due to the acquired knowledge of the AUS, provided the number of users is not too “large.” As the number of users increases, the TDMA-based reservation slot in our approach can be wasteful in terms of bandwidth. This can be alleviated by allowing contention in the reservation slot, following [9] and [26], e.g., this is left for future research. We do note that unlike most reservation-based MAC protocols (see [29, Sec. 4.3]), the data packet transmissions in our method are contention-based.

The paper is organized as follows. In Section II, the proposed protocol is described. In Section III, a steady-state performance analysis of the proposed protocol is carried out under the assumption of Poisson sources. Both infinite-packet-buffering case and one-packet buffering cases are investigated in Sec-

tion III. In Section IV, simulations results are presented in the context of a slotted CDMA network, where we verify the analytical results of Section III and compare the performance of the proposed BMDQ protocol with two conventional and one network-assisted MAC protocols. Some relevant derivations are presented in the Appendix.

#### *Nomenclature:*

AUS	Active user set.
DP	Data transmission period.
TP	Transmission period.
BM	Bit map.
BMDQ	Bit map dynamic queue protocol.
MAC	Medium access control.
MPR	Multiple packet reception.
$h$	length of TP.
$h_R$	length of relevant TP.
$h_I$	length of irrelevant TP.

## II. BIT-MAP-ASSISTED DYNAMIC QUEUE PROTOCOL

In the proposed protocol time is divided into TPs, each TP consisting of a BM slot (“zeroth” slot) in the beginning of the TP, followed by a DP composed of a variable number of data packet slots. All data packet slots are of the same size, whereas the BM slot is of a different size. At the start of a given TP, all users having packets to transmit are allowed to transmit one packet per user in that TP after they indicate their desire to transmit via the BM slot. The BM slot is contention-free following a TDMA scheme: Each user is assigned a fixed reservation period in a specified order in the BM slot, where it places its signature if it has a packet to transmit; otherwise, its reservation period is empty. Thus, the BM slot is of fixed size. Using the BM slot transmissions, the central controller determines the AUS using some signal detection methods. Once the AUS is known and if the AUS is not empty, the DP is constructed following the BM by applying the principles of dynamic queue protocol, where the access set consisting of users allowed access to the channel is controlled in every packet slot (see Section II-C for details). Otherwise, no DP exists for the current TP. The TP flow is illustrated in Fig. 1. The DP, and therefore the TP, ends when the central controller has determined that all packets (one per active user) slated for transmission at the beginning of the TP have been successfully transmitted. We assume that the central controller can identify the source of any successfully demodulated packets, and it can distinguish between empty and nonempty slots without error. If a transmitted packet is not received successfully, it is retransmitted in a later packet slot in the same TP. The packet generation process, which consists of both new packet origination and packet retransmission, is assumed to follow a Poisson distribution (for analysis purposes).

Since the BM slot is contention-free, just a short reservation period is required for each user to achieve a small detection error probability with simple implementation. We could allow contention and apply approaches such as [9], [26], CDMA, the dynamic tree protocol, or even the dynamic queue protocol, in the BM slot to minimize its duration and make it much smaller than the data packet duration; this aspect is outside the scope of this paper and will not be discussed. A possible advantage of using

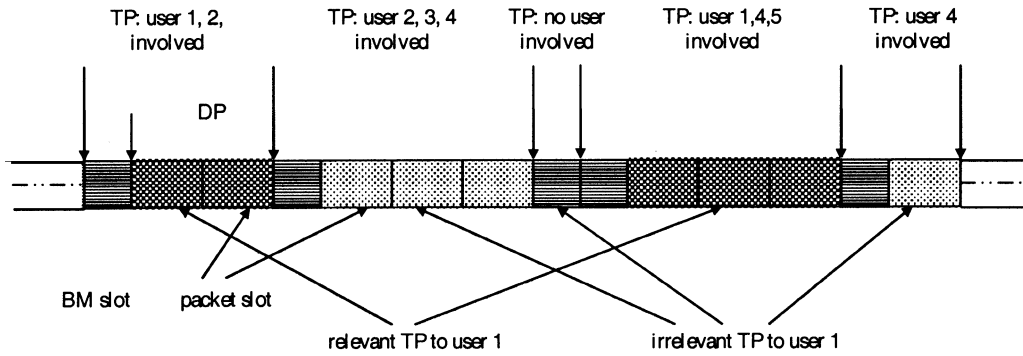


Fig. 1. TP flow. Each TP includes a BM slot and a DP composed of several packet slots; for a particular user, there are two types of TPs: relevant TPs (the user has data packets in the TP) and irrelevant TPs (the user has no data packets).

[9] or [26] is that user synchronism and negligible multipath delay spread are not required.

The structure of our proposed BM slot follows the structure of the bit-map method [1, p. 254]; therefore, we call it the BM slot, even though unlike the bit-map method [1], the reservation period for each user is not just of one bit duration and even though, unlike the bit-map method [1], the data packet transmissions in our method are contention-based.

In the following subsections, we describe the various parts of our protocol in greater detail.

#### A. MPR Channel Model

Following [14], [24], and [25], we consider a general model for MPR channels described below. We consider a network with  $J$  users transmitting data to a central controller through a common wireless channel. The transmission time is slotted, and each user generates data in the form of equal-sized packets. The slotted channel is characterized by an MPR matrix

$$\mathbf{C} = \begin{bmatrix} C_{1,0} & C_{1,1} & & & \\ C_{2,0} & C_{2,1} & C_{2,2} & & \\ \vdots & \vdots & \vdots & \ddots & \\ C_{J,0} & C_{J,1} & C_{J,2} & \cdots & C_{J,J} \end{bmatrix} \quad (1)$$

where  $C_{n,k}$  denotes the probability of having exactly  $k$  successes when there are  $n$  transmitted packets in a slot:

$$C_{n,k} = P\{k \text{ packets are successfully received} \mid n \text{ are transmitted}\} \quad (1 \leq n \leq J, 0 \leq k \leq n).$$

The capacity  $\eta$  of an MPR channel has been defined in [14] as the maximum expected number of successfully received packets in one slot

$$\eta := \max_{n=1, \dots, J} C_n = \max_{n=1, \dots, J} \sum_{k=1}^n k C_{n,k} \quad (2)$$

where  $C_n$  denotes the expected number of successfully received packets when there are  $n$  transmitted packets. By definition,  $\eta$  is the maximum throughput the MPR channel can offer, independent of MAC protocols.



Fig. 2. Structure of the BM slot.

#### B. User Detection in the BM Slot

The structure of the BM slot is illustrated in Fig. 2. We assume that the users are synchronized, the delay spread is negligible i.e., the channel is flat, and the time-slots are long enough to encompass the maximum propagation delay plus the delay spread. A common portion of a short  $m$ -sequence [19, Sec. 13.2.5] is used by every user as its signature to transmit in its reserved period. Therefore, the same matched filter can be used as the detector for all users. Let  $\alpha_j$  denote the  $j$ th users complex gain,  $\tau_j$  denote its propagation delay,  $\Delta$  denote the AUS,  $v_j(n)$  denote the chip-rate zero mean additive complex Gaussian noise with variance  $\sigma_v^2$ , and  $m(n)$  denote the  $m$ -sequence segment used in the reservation period. Then, the received signal in the user  $j$ s reserved period is given by

$$y_j(n) = \begin{cases} \alpha_j m(n - \tau_j) + v_j(n), & j \in \Delta \\ v_j(n), & j \notin \Delta. \end{cases} \quad (3)$$

Due to the good auto-correlation of the  $m$ -sequences [19], the signal can be synchronized at the detector. Let  $N$  denote the length of the  $m$ -sequence segment as well as that of the  $j$ th users reserved period in the BM slot in chips, let  $m_{\tau_j}(n)$  denote the synchronized copy of the  $m$ -sequence segment having unit chip energy, and let  $\tilde{v}_j$  be a zero mean complex Gaussian random variable with variance  $\sigma_v^2/N$ . Then, the output of the matched filter for  $j$ th user detection is given by

$$z_j = \frac{1}{N} \sum_{n=0}^{N-1} y_j^*(n) m_{\tau_j}(n) = \begin{cases} \alpha_j + \tilde{v}_j, & j \in \Delta \\ \tilde{v}_j, & j \notin \Delta. \end{cases} \quad (4)$$

The optimal detector for user  $j$  depends upon the statistical properties of  $\alpha_j$ . For simulation results presented in Section IV, we model  $\alpha_j$  as a complex random variable with known amplitude (power control is effective) and uniformly distributed phase

over  $[0, 2\pi)$ . For this and several other distributions of  $\alpha_j$ , the optimal detector for user  $j$  is given by [8], [28]

$$|z_j| \begin{cases} > H_{j,1} \\ < H_{j,0} \end{cases} T \quad (5)$$

where  $T$  is a threshold selected to achieve a given false alarm probability  $P_F$ ,  $H_{j,1}$  denotes the hypothesis corresponding to  $j \in \Delta$ , and  $H_{j,0}$  corresponds to  $j \notin \Delta$ .

For given  $N$  and  $T$ , we can obtain the detection probability  $P_D$  and the false alarm probability  $P_F$ . Clearly, we want  $P_D \rightarrow 1$  and  $P_F \rightarrow 0$  to obtain an accurate knowledge of the AUS. See Section IV-A-1 for further details.

### C. Structure of DP

Let  $K$  ( $1 \leq K \leq J$ ) denote the number of active users in the beginning of a DP. We choose to maximize the conditional throughput in every packet slot given the number of active users at the beginning of the slot, i.e., we maximize the expected number of successfully transmitted packets in a slot given the number of active users. When  $K = J$ , the conditional throughput equals the channel MPR capacity  $\eta$ . Let [10]

$$n_0 := \min \left\{ \arg \max_{n=1, \dots, J} \sum_{k=1}^n k C_{n,k} \right\}. \quad (6)$$

In (6), there may be more than one value of  $n$  leading to the maximum; we pick the smallest such  $n$ . Clearly, under a heavy traffic load,  $n_0$  packets should be transmitted simultaneously to achieve the channel capacity  $\eta$ . Similarly, we can define

$$n_i := \min \left\{ \arg \max_{n=1, \dots, n_{i-1}-1} \sum_{k=1}^n k C_{n,k} \right\}, \quad n_i \geq 1. \quad (7)$$

Therefore, with the knowledge of  $n_0$ , we can find  $n_1$ , and this process can be iterated to find  $n_{i+1}$  from  $n_i$ . The iteration stops when  $n_i$  equals 1. Define a look-up vector  $\mathbf{N}_{opt} = [n_0, n_1, \dots, 1]$ .

We determine the access set and construct the DP according to  $\mathbf{N}_{opt}$  as follows.

- 1) Let the waiting list be composed of the users in the AUS. Therefore, initially, the number of waiting users  $n = K$ .
- 2) Let the size of the access set

$$N_n = \begin{cases} n_0, & n \geq n_0 \\ n_i, & n_{i-1} > n \geq n_i \end{cases} \quad (8)$$

and let the first  $N_n$  users in the waiting list access the channel in the current slot.

- 3) If the slot is empty, remove all the users in the access set from the waiting list, and let  $n = n - N_n$ . If the slot is not empty and  $k$  packets are successfully received, remove these  $k$  users from the waiting list, and let  $n = n - k$ .
- 4) Repeat steps 2) and 3) until  $n = 0$ .

*Remark 1:* The first part in Step 3) is needed to combat the effect of a false alarm in user detection in the BM slot. •

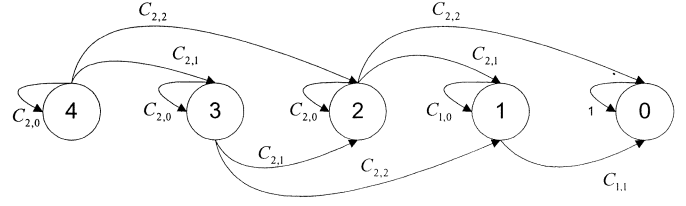


Fig. 3. State diagram of Markov chain when  $K = 4$ . If  $n > 1$ ,  $N_n = 2$ ; otherwise,  $N_n = 1$ .

*Remark 2:* An equivalent formulation of Step 2), (6), and (7) is as follows. Let

$$m_n := \min \left\{ \arg \max_{p=1, \dots, n} \sum_{k=1}^p k C_{p,k} \right\} \quad (9)$$

i.e.,  $m_n$  denotes the number of simultaneous transmissions to optimize throughput in a given slot given  $n$  active users. In Step 2), set  $N_n = m_n$ . Given  $n$  active users, if  $n_{i-1} > n \geq n_i$ , then

$$\begin{aligned} n_i &:= \min \left\{ \arg \max_{p=1, \dots, n_{i-1}-1} \sum_{k=1}^p k C_{p,k} \right\} \\ &= \min \left\{ \arg \max_{p=1, \dots, n} \sum_{k=1}^p k C_{p,k} \right\} =: m_n. \end{aligned}$$

Similarly, if  $n \geq n_0$ , then in addition,  $m_n = n_0$ . Thus, we have an equivalent formulation. •

According to above scheme, the number of users in the waiting list forms a Markov chain. At the beginning of DP, the network is in state  $K$ . When the network reaches the state 0, which is the absorbing state, the DP ends. A state diagram of this Markov chain is illustrated in Fig. 3. Clearly, the length  $L$  of DP is a random variable whose distribution is associated with the number of active users  $K$ . We can determine the expected length of DP ( $\bar{L}_K = E\{L | K \text{ active users}\}$ ) as the absorbing time of this finite state discrete Markov chain, which is initiated in state  $K$ . In general, the transition probability from state  $l$  to state  $m$  is given by

$$p_{l,m} = \begin{cases} C_{N_l, l-m}, & 0 \leq l-m \leq N_l, m \leq J \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

where  $N_l$  is given by (8), and  $C_{n,k}$  is defined in (1). Let

$$\mathbf{L} = [\bar{L}_1 \bar{L}_2 \cdots \bar{L}_{J-1} \bar{L}_J]^T. \quad (11)$$

Then, from (10) and (11), we have

$$(\mathbf{I} - \mathbf{P}) \cdot \mathbf{L} = \mathbf{1} \quad (12)$$

where  $\mathbf{P}$  is the transition probability matrix with the entries specified by (10), and  $\mathbf{I}$  and  $\mathbf{1}$  denote an identity matrix and a vector with all 1 entries, respectively. From (12), we can solve for  $\bar{L}_K$ , ( $K = 1, \dots, J$ ).

### D. Procedure of the BMDQ Protocol

We now summarize the basic procedure of the BMDQ protocol. The following steps are executed in the  $i$ th TP.

- 1) Reserve the zeroth slot (which has a length different from that of the packet slot) for BM. Determine the AUS using the BM slot transmissions.

- 2) Form a waiting list with all the users in the AUS in a randomized order. Let the number of waiting users  $n = K$  if there are  $K$  users in the AUS. If  $n = 0$ , go to step 5), else continue.
- 3) Determine the access set size  $N_n$  via (8). Let the first  $N_n$  users in the waiting list access the channel, namely, transmit their packets in the current packet slot, one packet per user.
- 4) If the slot is empty, remove all the users in the access set from the waiting list, and let  $n = n - N_n$ . If the slot is not empty and  $k$  packets are successfully received, remove these  $k$  users from the waiting list, and let  $n = n - k$ .
- 5) Repeat steps 3) and 4) until  $n = 0$ . This ends the DP of the  $i$ th TP and starts the  $(i + 1)$ th TP.

### III. PERFORMANCE ANALYSIS

For evaluating the MAC protocols in this paper, we are mainly concerned with their long-term (steady-state) behavior. In this section, the steady-state performance measures such as throughput and average delay are investigated. The infinite buffering case is considered in Section III-A, and the one-packet-buffering case is presented in Section III-B. Throughout, it is assumed that all users' packets follow a Poisson distribution with rate  $\lambda$  (packets per packet duration). In our analysis, we follow the general approach of [8]. Specifics differ, however. Our analysis is based on certain simplifying assumptions (which are stated as needed), and the main justification for them is that the resulting expressions yield useful qualitative results, which, as shown in Section IV, also agree well with the simulations-based results not requiring the simplifying assumptions.

#### A. Infinite-Buffering Case

In the following analysis, we assume that the users' buffers are infinite. For a given user, all the generated packets remain in its buffer until they are successfully transmitted. No packets are discarded, and the first generated one will be transmitted first. If the buffer is not empty, the data packets generated in the current TP will not necessarily be transmitted in the next TP. Instead, they will be transmitted in a later TP after all the packets generated before them have been successfully transmitted.

1) *Steady-State Analysis: System Stability:* Following [8], we view the traffic in the channel as a flow of TPs (called epochs in [8]) and assume that every user's buffer is fed by data packets with independent and identical distribution (i.i.d.). For a particular user, there are two types of TPs: relevant TPs and irrelevant TPs. In the relevant TP, this user belongs to the AUS and transmits its data packet in one packet slot during the DP. Correspondingly, in the irrelevant TP, this user is not active. Let  $q_m$  denote the number of data packets in the particular user's buffer at the beginning of the  $m$ th TP. Therefore, at the beginning of the relevant TPs, the buffer of this user is not empty ( $q_m > 0$ ), whereas at the beginning of the irrelevant TPs, it is empty ( $q_m = 0$ ). The sequence  $q_m$  constitutes an embedded Markov chain [8]. Let  $v(q_m)$  denote the number of new data packets arriving in the user's buffer during the  $m$ th TP. Then, according to the state transitions of the above Markov chain, we have

$$q_{m+1} = \begin{cases} q_m - 1 + v(q_m), & q_m > 0 \\ v(q_m), & q_m = 0. \end{cases} \quad (13)$$

If the user's buffer is fed by a Poisson source with rate  $\lambda$ , then  $v(q_m)$  becomes a random variable whose distribution depends on the length of  $m$ th TP. From Section II, we know that the length  $L$  of the DP, and, therefore, the length  $h$  of the TP, is also a random variable whose distribution depends on the number of active user's  $K$  and obeys different steady-state distribution for the two types of TPs. Let  $P_e$  denote the steady-state probability of a user's buffer being empty at the beginning of a TP, namely,  $P_e = \lim_{m \rightarrow \infty} \Pr\{q_m = 0\}$ . Then, the steady-state probabilities  $P_R(K)$  and  $P_I(K)$  for the number of active user's  $K$  in relevant TP and irrelevant TP, respectively, are given by

$$\begin{aligned} P_R(K) &= B(K - 1, J - 1, 1 - P_e) \\ &= \binom{J - 1}{K - 1} (1 - P_e)^{K-1} P_e^{J-K} \\ &\text{for } K = 1, \dots, J \end{aligned} \quad (14)$$

$$\begin{aligned} P_I(K) &= B(K, J - 1, 1 - P_e) \\ &= \binom{J - 1}{K} (1 - P_e)^K P_e^{J-K-1} \\ &\text{for } K = 0, \dots, J - 1 \end{aligned} \quad (15)$$

where  $B(u, U, s)$  denotes the probability mass at the value  $u$  of a Binomial random variable with total  $U$  trials and a success probability  $s$ . In this subsection, for simplification, we assume that  $P_D \rightarrow 1$  and  $P_F \rightarrow 0$  in user detection in the BM slot. Let  $L_B$  denote the length of the BM slot (it can be a noninteger since all lengths are measured in terms of data packet durations). Let

$$P_{h,K}(L_B + l) := P(h = L_B + l | K \text{ active users}) \quad (16)$$

denote the probability of the length  $h$  of TP being  $L_B + l$  when there are  $K$  active users. Let  $h_R$  and  $h_I$  denote the length of the relevant and irrelevant TP, respectively. Following (16), define

$$\begin{aligned} P_{h_R,K}(L_B + l) \\ := P(h = L_B + l | K \text{ active users, relevant TP}) \end{aligned} \quad (17)$$

and

$$\begin{aligned} P_{h_I,K}(L_B + l) \\ := P(h = L_B + l | K \text{ active users, irrelevant TP}) \end{aligned} \quad (18)$$

as conditional versions of (16), conditioned on the TP being relevant and irrelevant, respectively. It then follows that

$$\begin{aligned} P_{h,K}(L_B + l) &= (1 - P_e)P_{h_R,K}(L_B + l) \\ &\quad + P_e P_{h_I,K}(L_B + l), \end{aligned} \quad (19)$$

$$E\{h\} = (1 - P_e)E\{h_R\} + P_e E\{h_I\}. \quad (20)$$

The TP lengths  $h_R$  and  $h_I$  obey the distributions

$$P_{h_R}(L_B + l) = \sum_{K=1}^J P_R(K) P_{h_R,K}(L_B + l), \quad l = 1, 2, \dots \quad (21)$$

$$P_{h_I}(L_B + l) = \sum_{K=0}^{J-1} P_I(K) P_{h_I,K}(L_B + l), \quad l = 0, 1, \dots \quad (22)$$

Let the steady-state probability generating function of  $q_m$  be denoted by  $Q(z)$ , given by

$$Q(z) = \lim_{m \rightarrow \infty} \sum_{k=0}^{\infty} \Pr\{q_m = k\} z^k = \lim_{m \rightarrow \infty} E[z^{q_m}]. \quad (23)$$

Define the functions

$$F(z) = \lim_{m \rightarrow \infty} E \left[ z^{v(q_m)} \middle| q_m = 0 \right] \quad (24)$$

$$G(z) = \lim_{m \rightarrow \infty} E \left[ z^{v(q_m)} \middle| q_m > 0 \right]. \quad (25)$$

Then, following the procedure of [8, Lemma 1],  $Q(z)$ ,  $F(z)$ , and  $G(z)$  are given by

$$Q(z) = P_e \frac{zF(z) - G(z)}{z - G(z)} \quad (26)$$

$$G(z) = \sum_{l=1}^{\infty} e^{(L_B+l)(z\lambda-\lambda)} P_{h_R}(L_B+l) \quad (27)$$

$$F(z) = \sum_{l=0}^{\infty} e^{(L_B+l)(z\lambda-\lambda)} P_{h_I}(L_B+l) \quad (28)$$

where  $P_{h_R}(L_B+l)$  and  $P_{h_I}(L_B+l)$  are given by (21) and (22). Evaluating (26) at  $z = 1$  and applying L'Hopital's rule, we have

$$1 = Q(1) = P_e \frac{1 + F'(1) - G'(1)}{1 - G'(1)} \quad (29)$$

where  $F'(1)$  and  $G'(1)$  are the first derivative of  $F(z)$  and  $G(z)$ , respectively, w.r.t.  $z$  evaluated at  $z = 1$

$$\begin{aligned} G'(1) &= \lambda E\{h_R\} = \lambda \sum_{K=1}^J P_R(K)(L_B + \bar{L}_{KR}) \\ &= \lambda \left( L_B + \sum_{K=1}^J P_R(K) \bar{L}_{KR} \right) \\ F'(1) &= \lambda E\{h_I\} = \lambda \left[ P_I(0)L_B + \sum_{K=1}^{J-1} P_I(K)(L_B + \bar{L}_{KI}) \right] \\ &= \lambda \left( L_B + \sum_{K=1}^{J-1} P_I(K) \bar{L}_{KI} \right). \end{aligned} \quad (30)$$

$\bar{L}_K$  is defined in Section II-C [see (11)] as  $E\{L | K \text{ active users}\}$ ,  $L = \text{length of DP}$ ,  $\bar{L}_{KI} := E\{L | K \text{ active users, irrelevant TP}\}$ , and  $\bar{L}_{KR} := E\{L | K \text{ active users, relevant TP}\}$ . From (12), we can solve for  $\bar{L}_K$  for  $K = 1, 2, \dots, J$ . Similarly, for the irrelevant TP, we can determine  $\bar{L}_{KI}$  as the absorbing time of a similar finite state discrete Markov chain, which is initiated in state  $K$ , but having states  $0, 1, \dots, J-1$  instead of  $0, 1, \dots, J$ . Letting  $\mathbf{L}_I := [\bar{L}_{1I} \ \bar{L}_{2I} \ \dots \ \bar{L}_{(J-1)I}]^T$ , we have

$$(\mathbf{I} - \mathbf{P}_I) \cdot \mathbf{L}_I = \mathbf{1} \quad (31)$$

where  $\mathbf{P}_I$  is the  $(J-1) \times (J-1)$  submatrix of  $\mathbf{P}$  in the left upper corner, and  $\mathbf{I}$  and  $\mathbf{1}$  denote an identity matrix and a vector with all 1 entries, respectively. From (31), we can solve for  $\bar{L}_{KI}$ , ( $K = 1, \dots, J-1$ ). Note that

$$\bar{L}_K = \begin{cases} (1 - P_e) \bar{L}_{KR} + P_e \bar{L}_{KI}, & K = 1, \dots, J-1 \\ \bar{L}_{KR}, & K = J. \end{cases} \quad (32)$$

Since both  $\mathbf{P}$  in (12) and  $\mathbf{P}_I$  in (31) are lower triangular matrices by construction, and  $\mathbf{P}_I$  is a submatrix of  $\mathbf{P}$ , it can be established that  $\bar{L}_{KI} = \bar{L}_K$  for  $1 \leq K \leq J-1$ ; hence,  $\bar{L}_{KR} = \bar{L}_K$  for  $1 \leq K \leq J$ . Therefore, we only need to determine  $\bar{L}_K$ .

From (29) and (30), we have the equation

$$D(P_e) := P_e(1 + \lambda E\{h_I\} - \lambda E\{h_R\}) - (1 - \lambda E\{h_R\}) = 0. \quad (33)$$

Note that  $E\{h_R\}$  and  $E\{h_I\}$  are functions of  $P_e$ . For steady-state to exist, there must exist a solution of (33) for  $P_e$  in  $[0, 1]$ . We have  $D(1) = \lambda E\{h_I\} \geq \lambda L_B > 0$ . Therefore, if  $D(0) \leq 0$ , then due to the continuity of  $D(P_e)$  in  $P_e$ , we have a solution of (33) for  $P_e$  in  $[0, 1]$ . We have  $D(0) = \lambda E\{h_R\}|_{P_e=0} - 1$ . Now

$$\begin{aligned} E\{h_R\}|_{P_e=0} &= \left( L_B + \sum_{K=1}^J P_R(K) \bar{L}_{KR} \right) \Big|_{P_e=0} \\ &= \left( L_B + \sum_{K=1}^J P_R(K) \bar{L}_K \right) \leq L_B + \bar{L}_J \end{aligned} \quad (34)$$

where we have used the fact that  $\bar{L}_i \leq \bar{L}_k$  for  $i < k$ . Hence,  $D(0) \leq \lambda(L_B + \bar{L}_J) - 1$ . This yields a sufficient condition for a steady-state solution for  $P_e$  in  $[0, 1]$  (which may be interpreted as a system stability condition) as

$$\lambda \leq 1/(L_B + \bar{L}_J). \quad (35)$$

2) *Throughput and Traffic Load Analysis:* In the steady state, the probability  $P_u(K)$  that the number of active users at the start of a TP is  $K$  is given by ( $K = 0, 1, \dots, J$ )

$$P_u(K) = B(K, J, 1 - P_e) := \binom{J}{K} (1 - P_e)^K P_e^{J-K}. \quad (36)$$

If the detection errors in the BM slot (see Section II-B) are not negligible (i.e., detection probability  $P_D < 1$ ), then we have ( $K = 0, 1, \dots, J$ )

$$\begin{aligned} &P(K \text{ detected users} | j \text{ active users}) \\ &= P_u(j) \left[ \sum_{l=0}^{\min\{j, K\}} B(l, j, P_D) B(K-l, J-j, P_F) \right] \end{aligned} \quad (37)$$

where  $B(l, j, P_D)$  is the probability of  $l$  correctly detected active users out of  $j$  active users, and  $B(K-l, J-j, P_F)$  is the probability of  $K-l$  (falsely) detected active users out of  $J-j$  inactive users with false alarm probability  $P_F$ . Therefore, the

probability that the number of detected active users at the start of a TP is  $K$  is given by ( $K = 0, 1, \dots, J$ )

$$\tilde{P}_u(K) = \sum_{j=0}^J P_u(j) \cdot \left[ \sum_{l=0}^{\min\{j, K\}} B(l, j, P_D) B(K-l, J-j, P_F) \right]. \quad (38)$$

The system throughput  $R$  is defined as the expected number of successfully transmitted packets per packet slot duration, given by

$$R := \frac{\text{expected number of successfully transmitted packets/TP}}{\text{expected length of TP}}.$$

Then, from (38), we obtain

$$\begin{aligned} R &= \frac{\sum_{K=0}^J P_u(K) \left[ \sum_{l=0}^K l \cdot B(l, K, P_D) \right]}{L_B \tilde{P}_u(0) + \sum_{K=1}^J (L_B + \bar{L}_K) \tilde{P}_u(K)} \\ &= \frac{J(1 - P_e) P_D}{L_B + \sum_{K=1}^J \bar{L}_K \tilde{P}_u(K)}. \end{aligned} \quad (39)$$

Note that when detection errors are taken into account,  $P_e$  and  $\bar{L}_K$  in (39) may have different values than derived in Section III-A1.

*Proposition 1:* As  $P_D \rightarrow 1$  and  $P_F \rightarrow 0$ , the throughput is given by

$$R = \frac{J(1 - P_e)}{L_B + \sum_{K=1}^J \bar{L}_K P_u(K)} = J\lambda. \quad (40)$$

*Proof:* See the Appendix.

From (35) and (40), under high SNR ( $P_D \rightarrow 1$  and  $P_F \rightarrow 0$ ), the network stability condition can be also expressed as

$$R \leq J/(L_B + \bar{L}_J). \quad (41)$$

Therefore, the maximum steady-state throughput provided by the BMDQ protocol is  $R_{\max} = J/(L_B + \bar{L}_J)$  at  $P_e = 0$ . By definition, the throughput is upper-bounded by the MPR channel capacity  $\eta$ . From the structure of the DP, in heavy traffic under high SNR ( $P_D \approx 1$ ), we have  $J/\bar{L}_J \rightarrow \eta$ . Therefore, if  $L_B \ll \bar{L}_J$ ,  $R_{\max} \rightarrow \eta$ .

Using the same Markov chain that was used for determining the expected length of DP, we can similarly determine the expected number of total transmissions (including retransmissions) in DP. Let

$$\mathbf{\Gamma} = [\bar{\Gamma}_1 \bar{\Gamma}_2 \cdots \bar{\Gamma}_{J-1} \bar{\Gamma}_J]^T \quad (42)$$

where  $\bar{\Gamma}_K$  denotes the expected number of transmissions in DP when the Markov chain is initiated in state  $K$ . Then, we have

$$(\mathbf{I} - \mathbf{P}) \cdot \mathbf{\Gamma} = \mathbf{N} \quad (43)$$

where  $\mathbf{N} = [N_1 \ N_2 \ \cdots \ N_J]^T$ ,  $N_l$  ( $l = 1, \dots, J$ ) is given by (8),  $\mathbf{P}$  denotes the transition probability matrix with the entries specified by (10), and  $\mathbf{I}$  denotes an identity matrix.

The traffic load  $G$  is defined as the expected number of transmissions (including original packets as well as retransmissions) per packet slot duration, given by

$$G := \frac{\text{expected number of transmissions/TP}}{\text{expected length of TP}}.$$

From (43), we can solve for  $\bar{\Gamma}_K$  ( $K = 1, \dots, J$ ). Together with  $\bar{L}_K$  and  $\tilde{P}_u(K)$ , we have

$$G = \frac{\sum_{K=1}^J \bar{\Gamma}_K \tilde{P}_u(K)}{L_B + \sum_{K=1}^J \bar{L}_K \tilde{P}_u(K)} \stackrel{P_D \rightarrow 1}{=} \frac{\sum_{K=1}^J \bar{\Gamma}_K P_u(K)}{L_B + \sum_{K=1}^J \bar{L}_K P_u(K)}. \quad (44)$$

Therefore, given  $P_e$ , the throughput and the traffic load can be calculated. They are related through (40) and (44).

3) *Delay Analysis:* The total average delay for a data packet in the system is given by

$$D_I = E\{s_I\} + E\{w_I\} \quad (45)$$

where  $E\{s_I\}$  is the expected delay in the serving TP in which this particular packet is transmitted, and  $E\{w_I\}$  is the expected delay for waiting in the buffer. As discussed in [8], if the user population is large and SNR is high, a users buffer can also be approximately modeled as an M/G/1 queue with vacation, in which the relevant TPs and irrelevant TPs play the roles of service time and vacation time, respectively. Therefore, the expected delay for a data packet waiting in the M/G/1 queue with vacation is given by [21], [22]

$$E\{w_I\} = \frac{\lambda E[h_R^2]}{2(1 - \lambda E\{h_R\})} + \frac{E[h_I^2]}{2E\{h_I\}} \quad (46)$$

where the distributions of  $h_R$  and  $h_I$  (lengths of relevant and irrelevant TPs, respectively) are given by (21) and (22), respectively. However, unlike [8], [21], and [22], where the data packets can only be successfully transmitted at the end of the serving TP, in our case, a data packet is possibly transmitted with success at the end of any packet slot in the DP.

*Proposition 2:* In the proposed BMDQ protocol, the expected delay for a particular data packet in its serving TP is given by

$$E\{s_I\} = \frac{1}{2} (1 + L_B + E\{h_R\}). \quad (47)$$

*Proof:* See the Appendix.

Substituting (46) and (47) in (45), we obtain the expression for the average delay for a data packet as

$$D_I = \frac{1}{2} (1 + L_B + E\{h_R\}) + \frac{\lambda E[h_R^2]}{2(1 - \lambda E\{h_R\})} + \frac{E[h_I^2]}{2E\{h_I\}}. \quad (48)$$

## B. One-Packet-Buffering Case

In this subsection, we assume that every user has only a one-packet buffer (the case discussed in [10]). If a user generates more than one data packet in the current TP, then only the first

packet is kept in the buffer and is transmitted in the next TP while others are discarded.

1) *Throughput and Traffic Load*: In the one-packet-buffering case, mimicking the analysis for the infinite-buffering case, we arrive at the same conclusions for the network stability condition, throughput, and traffic load, which are given by (41), (40) and (44), respectively.

2) *Delay and Packet-Loss-Rate*: Similar to (45), the average delay for a packet can be expressed as

$$D_1 = E[s_1] + E\{w_1\} \quad (49)$$

where  $E[s_1]$  is the expected delay in the serving TP, and  $E\{w_1\}$  is the expected delay for waiting in the buffer for one-packet-buffering case. The expected delay in the serving TP is the same as that in infinite-buffering case, given by

$$E[s_1] = \frac{1}{2} (1 + L_B + E\{h_R\}). \quad (50)$$

Because in one-packet-buffering case, the packet transmitted in the current TP is the first generated packet in the previous TP, it follows that

$$E\{w_1\} = E[h - t_g] \quad (51)$$

where  $t_g$  denotes the generation time for the first packet w.r.t. the start of the previous TP, and  $h$  is the length of the previous TP.

*Proposition 3*: If the users buffer is fed with a Poisson source with intensity  $\lambda$ , then

$$E\{w_1\} \approx \frac{1}{1 - P_e} E\{h\} - \frac{1}{\lambda} = \left( \frac{1}{1 - P_e} + \frac{1}{\ln P_e} \right) E\{h\}. \quad (52)$$

*Proof*: See the Appendix.

Substituting (50) and (52) in (49), we obtain the expression for average delay as

$$D_1 \approx \frac{1}{2} (1 + L_B + E\{h_R\}) + \left( \frac{1}{1 - P_e} + \frac{1}{\ln P_e} \right) E\{h\}. \quad (53)$$

Since a user has a one-packet buffer, it is inevitable that some packets have to be discarded in heavy traffic load. We define the packet-loss-rate (PLR) as

$$\beta := \frac{\text{expected number of discarded packets/TP}}{\text{expected number of generated packets/TP}}.$$

Assuming a Poisson source with intensity  $\lambda$ , the expected number of generated packets per TP is approximately  $J\lambda E\{h\}$ . Clearly, the expected number of transmitted packets is equal to  $J(1 - P_e)$ . Using the approximation (71) in the Appendix, we have

$$\beta = \frac{J\lambda E\{h\} - J(1 - P_e)}{J\lambda E\{h\}} = 1 + \frac{1 - P_e}{\ln P_e}. \quad (54)$$

In the one-packet-buffering case, the packets have shorter average delay than that in infinite-buffering case. However, some packets have to be discarded in heavy traffic because there are not enough buffers for them.

## IV. ANALYTICAL AND SIMULATION EXAMPLES

### A. System

Assuming that the MPR capability is provided by spread spectrum, we consider a slotted CDMA network with  $J$  users. The user packets have fixed length of  $L_p$  bits and each packet is spread by a specific code with processing gain  $P$ . In each packet, up to  $t$  errors can be corrected due to a block error control coding. Moreover, the system is operated in a noisy environment where the variance of the additive white Gaussian noise (AWGN) is  $\sigma_v^2$ . Under the Gaussian assumption on the multi-access interference from users with equal power and SNR  $= 1/\sigma_v^2$ , the bit-error-rate (BER)  $p_e$  of a packet received in the presence of  $n - 1$  interfering packets is given by [23, p. 634]

$$p_e(n - 1) = Q \left( \sqrt{\frac{3P}{(n - 1) + 3P\sigma_v^2}} \right) \quad (55)$$

where  $Q(x) := \int_x^\infty (1/\sqrt{2\pi}) e^{-y^2/2} dy$  is the Marcum's  $Q$ -function [19]. If errors occur independently in a packet, the probability of receiving a packet successfully is given by

$$p_s(n - 1) = \sum_{i=0}^t B(i, L_p, p_e(n - 1)). \quad (56)$$

By the definition of  $C_{n,k}$ , we have

$$C_{n,k} = B(k, n, p_s(n - 1)). \quad (57)$$

Therefore, we can construct the MPR matrix  $\mathbf{C}$  for such a network using (1) and (57).

1) *Length of BM*: We will model the propagation channel as a nonfading channel with power control but arbitrary phase. Therefore, in (3), we take  $\alpha_j = Ae^{-j\phi_j}$ , where  $A$  is a constant and  $\phi_j$ s, ( $j = 1, 2, \dots, J$ ) are mutually independent random variables uniformly distributed over  $[0, 2\pi)$ . By [8], the detection probability  $P_D$  and the false alarm probability  $P_F$  for detector (5) are given by

$$P_D = Q \left( \frac{A}{\sigma_v/\sqrt{N}}, \frac{T}{\sigma_v/\sqrt{N}} \right), \quad P_F = \exp \left( -\frac{T^2}{2\sigma_v^2/N} \right) \quad (58)$$

where  $Q(\alpha, \beta) := \int_\beta^\infty z e^{-(z^2 + \alpha^2)/2} I_0(\alpha z) dz$  is the Marcum's  $Q$ -function defined on [28, p. 344], and  $I_0(\cdot)$  is a modified Bessel function of the first kind.

Without loss of generality, we take  $A = 1$  and scale the noise variance  $\sigma_v^2$  to achieve a desired SNR. If we need  $P_F = 0.01$  and  $P_D \geq 0.99$ , then given SNR  $= 1/\sigma_v^2$ , the plots of  $N$  and corresponding  $T$  versus SNR, respectively, are shown in Fig. 4(a) and (b).

Given  $N$ , the length of BM ( $L_B$ ) (in terms of data packets) can be obtained as

$$L_B = JN/(L_p P). \quad (59)$$

2) *Normalized Throughput*: As pointed out in [10], while spread spectrum and error control coding strengthen the chan-



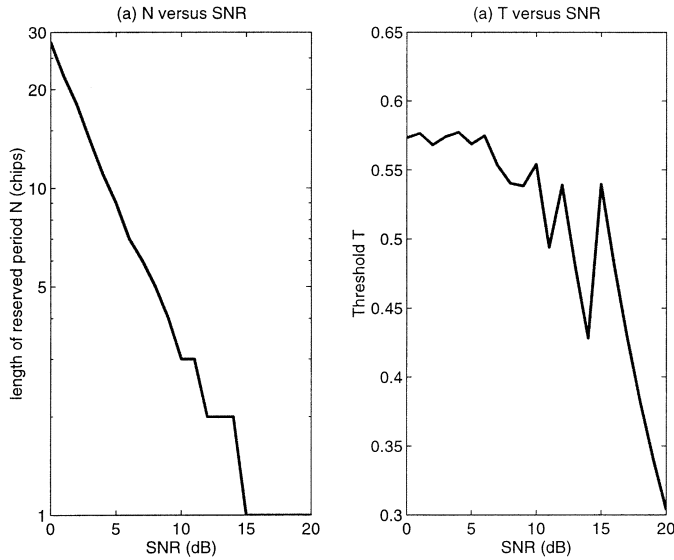


Fig. 4. Length of reserved period and corresponding threshold with  $P_D \geq 0.99$  and  $P_F = 0.01$ .

nels MPR capacity, they consume bandwidth. Following [10], we define a normalized throughput as the average number of information bits successfully transmitted per second per Hertz. It is given by

$$R_n = \frac{r_c}{P} R \quad (60)$$

where  $P$  is spreading gain, and  $r_c$  is the maximum coding rate that can be calculated from correctable number  $t$  of errors as

$$r_c = 1 + \alpha \log_2(\alpha) + (1 - \alpha) \log_2(1 - \alpha), \quad \alpha = \frac{2t + 1}{L_p}. \quad (61)$$

## B. Simulations

We tested the proposed BMDQ protocol on the simulated slotted data communication system as described in Section IV-A with the parameters  $J = 10$ ,  $L_p = 250$ ,  $P = 8$ ,  $t = 5$ . The simulations were carried under two SNRs: 10 and 20 dB. For each SNR, the duration of reserved period  $N$  can be obtained from Fig. 4, and then, the length of BM can be calculated from (59). Specifically, when SNR = 10 dB and desired  $P_F = 0.01$ , we obtain  $N = 3$  chips and  $L_B = (10 \times 3)/(250 \times 8) = 0.015$  packets with the corresponding  $P_D = 0.9948$ . When SNR = 20 dB and desired  $P_F = 0.01$ , we obtain  $N = 1$  chips and  $L_B = (10 \times 1)/(250 \times 8) = 0.005$  packets with the corresponding  $P_D \approx 1$ . Since  $N$  turns out to be so small, for all simulations presented (unless otherwise noted), we take  $N = 7$  [ $L_B = (10 \times 7)/(250 \times 8) = 0.035$  packets] with a fixed threshold  $T$  in (5) for all SNRs leading to  $P_D = 0.9999$ ,  $P_F = 2.15 \times 10^{-5}$  for SNR = 10 dB, and  $P_D \approx 1$ ,  $P_F = 10^{-14}$  for SNR = 20 dB. Recall that for synchronization purposes, one may need a “reasonable” length  $N$ , whereas for detection purposes, “small”  $N$  suffices. In the simulations, each user’s buffer was fed with a Poisson source with intensity  $\lambda$ . Under each SNR, 15 cases were considered, where in each case, the user’s buffer was fed with a different

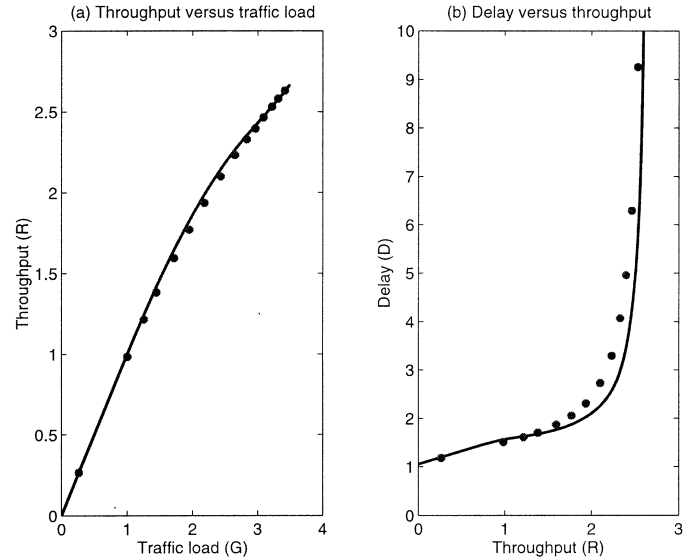


Fig. 5. Experimental and analytical performance results in infinite-buffering case when SNR = 10 dB.

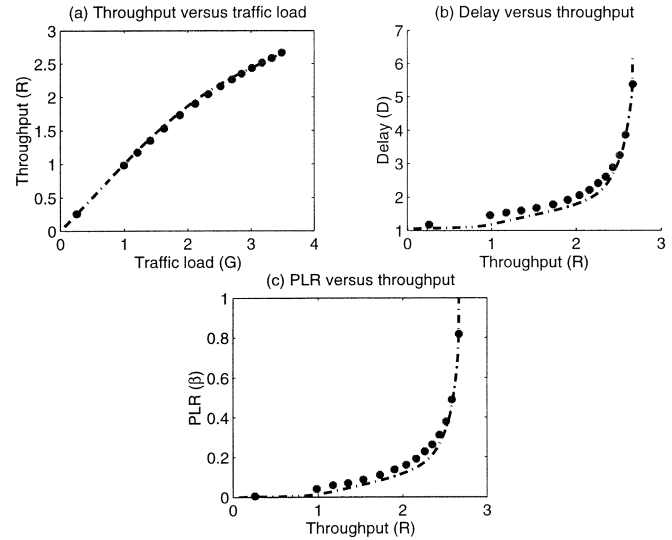


Fig. 6. Experimental and analytical performance results in one-packet-buffering case when SNR = 10 dB.

$\lambda$ . For each case, the results are obtained by averaging over 50 independent runs, where in each run, the system was run for a time period equivalent to 1000 TPs.

Fig. 5 shows the simulation results for the infinite-buffer case, whereas Fig. 6 is for the one-packet-buffer case, when SNR = 10 dB. In both figures, the solid or dash-dot lines are obtained from the analytical results, whereas each star point is the simulation result. Figs. 7 and 8 show the simulation results for SNR = 20 dB. Note that in Section II, the analytical results have been derived under the assumption of perfect detection ( $P_D = 1$ ) for the AUS, whereas the simulations are carried out with the specific  $P_D$  and  $P_F$ . Clearly, some loss in the throughput and increase in PLR is expected due to missed detections ( $P_D < 1$ ), and some additional delay is expected due to false alarm ( $P_F > 0$ ). Figs. 5–8 verify this trend. The differences between the analytical and the simulation results may also partly arise due to the approximations used in calculating the analytical expression for the average delay. For the infinite-

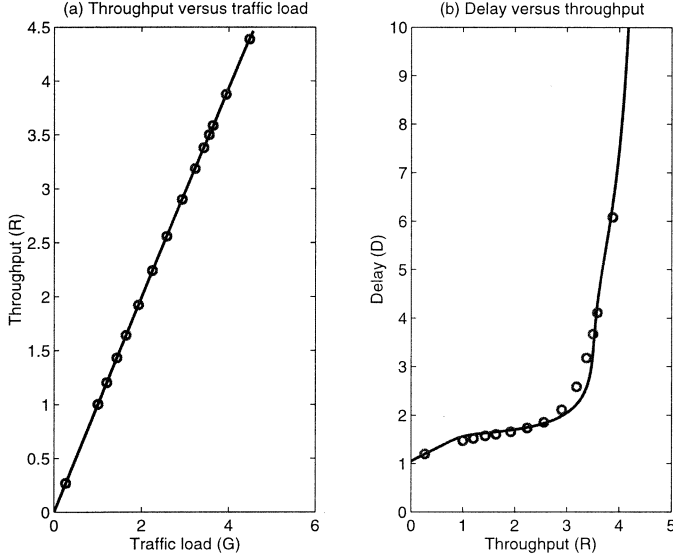


Fig. 7. Experimental and analytical performance results in infinite-buffering case when SNR = 20 dB.

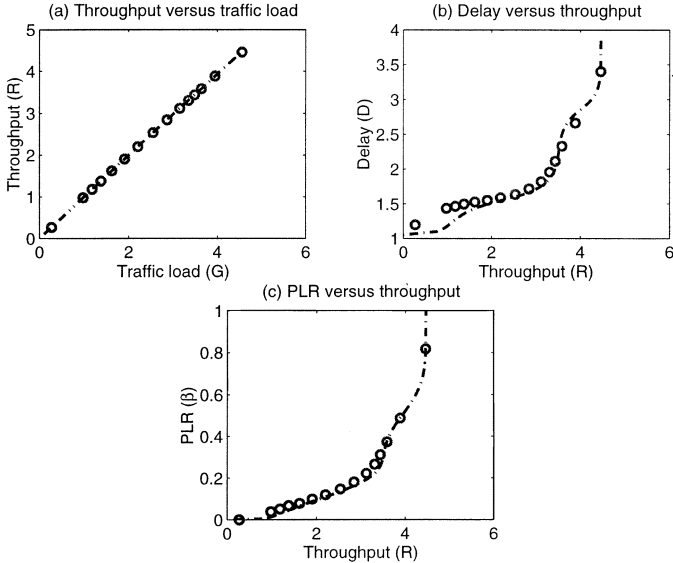


Fig. 8. Experimental and analytical performance results in one-packet-buffering case when SNR = 20 dB.

buffering case, the analytical delays are given by (48). By definition

$$E[h_R^2] = \sum_{l=L_B+1}^{\infty} \left( \sum_{K=1}^J P_R(K) P_{h_R, K}(l) \right) l^2. \quad (62)$$

However, because the probabilities  $P_{h_R, K}(l)$  are hard to calculate, we used the approximation (recall that  $\bar{L}_K = \bar{L}_{KR}$ )

$$E[h_R^2] \approx \sum_{K=1}^J P_R(K) (L_B + \bar{L}_K)^2. \quad (63)$$

Similar approximation was made in calculating  $E[h_T^2]$ . For the one-packet-buffering case, we made additional two approximations specified in (71) and (74) in the Appendix.

Finally, in Fig. 9, we show the performance results for two different lengths of the BM slot when SNR = 10 dB:  $N = 3$  is obtained from Fig. 4 with the desired  $P_F = 0.01$ , yielding

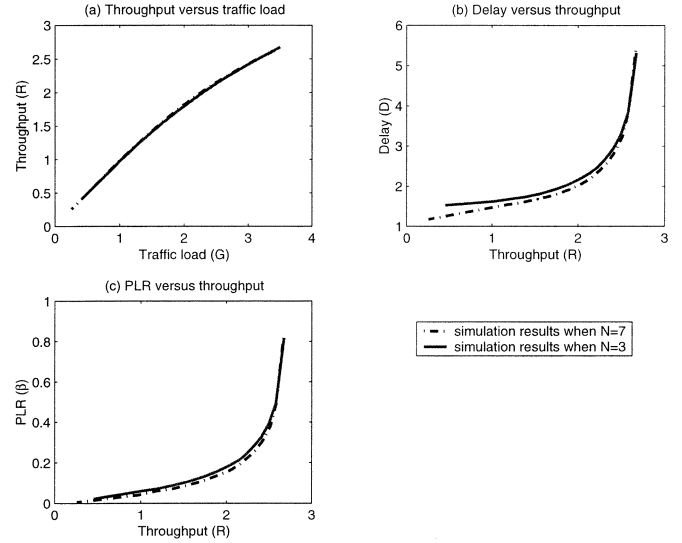


Fig. 9. Comparison of experimental performance results in one-packet-buffering case for different BM slot lengths. SNR = 10 dB.

$P_D = 0.9948$ , whereas  $N = 7$  is arbitrarily chosen to be of “sufficient” length with a fixed threshold  $T$  in (5), yielding  $P_D = 0.9999$  and  $P_F = 2.15 \times 10^{-5}$ . It is seen from Fig. 9 that in spite of a longer BM slot, we have better performance for  $N = 7$  since both  $P_D$  and  $P_F$  are “improved.”

### C. Comparison With Other MAC Protocols

Consider the same simulated network as in Section IV-B under SNR = 10 dB. From (2), the MPR capacity of this channel is  $\eta = 2.8990$ , which is achieved by simultaneously transmitting  $n_0 = 4$  packets in one packet slot. We compare the performance of the proposed BMDQ protocol with that of other MAC protocols in such a scenario.

1) *Comparison With Conventional MAC Protocols:* We select spread ALOHA [6] and slotted ALOHA with multiuser detection (MUD) [7] as representatives of the conventional protocols. While spread ALOHA is an unslotted ALOHA protocol with MPR capability obtained from spread spectrum techniques, the slotted ALOHA with MUD is a slotted ALOHA with MPR capability from MUD techniques. Their performance analysis based on the MPR matrix model (1) has been investigated in the Appendix. Fig. 10(a) shows the throughput versus traffic load for the proposed BMDQ and these two ALOHA protocols. Fig. 10(b) shows the normalized throughput versus traffic load in which the normalized throughputs are calculated according to (60). Comparison of the delay performance is shown in Fig. 10(c). In Fig. 10, the throughput and delay curves for spread ALOHA and those for slotted ALOHA with MUD are plotted according to the corresponding expressions modified from the existing expressions in [6] and [7] (see the Appendix).

As seen in Fig. 10(a) and (b), the proposed BMDQ protocol has a higher maximum stable throughput and can achieve the same throughput with lower traffic load. From Fig. 10(c), we see that the delay performance of the BMDQ protocol is a little bit worse in light traffic load than that for the spread ALOHA and slotted ALOHA with MUD. However, the differences are small. Note that the expressions for the delay for the two ALOHA protocols in the Appendix were derived under some assumptions

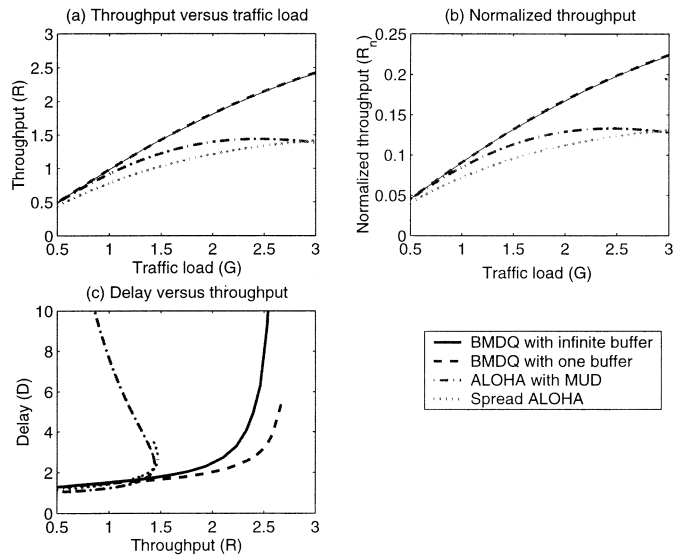


Fig. 10. Performance comparison for ALOHA (based on analytical expressions) and BMDQ. In (a) and (b), the curves for BMDQ with infinite buffer and BMDQ with one buffer are overlaid. SNR = 10 dB.

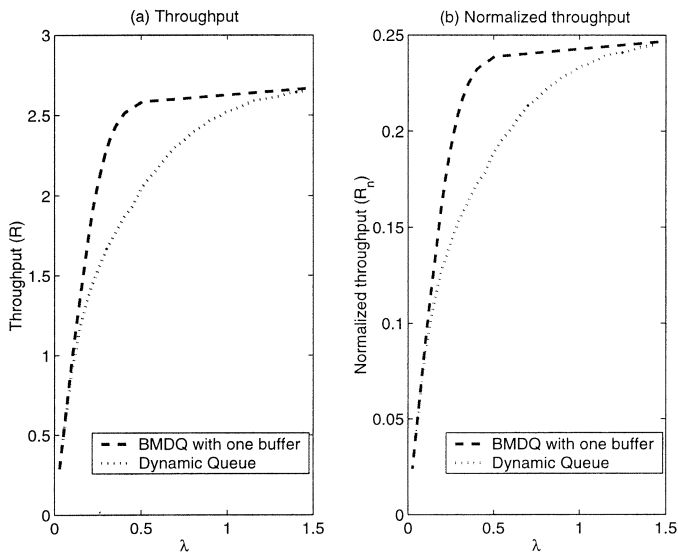


Fig. 11. Comparison of throughput for BMDQ and dynamic queue in the one-packet-buffering case.  $\lambda$  denotes the intensity of a single Poisson-distributed user. SNR = 10 dB. [Note that since we have a one-packet buffer, for higher  $\lambda$ s, there are lots of discarded packets; see also Fig. 13.]

favorable to ALOHA. We conclude that the proposed BMDQ protocol has a delay performance comparable with that of the two ALOHA protocols. Overall, the proposed BMDQ protocol outperforms the ALOHA protocols. The main reason that accounts for this difference is that the BMDQ is a network-assisted protocol, whereas the ALOHA protocols are not.

2) *Comparison With Dynamic Queue Protocol:* Now, we consider the dynamic queue protocol [10]: a network-assisted protocol explicitly designed for networks with MPR capability. We compare the throughput and normalized throughput of the BMDQ and the dynamic queue protocols in the one-packet-buffering case. Execution of the dynamic queue protocol requires knowledge of the probability that a user has a packet to transmit in a given TP. This knowledge can be inferred from the knowledge of  $\lambda$ , and in our simulations, the dynamic

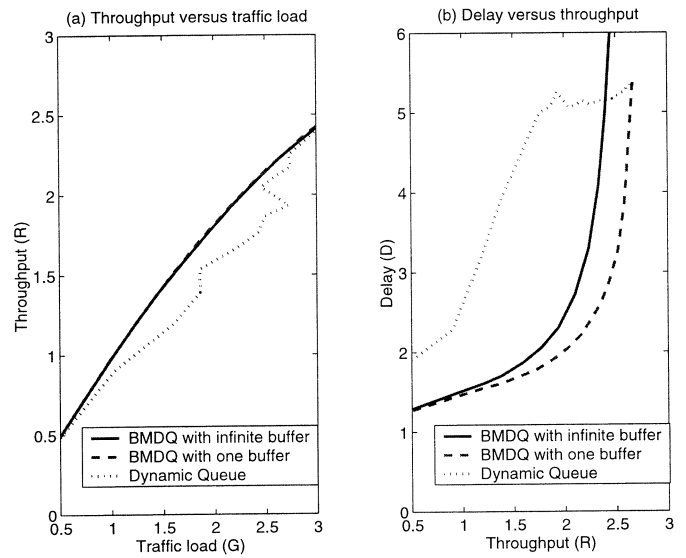


Fig. 12. Performance comparison for BMDQ and dynamic queue. SNR = 10 dB.

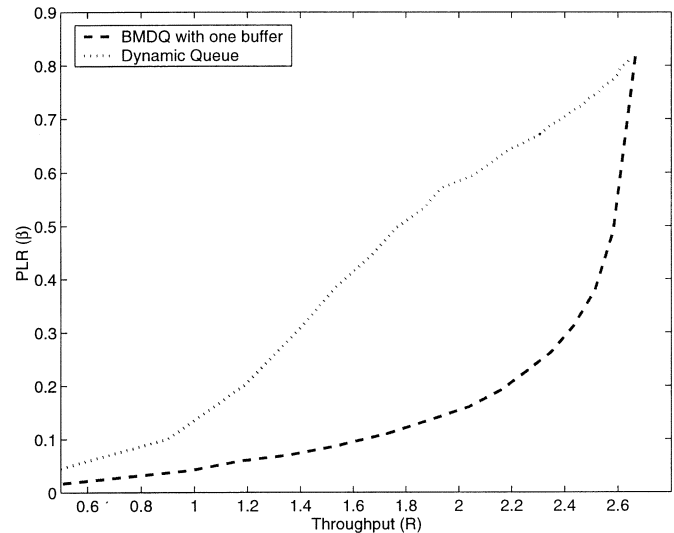


Fig. 13. Comparison of PLR for BMDQ and dynamic queue. SNR = 10 dB.

queue protocol was given this information. Fig. 11(a) shows the throughput versus  $\lambda$  for the two protocols. The normalized throughputs, which have been obtained via (60), are shown in Fig. 11(b). From Fig. 11, we see that when  $\lambda$  becomes large, the throughput performance of dynamic queue approaches that of BMDQ. This is so because when  $\lambda$  is large, for the dynamic queue protocol, the steady-state probability  $\tilde{q} := 1 - P_e \rightarrow 1$ . When  $\tilde{q} \rightarrow 1$ , the Markov chain of dynamic queue becomes almost the same as that of the BMDQ. Due to the existence of the BM slot in BMDQ, it is reasonable to expect the dynamic queue to outperform BMDQ for  $\tilde{q} \rightarrow 1$  and to have a little higher maximum stable throughput since the presence of the BM slot wastes resources. In this example, since the overhead cost induced by the BM is very small (0.035 packets), the maximum stable throughputs for the two protocols are almost the same as  $\tilde{q} \rightarrow 1$ . Fig. 12(a) shows the throughput versus traffic load for the two protocols, and Fig. 12(b) shows the average delay versus throughput. Comparison of PLR versus throughput is shown in Fig. 13. It is seen that for this example, the BMDQ

protocol is superior to the dynamic queue protocol, achieving the same throughput with lighter traffic load and shorter delay and achieving the same throughput with lower PLR.

## V. CONCLUSIONS

A novel network-assisted MAC protocol known as the BMDQ protocol was proposed for multiaccess wireless networks with MPR capability. The traffic in the channel is viewed as a flow of transmission periods (TPs). Each TP has a bit-map (BM) slot that is contention-free at the beginning followed by a data transmission period (DP), which is contention-based. The BM slot is reserved for user detection so that accurate knowledge of active user set (AUS) can be obtained. Then, given the knowledge of the AUS and the channel MPR matrix, the access set in each packet slot in the DP is chosen to maximize the conditional throughput of every packet slot. Therefore, an optimal utilization of the channel MPR capability is achieved, namely, obtaining high stable throughput with low traffic load and short delay. Compared with some existing conventional and network-assisted MAC protocols, it can achieve the same stable throughput with lower traffic load and shorter delay. The performance of the proposed protocol was illustrated by both (approximate) analysis and simulation results.

## APPENDIX

Here, we provide proofs of Props. 1–3 and derive some performance expressions used in Section IV-C.

*Proof of Proposition 1:* When  $P_D \rightarrow 1$  and  $P_F \rightarrow 0$ ,  $\tilde{P}_u(K) = P_u(K)$ , and we have

$$R = \frac{J(1 - P_e)}{L_B + \sum_{K=1}^J \bar{L}_K P_u(K)} = \frac{J(1 - P_e)}{E\{h\}}. \quad (64)$$

From (33), we have

$$D(P_e) = P_e(1 + \lambda E\{h_I\} - \lambda E\{h_R\}) - (1 - \lambda E\{h_R\}) = 0. \quad (65)$$

Therefore

$$\lambda = \frac{1 - P_e}{(1 - P_e)E\{h_R\} + P_e E\{h_I\}} = \frac{1 - P_e}{E\{h\}}. \quad (66)$$

Hence

$$R = \frac{J(1 - P_e)}{E\{h\}} = J\lambda. \quad (67)$$

*Proof of Proposition 2:* For a particular data packet, its serving TP is a relevant TP for its user. As discussed in Section III-A1, the length  $L$  of the DP in the serving TP is a random variable whose distribution depends on the number of active users  $K$ . Following (21),  $L$  obeys the distribution

$$P_L(\tilde{l}) = \sum_{K=1}^J P_R(K) P_{h_{R,K}}(L_B + \tilde{l}), \quad \tilde{l} = 1, 2, \dots \quad (68)$$

where  $P_R(K)$  is given by (14),  $P_{h_{R,K}}(L_B + \tilde{l})$  is specified in (17),  $h_R$  is the length of the relevant TP, and  $h_R = L_B + L$ . Since we transmit the users' packets in a randomized order in the BMDQ protocol, it is reasonable to assume that for a particular

user, its data packet is successfully transmitted at the end of the  $d$ th packet slot in the DP with probability  $1/\tilde{l}$  when the length of DP is  $\tilde{l}$ . Hence, the expected delay in the serving TP for a particular packet is given by

$$\begin{aligned} E\{s_I\} &= L_B + \sum_{\tilde{l}=1}^{\infty} \left( P_L(\tilde{l}) \sum_{d=1}^{\tilde{l}} \frac{1}{\tilde{l}} d \right) \\ &= L_B + \sum_{\tilde{l}=1}^{\infty} \left( P_L(\tilde{l}) \frac{\tilde{l}+1}{2} \right) \\ &= L_B + \frac{1}{2} (1 + E\{L \mid \text{relevant TP}\}). \end{aligned} \quad (69)$$

Since  $E\{L \mid \text{relevant TP}\} + L_B = E\{h_R\}$ , we have

$$E\{s_I\} = \frac{1}{2} (1 + L_B + E\{h_R\}). \quad (70)$$

*Proof of Proposition 3:* Any given user's buffer is fed with a Poisson source with intensity  $\lambda$ . For relating  $P_e$  to  $\lambda$ , we use the following (crude) approximation:

$$P_e \approx e^{-\lambda E\{h\}}. \quad (71)$$

Simulation results presented in Section IV-B suggest that this approximation is reasonable. According to the Poisson distribution, the probability that the first packet is generated before time  $t_g$  is given by

$$\Pr(t < t_g) = 1 - e^{-\lambda t_g}. \quad (72)$$

Therefore, given that at least one packet is generated in the TP, the conditional probability density of the first packet being generated at time  $t_g$  is given by

$$p(t_g) = \frac{d}{dt_g} \frac{\Pr(t < t_g)}{1 - P_e} = \frac{\lambda e^{-\lambda t_g}}{1 - P_e}. \quad (73)$$

Therefore, using some (crude) approximation, we have

$$\begin{aligned} E\{w_1\} &= E[h - t_g] \approx E\{h\} - \int_0^{E\{h\}} t_g p(t_g) dt_g \\ &= E\{h\} - \frac{1}{1 - P_e} \int_0^{E\{h\}} t_g \lambda e^{-\lambda t_g} dt_g \\ &= E\{h\} + \frac{1}{1 - P_e} \left[ e^{-\lambda E\{h\}} E\{h\} - \frac{1}{\lambda} (1 - e^{-\lambda E\{h\}}) \right]. \end{aligned} \quad (74)$$

Substituting (71) in (74), we have

$$E\{w_1\} \approx \frac{1}{1 - P_e} E\{h\} - \frac{1}{\lambda} = \left( \frac{1}{1 - P_e} + \frac{1}{\ln P_e} \right) E\{h\}. \quad (75)$$

*Performance of Spread ALOHA:* In the spread ALOHA system [6], each bit of the packets from different users is spread with the same short  $m$ -sequence. Because of the good auto-correlation of the  $m$ -sequence, the bits (symbols) from two different users become coherent only when their relative delay is less than two chip durations. A collision occurs only when there are coherent bits. Therefore, if all packets are spread with an  $m$ -sequence of  $P$  chips, at most  $P/2$  packets can be simultaneously transmitted with success. In the model of [6], a

given packet is correctly received if no other packet is generated in the vulnerable period of two chip durations. The probability of no other traffic being initiated during the vulnerable period is given by  $\exp(-2G/P)$ , where  $2G/P$  is the average number of packets generated in the two chip intervals, and  $G$  is the offered traffic load. Therefore, the throughput  $R_{SA}$  for spread ALOHA is given by [6]

$$R_{SA} = Ge^{-2G/P}. \quad (76)$$

However, the above expression is based on a noiseless collision channel model (i.e., not an MPR model), and it also ignores any noise. Since a given packet is correctly received not with certainty, but with probability  $C_{1,1}$  in the MPR channel model, the throughput is given by

$$R_{SA} = Ge^{-2G/P}C_{1,1}. \quad (77)$$

The maximum throughput  $(R_{SA})_{\max} = (P/2e)C_{1,1}$  is obtained when  $G = P/2$ .

Clearly, a packet needs to be transmitted or retransmitted on the average  $G/R_{SA}$  times until we are successful. Therefore, the average delay (in terms of packet duration) for a particular packet is approximately given by

$$D_{SA} = (G/R_{SA}) + (G/R_{SA} - 1)\bar{\tau} \quad (78)$$

where  $\bar{\tau}$  is the average interval between two (re-)transmissions. According to the (pure) ALOHA protocol, the probability for the interval  $\tau$  between two (re-)transmissions is given by [19]

$$p(\tau) = \alpha e^{-\alpha\tau} \quad (79)$$

where  $\alpha$  is a preset parameter with  $\alpha = 1/\bar{\tau}$ . Therefore, we have

$$D_{SA} = (G/R_{SA}) + \frac{1}{\alpha} ((G/R_{SA}) - 1) \geq G/R_{SA} \quad (80)$$

where the equality holds only when we let  $1/\alpha = \bar{\tau} \rightarrow 0$  by selecting  $\alpha$  as a large number. •

*Performance of Slotted ALOHA With MUD:* In [7], a slotted ALOHA packet communication system utilizing MUD has been presented. In this system, slotted ALOHA is still adopted as the access protocol. However, with the help of the MUD techniques, a packet may be successfully transmitted, even if there are concurrent transmissions. In [7], a general MPR channel such as (1) was not considered; instead, in terms of (1), [7] assumed that for some  $M \geq 1$ ,  $C_{K,m} = 1$  if  $K \leq M$  and  $m = K$ , and  $C_{K,m} = 0$  otherwise. Consider a slotted ALOHA network with MUD capability, MPR channel, and  $J$  users. Since in slotted ALOHA, the number of active users  $K$  (= number of packets) in a slot obeys a Poisson distribution [1]

$$P_A(K) = \frac{G^K}{K!} e^{-G}, \quad K = 0, 1, 2, \dots \quad (81)$$

where  $G$  denotes the traffic load of the network. According to the MPR channel model (1), the expected number of successfully received packets when there are  $K$  active users (=  $K$  transmitted packets) is given by

$$C_K = \sum_{m=1}^K m C_{K,m}. \quad (82)$$

Therefore, the throughput for this slotted ALOHA is given by

$$R_{AM} = \sum_{K=1}^J P_A(K) C_K = \sum_{K=1}^J \left( \frac{G^K}{K!} e^{-G} \sum_{m=1}^K m C_{K,m} \right). \quad (83)$$

Clearly

$$R_{AM} \leq \sum_{K=1}^J \left( \frac{G^K}{K!} e^{-G} \eta \right) < \eta. \quad (84)$$

It is of interest to compare (83) with the corresponding [7, expression (3)]. As discussed earlier, [7] assumes a "perfect" system with a MUD that can detect up to  $M$  users, leading to  $C_{K,m} = 1$  if  $m = K \leq M$ , and  $C_{K,m} = 0$  otherwise. Using these values for  $C_{K,m}$  in (82), we obtain  $C_K = K$  if  $K \leq M$ , and  $C_K = 0$  otherwise. Using this result in (83), we obtain for the model of [7]

$$\begin{aligned} R_{AM} &= \sum_{K=1}^J P_A(K) C_K = \sum_{K=1}^M \frac{G^K}{K!} e^{-G} K \\ &= Ge^{-G} \sum_{K=0}^{M-1} \frac{G^K}{K!}. \end{aligned} \quad (85)$$

Expression (85) is the same as [7, (3)].

Similarly, the average delay for a particular packet is given by

$$D_{AM} = (G/R_{AM}) + (G/R_{AM} - 1)\bar{\tau}. \quad (86)$$

However, in a slotted ALOHA network, the interval between transmissions is discrete. Therefore, based on (79), the probability for the interval is given by

$$p(n) = \int_{n-1}^n \alpha e^{-\alpha\tilde{\tau}} d\tilde{\tau} = e^{-\alpha(n-1)} - e^{-\alpha n}, \quad n = 1, 2, \dots \quad (87)$$

Therefore

$$\bar{\tau} = (e^\alpha - 1) \sum_{n=1}^{\infty} n e^{-\alpha n} = \frac{1}{1 - e^{-\alpha}}. \quad (88)$$

Substituting (88) into (86), we have

$$D_{AM} = G/R_{AM} + \frac{1}{1 - e^{-\alpha}} (G/R_{AM} - 1) \geq 2G/R_{AM} - 1 \quad (89)$$

where the equality holds only when we let  $e^{-\alpha} \rightarrow 0$  by selecting  $\alpha$  as a large number. •

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