

A TOA-Based Location Algorithm Reducing the Errors Due to Non-Line-of-Sight (NLOS) Propagation

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Abstract—An effective location algorithm, which considers non-line-of-sight (NLOS) propagation, is presented. By using a new variable to replace the square term, the problem becomes a mathematical programming problem, and then the NLOS propagation's effect can be evaluated. Compared with other methods, the proposed algorithm has high accuracy.

Index Terms—Maximum-likelihood (ML) estimation, non-line-of-sight (NLOS) propagation, time difference of arrival (TDOA), time of arrival (TOA).

I. INTRODUCTION

IN LAND cellular wireless location systems, one of the key parts is the location algorithm. Generally, a number of receivers distributed separately are used to receive the transmitted signal from a source and make the measurements accurately for the time of arrival (TOA) or the time difference of arrival (TDOA) [1]–[3]. In the multipath propagation environment, TOA is the measured propagation delay of the earliest distinguished path in the receivers. With the data of TOA or TDOA, the location algorithms are used to estimate the position of the source in the location service center.

The problem of location estimation is simplified when the receivers are distributed along a straight line, and many optimum processing techniques for this situation have been proposed [4]–[6]. But when receivers are distributed arbitrarily, the estimation becomes more complex. In this case, Chan and Ho proposed a two-step maximum-likelihood (ML) TDOA-based location algorithm, which has high accuracy when the non-line-of-sight (NLOS) propagation interference is not very serious [11].

Although other methods [7]–[10] have also been studied, these algorithms do not consider the inferences of NLOS propagation on the location estimation. Because NLOS propagation always exists in cities or other builtup environments so that actually the signals arrive at the different receivers via NLOS propagation, the influences of NLOS propagation on the location estimation must be taken into account. The NLOS error is roughly estimated in [12]. It is indicated that a large deviation of measurement would be caused by NLOS errors, and these errors would degrade the mean location accuracy

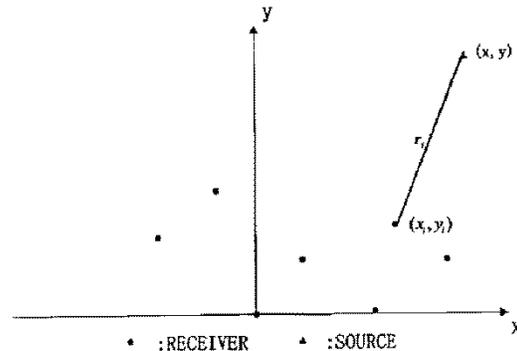


Fig. 1. Localization in a 2-D plane.

dramatically. The algorithm in [13], which incorporates known statistics of NLOS into a probability density function (pdf) model, could reduce the NLOS errors. However, not only is the pdf difficult to set up, but also this pdf should vary flexibly with the changing of the NLOS errors. And since the numerical search in [13] is used to find the solution of location, large computation is needed to obtain the high estimation accuracy. Therefore, the efforts for finding an efficient method to reduce the NLOS errors are still needed. One method is to distinguish NLOS by counting the standard deviation of the TOA measurement; then the NLOS receivers can be excluded or given less weight in location algorithms [14]–[16]. An alternative approach is to exploit the property that the NLOS errors are always positive errors, then to search the true position by adding some constraints such as penalty function in [17] and [18]. The NLOS algorithm proposed in this paper is also one of these alternative approaches. According to the simulation and comparison, this method shows higher accuracy.

II. THE NLOS LOCATION ALGORITHM

Here, for simplification, we consider the location in a two-dimensional (2-D) plane. The extension to three-dimensional space can also be done with the same steps described as below. In Fig. 1, M receivers are distributed arbitrarily in a 2-D plane.

Assuming that (x, y) is the position of the source, (x_i, y_i) is the position of the i th receiver and d_i is the TOA measured in receiver i . TOA can be estimated by an extended Kalman filter [19] or other methods. Since in practice, especially in big cities

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or in mountainous areas, the signals from the source are usually unable to arrive at the receivers directly, they always take a longer path than the direct one. So by incorporating the influences of NLOS propagation on wireless location, there exists

$$r_i^2 \geq (x_i - x)^2 + (y_i - y)^2 \\ = K_i - 2x_i x - 2y_i y + x^2 + y^2, \quad i = 1, 2, \dots, M. \quad (1)$$

where $K_i = x_i^2 + y_i^2$, $r_i = cd_i$ is the measured distance between the source and the i th receiver, and c is the signal propagation speed. And by defining a new variable $R = x^2 + y^2$, we rewrite (1) through a set of linear expressions

$$-2x_i x - 2y_i y + R \leq r_i^2 - K_i, \quad i = 1, 2, \dots, M. \quad (2)$$

Let $\mathbf{Z}_a = [x, y, R]^T$, and express (2) in matrix form

$$\mathbf{G}_a \mathbf{Z}_a \leq \mathbf{h} \quad (3)$$

where

$$\mathbf{h} = \begin{bmatrix} r_1^2 - K_1 \\ r_2^2 - K_2 \\ \vdots \\ r_M^2 - K_M \end{bmatrix} \\ \mathbf{G}_a = \begin{bmatrix} -2x_1 & -2y_1 & 1 \\ -2x_2 & -2y_2 & 1 \\ \vdots & \vdots & \vdots \\ -2x_M & -2y_M & 1 \end{bmatrix}. \quad (4)$$

When LOS propagation exists between the source and all the receivers, (3) turn into equalities. In this case, the ML solution is given by [11]

$$\mathbf{Z}_a = (\mathbf{G}_a^T \mathbf{\Psi}^{-1} \mathbf{G}_a)^{-1} \mathbf{G}_a^T \mathbf{\Psi}^{-1} \mathbf{h} \quad (5)$$

where

$$\mathbf{\Psi} = E[\psi \psi^T] = 4c^2 \mathbf{B} \mathbf{Q} \mathbf{B} \\ \psi = \mathbf{h} - \mathbf{G}_a \mathbf{Z}_a \\ \mathbf{B} = \text{diag}\{r_1^0, \dots, r_M^0\} \quad (6)$$

\mathbf{Q} is the covariance matrix of measured noise, and r_1^0, \dots, r_M^0 are denoted as the true values of distances between the source and the receivers. In the estimation, we need to use matrix \mathbf{B} . But as shown in (6), the entries in the diagonal of \mathbf{B} are the unknown true distances from source to receivers. So, we can firstly use measured values r_1, \dots, r_M instead of true values r_1^0, \dots, r_M^0 for estimating an initial solution, then calculate the corresponding \mathbf{B} using this initial solution and afterwards get a further accurate result using again this new matrix \mathbf{B} . The process can be iterated until the results converge. Simulation shows the processing can quickly converge after several iterations in most cases. Denote the solution of this LOS algorithm as (x', y') .

The above TOA-based LOS location algorithm has high accuracy when the NLOS propagation is not heavy. As the NLOS propagation becomes heavy, its accuracy degrades because the equalities are not held in (3). So in the NLOS propagation case, we have to take into account the inequalities in (3). This means that we should find the ML estimate under the conditions given by (3) instead of searching it in full space, if the ML estimator

is still used here. Therefore, we have a mathematical programming problem as below

$$\min \{(\mathbf{h} - \mathbf{G}_a \mathbf{Z}_a)^T \mathbf{\Psi}^{-1} (\mathbf{h} - \mathbf{G}_a \mathbf{Z}_a)\}, \text{ such that } \mathbf{G}_a \mathbf{Z}_a \leq \mathbf{h} \quad (7)$$

where \mathbf{h} , \mathbf{G}_a , \mathbf{Z}_a , and $\mathbf{\Psi}$ have the same definitions as (4) and (6). For estimating $\mathbf{\Psi}$, we use r'_1, \dots, r'_M , which are calculated from (x', y') instead of true values r_1^0, \dots, r_M^0 as the diagonal entries of \mathbf{B} in (6).

It is obvious that (7) is a constrained linear least square problem, a type of quadratic programming (QP) problem. There are many algorithms developed to solve this type of problem [20]; here, the Matlab function quadprog is used to find the solution. When the satisfied solution of (7), \mathbf{Z}_a , is found, the covariance matrix of \mathbf{Z}_a can be calculated as [11]

$$\text{cov}(\mathbf{Z}_a) = (\mathbf{G}_a^T \mathbf{\Psi}^{-1} \mathbf{G}_a)^{-1}. \quad (8)$$

Since we have used the independence supposition of variables x , y , and R in the estimation of \mathbf{Z}_a though the variable R is dependent on the variable x and y , we should revise the results as follows. Let the estimation errors of x , y , and R be e_1 , e_2 , and e_3 . Here and below, denote the (i, j) th entry of a matrix \mathbf{M} as $[\mathbf{M}]_{i,j}$; then the entries in vector \mathbf{Z}_a become

$$[\mathbf{Z}_a]_1 = x^0 + e_1, \quad [\mathbf{Z}_a]_2 = y^0 + e_2 \\ [\mathbf{Z}_a]_3 = R^0 + e_3 \quad (9)$$

where x^0 , y^0 , and R^0 are denoted as the true values of x , y , and R . Let another error vector

$$\psi' = \mathbf{h}' - \mathbf{G}_a' \mathbf{Z}_p \quad (10)$$

where

$$\mathbf{h}' = \begin{bmatrix} [\mathbf{Z}_a]_1^2 \\ [\mathbf{Z}_a]_2^2 \\ [\mathbf{Z}_a]_3 \end{bmatrix}, \quad \mathbf{G}_a' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \quad (11)$$

and $\mathbf{Z}_p = \begin{bmatrix} x^2 \\ y^2 \end{bmatrix}$. Substituting (9) into (10), we have

$$[\psi']_1 = 2x^0 e_1 + e_1^2 \approx 2x^0 e_1 \\ [\psi']_2 = 2y^0 e_2 + e_2^2 \approx 2y^0 e_2, \quad [\psi']_3 = e_3. \quad (12)$$

Obviously, the above approximations are valid only when the errors e_1 , e_2 , and e_3 are fairly small. Subsequently, the covariance matrix of ψ' is

$$\mathbf{\Psi}' = E[\psi' \psi'^T] = 4\mathbf{B}' \text{cov}(\mathbf{Z}_a) \mathbf{B}' \\ \mathbf{B}' = \text{diag}\{x^0, y^0, 0.5\}. \quad (13)$$

As an approximation, elements x^0 and y^0 in matrix \mathbf{B}' can be replaced by the first two elements x and y in \mathbf{Z}_a . Similarly, the ML estimate of \mathbf{Z}_p is given by

$$\mathbf{Z}_p = (\mathbf{G}_a'^T \mathbf{\Psi}'^{-1} \mathbf{G}_a')^{-1} \mathbf{G}_a'^T \mathbf{\Psi}'^{-1} \mathbf{h}' \\ \approx (\mathbf{G}_a'^T \mathbf{B}'^{-1} (\text{cov}(\mathbf{Z}_a))^{-1} \mathbf{B}'^{-1} \mathbf{G}_a')^{-1} \\ \bullet (\mathbf{G}_a'^T \mathbf{B}'^{-1} (\text{cov}(\mathbf{Z}_a))^{-1} \mathbf{B}'^{-1}) \mathbf{h}'. \quad (14)$$

So the final position estimation $\mathbf{Z} = [x \ y]^T$ is

$$\mathbf{Z} = \sqrt{\mathbf{Z}_p}, \text{ or } \mathbf{Z} = -\sqrt{\mathbf{Z}_p}. \quad (15)$$

Here the sign of x should coincide with the sign of $[\mathbf{Z}_a]_1$ calculated by solving (7), and the sign of y coincides with the sign of $[\mathbf{Z}_a]_2$.

III. SIMULATION RESULTS AND COMPARISON

Simulations are done in two cases. In the first case, we assume that there are ten receivers, and the receivers are close to each other. Their positions are $(x_1 = 0 \text{ m}, y_1 = 0 \text{ m})$, $(x_2 = -500 \text{ m}, y_2 = 800 \text{ m})$, $(x_3 = 400 \text{ m}, y_3 = 600 \text{ m})$, $(x_4 = -200 \text{ m}, y_4 = 400 \text{ m})$, $(x_5 = 700 \text{ m}, y_5 = 300 \text{ m})$, $(x_6 = -700 \text{ m}, y_6 = 500 \text{ m})$, $(x_7 = 200 \text{ m}, y_7 = 500 \text{ m})$, $(x_8 = -400 \text{ m}, y_8 = 200 \text{ m})$, $(x_9 = 300 \text{ m}, y_9 = 300 \text{ m})$, and $(x_{10} = 100 \text{ m}, y_{10} = 800 \text{ m})$, and the source moves randomly in the square space $-500 \text{ m} \leq x_0, y_0 \leq 500 \text{ m}$

$$\begin{aligned} x_0 &= (1000 \cdot \text{rand} - 500) \text{ m} \\ y_0 &= (1000 \cdot \text{rand} - 500) \text{ m} \end{aligned} \quad (16)$$

where $\text{rand}(\cdot)$ is a random number from zero to one. And in the second case, we assume that there are still ten receivers but the distances between them are larger. Their positions are $(x_1 = 0 \text{ km}, y_1 = 0 \text{ km})$, $(x_2 = -5 \text{ km}, y_2 = 8 \text{ km})$, $(x_3 = 4 \text{ km}, y_3 = 6 \text{ km})$, $(x_4 = -2 \text{ km}, y_4 = 4 \text{ km})$, $(x_5 = 7 \text{ km}, y_5 = 3 \text{ km})$, $(x_6 = -7 \text{ km}, y_6 = 5 \text{ km})$, $(x_7 = 2 \text{ km}, y_7 = 5 \text{ km})$, $(x_8 = -4 \text{ km}, y_8 = 2 \text{ km})$, $(x_9 = 3 \text{ km}, y_9 = 3 \text{ km})$, and $(x_{10} = 1 \text{ km}, y_{10} = 8 \text{ km})$, and the source moves randomly in the square space $-5 \text{ km} \leq x_0, y_0 \leq 5 \text{ km}$

$$\begin{aligned} x_0 &= (10 \cdot \text{rand} - 5) \text{ km} \\ y_0 &= (10 \cdot \text{rand} - 5) \text{ km}. \end{aligned} \quad (17)$$

In both cases, let

$$r_i = r_i^0 + cn_i + N \cdot \text{rand}(\cdot) \quad (18)$$

where N is the possible maximum error introduced by NLOS, and assume that cn_i is a Gaussian random noise, which is zero mean with the same variance σ^2 . The covariance matrix of estimation noise in (6) is

$$\mathbf{Q} = \text{diag}(\sigma^2, \sigma^2, \dots, \sigma^2). \quad (19)$$

It can be easily proved that the LOS algorithm, which is used to obtain (x', y') in Section II, is the optimal estimator and the estimation error can theoretically reach Cramer–Rao lower bound when there exists LOS propagation (the proof is very similar to that in [11]). So we can evaluate the performance of the proposed NLOS algorithm through comparing its average location errors (ALEs) with those of the LOS algorithm. In the simulation, $\text{ALE} = E \left[\sqrt{(x - x^0)^2 + (y - y^0)^2} \right]$ is obtained from the average of 10000 independent runs. When the ALEs are changed with the number of receivers M , used in localization, taking $\sigma^2 = 100 \text{ m}^2$, $N = 300 \text{ m}$, the ALEs of the proposed NLOS algorithm and the LOS algorithm are given in Figs. 2 and 5, respectively, in case 1 and case 2. When $N = 300 \text{ m}$, $M = 10$, the ALEs of the NLOS algorithm and the LOS algorithm and their relationship with the power of Gaussian random noise σ^2 are shown in Figs. 3 and 6, respectively, in case 1 and case 2. And when $M = 10$, $\sigma^2 = 100 \text{ m}^2$, the ALEs of the NLOS algorithm and the LOS algorithm and their relationship

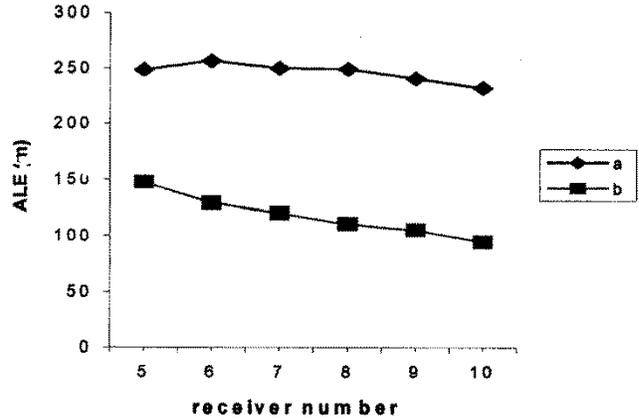


Fig. 2. Comparison of ALEs changed with M in the situation $N = 300 \text{ m}$, $\sigma^2 = 100 \text{ m}^2$ in case 1. (a) ALEs of TOA-based LOS algorithm. (b) ALEs of TOA-based NLOS algorithm.

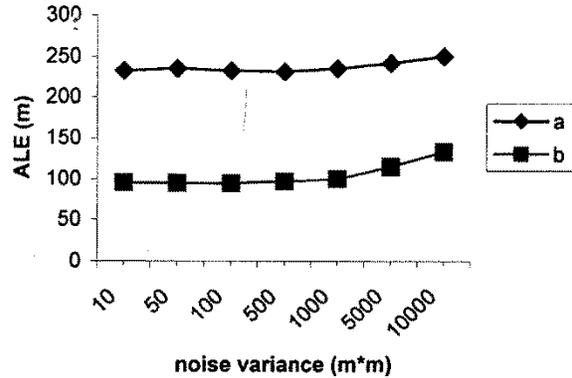


Fig. 3. Comparison of ALEs changed with σ^2 in the situation $N = 300 \text{ m}$, $M = 10$ in case 1. (a) ALEs of TOA-based LOS algorithm. (b) ALEs of TOA-based NLOS algorithm.

with the NLOS interference N are shown in Figs. 4 and 7, respectively, in case 1 and case 2. Shown in Figs. 2–4, the ALEs increase slightly as the number of receivers M decreases or the power of noise increases, while the ALEs increase largely as the NLOS interference increases. This means that NLOS interference plays a dominating role in the ALEs among the above three factors. Also, from Figs. 2–4, we know clearly that the ALEs of the NLOS algorithm are significantly improved compared with those of the LOS algorithm. The results in Figs. 5–7 are very similar to those in Figs. 2–4, except that the accuracy of the NLOS algorithm degrades a little in case 2. It can also be seen from Figs. 5–7 that the ALEs of the proposed NLOS algorithm are significantly improved compared with those of the LOS algorithm.

In [11], by comparing the proposed TDOA-based location algorithm with common penalty function SI method [8] and Taylor series method [9], [10], the author proved that the proposed algorithm has higher accuracy and the estimation error can theoretically reach Cramer–Rao lower bound, which is for any unbiased parameter estimation, when there is no or little

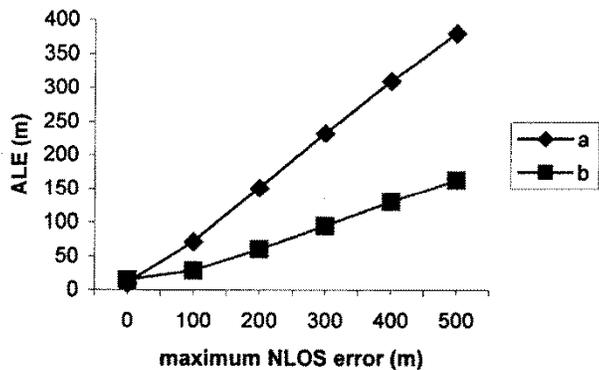


Fig. 4. Comparison of ALEs changed with N in the situation $\sigma^2 = 100 \text{ m}^2$, $M = 10$ in case 1. (a) ALEs of TOA-based LOS algorithm. (b) ALEs of TOA-based NLOS algorithm.

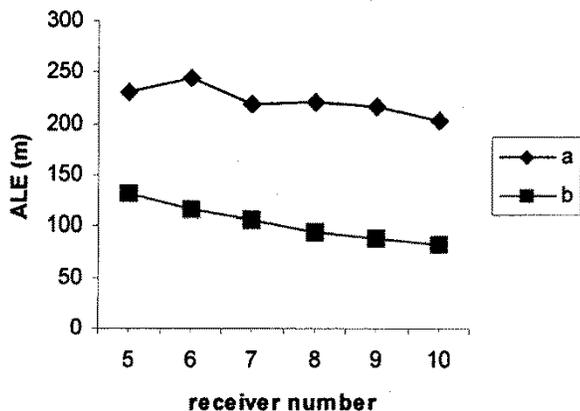


Fig. 5. Comparison of ALEs changed with M in the situation $N = 300 \text{ m}$, $\sigma^2 = 100 \text{ m}^2$ in case 2. (a) ALEs of TOA-based LOS algorithm. (b) ALEs of TOA-based NLOS algorithm.

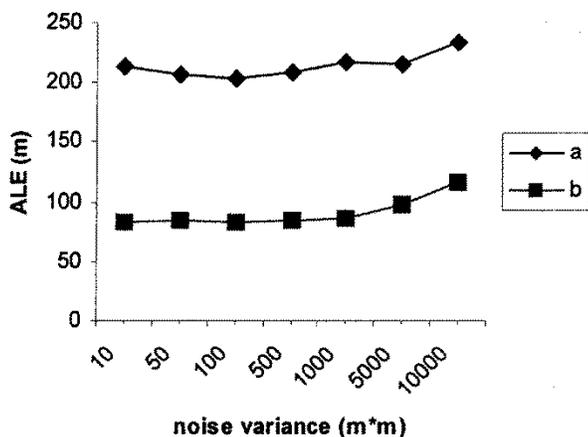


Fig. 6. Comparison of ALEs changed with σ^2 in the situation $N = 300 \text{ m}$, $M = 10$ in case 2. (a) ALEs of TOA-based LOS algorithm. (b) ALEs of TOA-based NLOS algorithm.

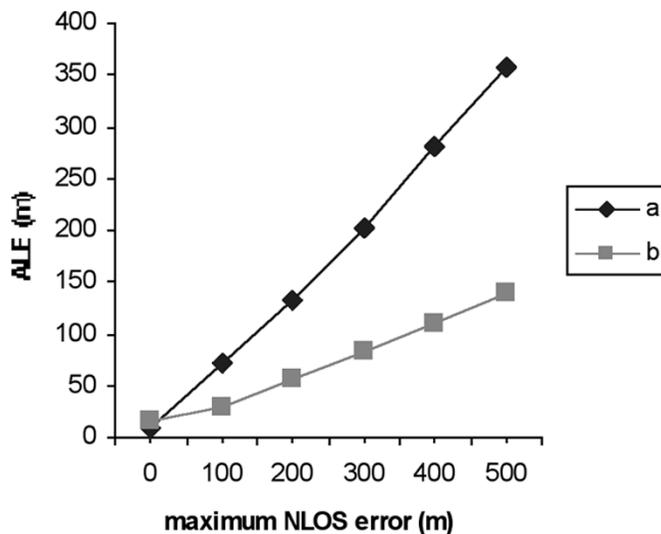


Fig. 7. Comparison of ALEs changed with N in the situation $\sigma^2 = 100 \text{ m}^2$, $M = 10$ in case 2. (a) ALEs of TOA-based LOS algorithm. (b) ALEs of TOA-based NLOS algorithm.

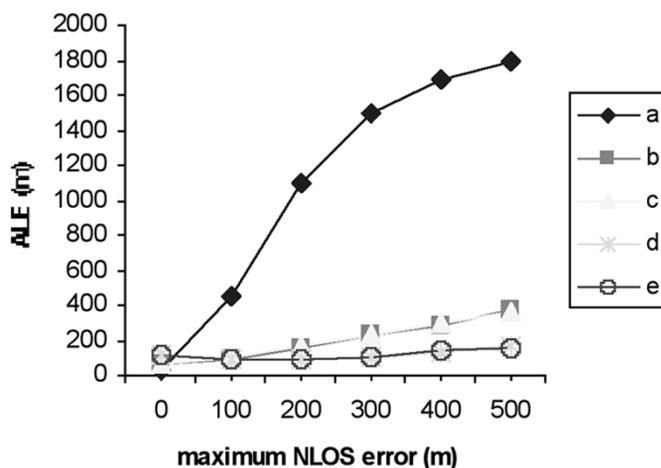


Fig. 8. Comparison of ALEs between TDOA-based algorithm in [11] and TOA-based algorithms proposed here. (a) ALEs of TDOA-based LOS algorithm when $\sigma^2 = 5 \text{ m}^2$. (b) ALEs of TOA-based LOS algorithm when $\sigma^2 = 500 \text{ m}^2$. (c) ALEs of TOA-based LOS algorithm when $\sigma^2 = 5000 \text{ m}^2$. (d) ALEs of TOA-based NLOS algorithm when $\sigma^2 = 500 \text{ m}^2$. (e) ALEs of TOA-based NLOS algorithm when $\sigma^2 = 5000 \text{ m}^2$.

NLOS propagation. So it can be safely said that the algorithm in [11] can achieve the optimum performance without NLOS propagation. Thus we choose the algorithm in [11] as the representative of the TDOA-based algorithm not considering the NLOS propagation's effect to compare with the TOA-based algorithm given in this paper.

The errors in the TOA measurement induced due to the motion of mobile and the delay between the receiving and transmitting signal in moving are assumed as the Gaussian random noise with zero average. We compare the simulation results in case 1. In the simulation of the TDOA-based algorithm [11], because the initial value of \mathbf{B} in (6) is difficult to obtain, we assume that the source's position is fixed at $x_0 = 800 \text{ m}$, $y_0 = 220 \text{ m}$ when the position of the source is not far away from the receivers. When the TOA-based algorithm is used, the source's

position still moves randomly in the square space: $-500 \text{ m} \leq x_0, y_0 \leq 500 \text{ m}$. This is even though, as shown in Fig. 8, if the NLOS interference is not very small, when the noise variance in the TOA-based algorithm is 100 or 1000 times greater than that in the TDOA-based algorithm, the position estimates of the TOA-based algorithm are still much better than that of the TDOA-based algorithm. Here, ten receivers are used, and the ALEs are also obtained from the average of 10 000 independent runs. In the case where the position is estimated to become a complex number, the real part of the estimate is taken to be the result in the TDOA-based algorithm.

IV. CONCLUSION AND DISCUSSION

In the presence of NLOS propagation, a location algorithm is proposed to reduce the induced errors. In the proposed algorithm, mathematical programming is used to find the ML estimate of the source position in the restricted domain defined by the inequalities induced due to NLOS propagation. The proposed algorithm has higher accuracy than the LOS algorithm because more restricted conditions are applied based on the results of the LOS algorithm. Because the ML estimator used in the proposed NLOS algorithm is evolved from the ML estimator with LOS propagation, some deviations might be induced in the very heavy NLOS propagation environment. Hence, if possible, another more suitable cost function should be set up in the proposed algorithm; then more accurate results are expected. This is one of the goals in our future work.

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