The Estimation of the Directions of Arrival of the Spread-Spectrum Signals With Three Orthogonal Sensors

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***Abstract*—An effective technique in estimating the directions of arrival (DOAs) of incoming signals using three orthogonal sensors is proposed. The model of channel with multipath transmissions for code-division multiple access (CDMA) users whose signals are modulated with binary phase-shift keying (BPSK) is used. Utilizing the maximum-likelihood (ML) method, the channel impulse response vectors of three sensors can be estimated, then all of the corresponding propagation delays and amplitude weights of three sensors' channels can be obtained respectively. By comparing the propagation delays of three sensors' channels, we can identify the same signal replica, then the DOI of it can be estimated by the relation of the three corresponding attenuation weights. Calculating results show this method can reach fairly high accuracy, and it needs only three sensors while in other techniques the required number of sensors is greater than the number of estimated signals.**

***Index Terms*—Direction of arrival (DOA), maximum-likelihood (ML) estimation, PN code, propagation delay and amplitude weight.**

I. Introduction

WITH the development of smart antenna and land cellular position location in recent years, lots of methods concerning the estimation of direction of arrival (DOA), which is one of the key components in those two techniques, were proposed. Some of the authors of [1]–[4] use the sensors array to receive the signals coming from various directions, then exploit the eigenstructure of the output covariance matrix to estimate their DOA. When incident signals are uncorrelated or partly correlated, these methods provide high resolution. But when estimating DOA of K signals, they require at least $K+1$ elements in sensors array. And if the signals are coherent, e.g., they are the multipath replicas of the same signal within the resolvable chip or symbol's duration, their performances will degrade severely. Many modifications to those algorithms were proposed in [5]–[10]. Most of them have overcome the above difficulty by modifying the covariance matrix through a preprocessing spatial smoothing scheme. But compared with the algorithms in [1]–[4], they require extra sensors, e.g., the forward/backward spatial smoothing technique proposed in [10] requires at least $3/2K$ sensors when estimating the DOA of K coherent signals. The authors of [11], [12] proposed an integrated approach that combines the ILSP-CMA algorithm with subspace-based algorithm to estimate the DOA of multipath components of a signal. It only requires $3/2\sqrt{K}$ sensors to estimate K signals in the presence of coherent signals.

Although the above algorithms have high resolution in high signal-to-noise ratio (SNR) conditions, the number of needed sensors will increase as the number of estimated signals increases. When the number of signals is fairly large, the number of sensors will become intolerant. Here we propose a new algorithm which can estimate the DOA of spread-spectrum signals with only three orthogonal sensors regardless of the number of signals. It exploits the underlying signal structure of direct-sequence CDMA (DS-CDMA) signals, and, therefore, transfers the estimation of DOA to the estimation of channel parameters.

First, we briefly present the signal and channel model proposed in [13], and, subsequently, the channel parameters estimation algorithm based on the model proposed in [14]. Then the DOA estimation algorithm that we developed and the simulation results are also presented.

II. Channel Parameters Estimation

A. Signal and Channel Model

In DS-CDMA system, users' signals are transmitted on the same frequency at the same time, they are distinguished from each other by unique spreading code. Due to their good self-correlation, cross correlation and part correlation [15], PN sequences are assigned to different users as their spreading codes. Assume that user's data is modulated with BPSK, then the baseband complex envelope of the transmitted signal of one user is given by

$$s(t)=\sqrt{2P}e^{j\phi}\sum_{i}e^{j\pi b_i}a(t-iT) \qquad (1)$$

where P is transmitted power, ϕ is the carrier phase relative to the local oscillator, $b_i \in \{0, 1\}$ is the transmitted symbol, $a(t)$ is the spreading waveform, namely, the PN code, and T is the symbol duration, the cycle duration of the PN code here.

$$a(t)=\sum_{n=0}^{N-1}\Pi_{T_c}(t-nT_c)a^{(n)}$$

where $\Pi_{T_c}(t)$ is a rectangular pulse, T_c is the chip duration $(T_c=T/N)$, and $\{a^{(n)}\}$ is a signature sequence.

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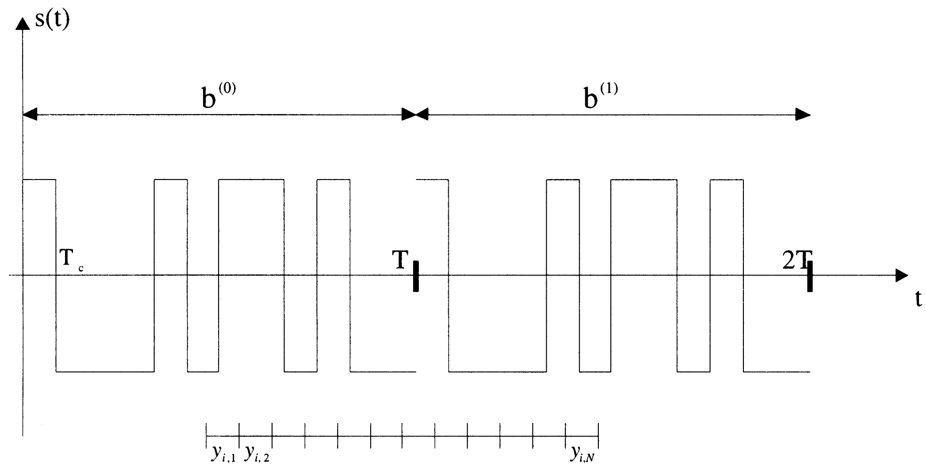

Fig. 1. Time sequence of observation vector and transmitted signal.

Assume that the channel impulse response $h(t)$ is time invariant, then the received signal is

$$r(t) = h(t)^* s(t) = \int_{-\infty}^{\infty} h(t-\tau)s(\tau)\,d\tau.$$

The discrete sample of the above received signal is the output of a matched filter. Because the chip waveform of the BPSK signal is a rectangular pulse, the matched filter can be implemented as an integrate-and-dump circuit. Then the discrete-time received signal is given by

$$r[n] = \frac{1}{T_c}\int_{nT_c}^{(n+1)T_c} r(t)\,dt. \tag{2}$$

The received signal $r[n]$ given by (2) is not wide-sense stationary (WSS), but by loading $r[n]$ into some buffers with length N, the received signal can be converted into a sequence of WSS random vectors [13]

$$\boldsymbol{y}_i = [r[iN] \;\; r[iN+1] \;\; \cdots \;\; r[iN+N-1]]^{\mathrm{T}} \in \mathbb{C}^N. \tag{3}$$

Though each observation vector $\boldsymbol{y}_i$ corresponds to one symbol duration, it will probably contain the end of the previous symbol and the beginning of the current symbol (see Fig. 1) because of the asynchronism between the transmitter and the receiver. So the model of received signal can be written as

$$\boldsymbol{y}_i = \sum_{k=1}^{K}\left(c_k^{(i-1)}\boldsymbol{u}_k^r + c_k^i\boldsymbol{u}_k^l\right) + \boldsymbol{\eta}_i.$$

where

$$\boldsymbol{\eta}_i = [\eta_{i,0}, \ldots, \eta_{i,N-1}]^T \in \mathbb{C}^N$$

is a Gaussian random vector whose elements are zero mean with variance $\sigma^2 = N_0/2T_c$ and are mutually independent. $c_k^{(i)} = e^{j\pi b_i}$ is the kth user's symbol at ith observation time, $\boldsymbol{u}_k^r$ and $\boldsymbol{u}_k^l$ are signal vectors, which depend on the user's PN code and associated channel impulse response, $\boldsymbol{u}_k^r, \boldsymbol{u}_k^l \in \mathbb{C}^N$.

Assume the channel parameters change slowly with time, then the channel can be seen as linear time invariant in sufficiently short intervals. Thus, the baseband channel impulse response can be expressed with a series of delta functions

$$h_k(t,\tau) = h_k(t) = \sum_{p=1}^{L}\alpha_{k,p}\delta(t-\tau_{k,p}).$$

where $\alpha_{k,p}$ is the amplitude weight associated with the propagation path of the kth user signal p; for simplification, the transmitted amplitude $\sqrt{2P_k}$ and random carrier phase ϕ_k are contained in this amplitude weight, $\tau_{k,p}$ is the corresponding propagation delay. Because we only need to know the propagation delay relative to the reference timing of local PN code, we can define

$$(\tau_{k,p}/T_c) \bmod N = v_{k,p} + \gamma_{k,p},$$
$$v_{k,p} \in \{0, \cdots, N-1\}, \;\; \gamma_{k,p} \in [0, 1).$$

When the baud rate of the transmitted signal is large enough to guarantee that the multipath spread is less than $0.5T$, the signal vectors can be written as [13]

$$\boldsymbol{u}_k^r = \sum_{p=1}^{L_k}\alpha_{k,p}\left[(1-\gamma_{k,p})\boldsymbol{a}_k^r(v_{k,p}) + \gamma_{k,p}\boldsymbol{a}_k^r(v_{k,p}+1)\right]$$

$$\boldsymbol{u}_k^l = \sum_{p=1}^{L_k}\alpha_{k,p}\left[(1-\gamma_{k,p})\boldsymbol{a}_k^l(v_{k,p}) + \gamma_{k,p}\boldsymbol{a}_k^l(v_{k,p}+1)\right]$$

where L_k is the number of multipath, and

$$\boldsymbol{a}_k^r(v) \equiv \left[a_k^{(N-v)} \;\; \cdots \;\; a_k^{(N-1)} \;\; 0 \;\; \cdots \;\; 0\right]^{\mathrm{T}}$$
$$\boldsymbol{a}_k^l(v) \equiv \left[0 \;\; \cdots \;\; 0 \;\; a_k^{(0)} \;\; \cdots \;\; a_k^{(N-v-1)}\right]^{\mathrm{T}}.$$

Then the observation vector $\boldsymbol{y}_i$ can be given by

$$\boldsymbol{y}_i = \sum_{k=1}^{K}\left(c_k^{(i-1)}\boldsymbol{U}_k^r + c_k^{(i)}\boldsymbol{U}_k^l\right)\boldsymbol{h}_k + \boldsymbol{\eta}_i \tag{4}$$

where $\boldsymbol{h}_k \in \mathbb{C}^N$ is the composite impulse response vector of the kth user channel

$$\boldsymbol{U}^r = [\boldsymbol{a}^r(0) \;\; \cdots \;\; \boldsymbol{a}^r(N-1)] \in \mathbb{C}^{N\times N}$$
$$\boldsymbol{U}^l = [\boldsymbol{a}^l(0) \;\; \cdots \;\; \boldsymbol{a}^l(N-1)] \in \mathbb{C}^{N\times N}.$$

If we can determine the range of the multipath delays before estimation (i.e., in IS-95, if we know the signal is transmitted by which base station, then we can know the range of the delays of this base station's signal), then through selecting suitable m columns, h_k becomes an m-dimension vector, and $\boldsymbol{U}^r$ and $\boldsymbol{U}^l$ become

$$\boldsymbol{U}_k^r = [\, \boldsymbol{a}_k^r(l) \quad \cdots \quad \boldsymbol{a}_k^r(l+m) \,] \in \mathbb{C}^{N \times m}$$
$$\boldsymbol{U}_k^l = [\, \boldsymbol{a}_k^l(l) \quad \cdots \quad \boldsymbol{a}_k^l(l+m) \,] \in \mathbb{C}^{N \times m}$$

where l is the beginning of the estimated or known range of multipath delays, $l+m$ is the end of the range.

In [13], the column number of the matrices $\boldsymbol{U}_k^r$, $\boldsymbol{U}_k^l$ is N. Considering the distributions of delay of multipath [14], here the column number of those matrices will be reduced to m. In practice, usually $m \ll N$, so the computation will also be greatly reduced.

B. Channel Parameters Estimation

Let the error vector

$$\boldsymbol{\psi} = \boldsymbol{y}_i - \boldsymbol{G}\boldsymbol{h}$$

where

$$\begin{aligned} \boldsymbol{G} &= \left[\, c_1^{(i-1)}\boldsymbol{U}_1^r + c_1^{(i)}\boldsymbol{U}_1^l \quad \cdots \quad c_K^{(i-1)}\boldsymbol{U}_K^r + c_K^{(i)}\boldsymbol{U}_K^l \,\right] \\ \boldsymbol{h} &= [\, \boldsymbol{h}_1 \quad \cdots \quad \boldsymbol{h}_K \,]^{\mathrm{T}}. \end{aligned} \tag{5}$$

Because m_k, the column number of $\boldsymbol{U}_k^r$, $\boldsymbol{U}_k^l$, can be different mutually, the length of $\boldsymbol{h}_i (i = 1, \ldots, K)$ in (5) can also be different. From (4)

$$\begin{aligned} &\boldsymbol{\psi} = \boldsymbol{\eta}_i \\ &\boldsymbol{\Psi} = \mathrm{E}\left[\boldsymbol{\psi}\boldsymbol{\psi}^{\mathrm{T}}\right] = \boldsymbol{Q} \\ &\boldsymbol{Q} = diag\{\sigma^2, \ldots, \sigma^2\} \in \mathbb{C}^{N \times N} \\ &\sigma^2 = N_0/2T_c. \end{aligned} \tag{6}$$

Then the maximum-likelihood (ML) estimate of $\boldsymbol{h}$ is given by [16]

$$\begin{aligned} \hat{\boldsymbol{h}} &= \arg\min \left\{ (\boldsymbol{y}_i - \boldsymbol{G}\boldsymbol{h})^{\mathrm{T}} \boldsymbol{\Psi}^{-1} (\boldsymbol{y}_i - \boldsymbol{G}\boldsymbol{h}) \right\} \\ &= \left(\boldsymbol{G}^{\mathrm{T}} \boldsymbol{Q}^{-1} \boldsymbol{G} \right)^{-1} \boldsymbol{G}^{\mathrm{T}} \boldsymbol{Q}^{-1} \boldsymbol{y}_i. \end{aligned} \tag{7}$$

Although $\boldsymbol{G}$ contains the transmitted symbol $c_k^{(i)}$ defined by (5), $c_k^{(i)}$ can be known beforehand in the estimation process if we transmit a series of known symbols, e.g., in the IS-95 forward link, the symbols of pilot channel are kept as 1, or $c_k^{(i)}$ can be estimated as done in [14]. So (7) can be used to estimate $\boldsymbol{h}$. After obtaining $\hat{\boldsymbol{h}}$, according to the definition of $\boldsymbol{h}$ in (5), we can calculate the corresponding amplitude weight $\hat{\alpha}_{k,p}$ and propagation delay $\hat{\tau}_{k,p}$

$$\begin{aligned} &[\alpha_{k,v}, \gamma_{k,v}] \\ &\quad = \underset{\alpha \in \mathbb{C},\, \gamma \in [0,1)}{\arg\min} \left(\left| \hat{h}_{k,v} - (1-\gamma)\alpha \right|^2 + \left| \hat{h}_{k,v+1} - \gamma\alpha \right|^2 \right) \end{aligned} \tag{8}$$

where $[\alpha_{k,v}, \gamma_{k,v}]$ is the channel parameters of the vth possible path of the kth user's signal, $\hat{h}_{k,v}$ is the vth element of $\hat{\boldsymbol{h}}_k$, and $\hat{h}_{k,v+1}$ is the $(v+1)$th element of $\hat{\boldsymbol{h}}_k$. The solution is given by

$$\begin{aligned} \gamma_{k,v} &= \begin{cases} 1/2 + \beta + \sqrt{\beta^2 + 1/4}, & \beta \geq 0 \\ 1/2 + \beta - \sqrt{\beta^2 + 1/4}, & \beta < 0 \end{cases} \\ \alpha_{k,v} &= \frac{(1-\gamma_{k,v})\hat{h}_{k,v} + \gamma_{k,v}\hat{h}_{k,v+1}}{(1-\gamma_{k,v})^2 + \gamma_{k,v}^2} \end{aligned} \tag{9}$$

where

$$\beta = \frac{\mathrm{Re}\left\{ \hat{h}_{k,v} \cdot \hat{h}_{k,v+1} \right\}}{\left| \hat{h}_{k,v} \right|^2 + \left| \hat{h}_{k,v+1} \right|^2}. \tag{10}$$

When all possible $[\alpha_{k,v}, \gamma_{k,v}]$ are calculated, we search for the strongest path

$$\hat{v} = \underset{v \in \{0, 1, \ldots, N-1\}}{\arg\max} |\alpha_{k,v}|.$$

Then we compute the desired channel parameters

$$\hat{\tau}_{k,p} = (\hat{v} + \gamma_{\hat{v}})T_c, \qquad \hat{\alpha}_{k,p} = \alpha_{\hat{v}}. \tag{11}$$

After this strongest path is extracted from $\hat{h}_k$, the above process is repeated to search for the next strongest path until $|\hat{\alpha}_{k,p}|$ falls below a certain significance level, i.e., the square root of the power of the additive Gaussian noise.

Here we use ML algorithm to estimate the channel impulse response vector $\boldsymbol{h}$ for it provides an explicit solution form. We can also use the subspace-based channel estimation algorithm proposed in [13]. It is also an efficient approach with high accuracy.

It is obvious that in the above process, we can only separate the resolvable paths, the differences of whose propagation delays are greater than one chip duration, if the chip duration T_c is used as the duration of the integrator in (2). But we can adjust the duration of the integrator to be small enough so that the coherent paths can be separated, e.g., if we adopt $1/8T_c$ as the duration of the integrator, the coherent paths whose propagation delays differences are greater than $1/8T_c$ can be separated. To avoid the computation volume of the ML estimator, we can also use extended Kalman filter (EKF) to estimate the channel parameters [14]. For this method, we need to know the approximate values of the propagation delays, namely, the value of v in (11). We can use the RAKE receiver or other techniques to obtain these approximate propagation delay values of all of the signal replicas [17].

III. DOA Estimation

Assume three orthogonal sensors x, y, z lie in the three coordinate axes as depicted in Fig. 2. The lengths of these three sensors are the same. Their intersection is the origin point of the coordinate as well as the middle point of each sensor. Because each sensor is a dipolar antenna, the fields produced by this antenna are similar to that by a static dipolar. That means, as a receiver antenna, the effective beginning time of this sensor should be the time of producing equal value but opposite induced potential at the two ends of the sensor. Namely, the current is induced in the sensor due to the difference of induced potential between two ends of the sensor when the electromagnetic wave arrives at the middle point of the sensor. Since the middle points of three sensors intersect at one point, so the arrival time of the signal will be the same at these three sensors from the point of view of production of induced current in the sensors.

Assume that the amplitudes of the received signal in these three sensors are $x(t)$, $y(t)$, and $z(t)$, respectively. Here $x(t)$,

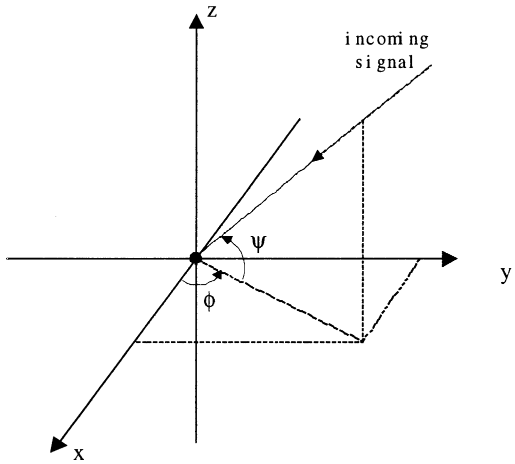

Fig. 2. DOA of an incoming signal.

$y(t)$, and $z(t)$ can be negative (negative means the direction of current induced by the signal in the sensor is opposite to the assumed positive direction). If the linear time-invariant (LTI) approximation is still retained, then

$$x(t)=\sum_{k=1}^{K}(h_k^x(t)*s_k(t))=\sum_{k=1}^{K}\int_{-\infty}^{\infty}h_k^x(t-\tau)s_k(\tau)\,d\tau$$
$$y(t)=\sum_{k=1}^{K}(h_k^y(t)*s_k(t))=\sum_{k=1}^{K}\int_{-\infty}^{\infty}h_k^y(t-\tau)s_k(\tau)\,d\tau$$
$$z(t)=\sum_{k=1}^{K}(h_k^z(t)*s_k(t))=\sum_{k=1}^{K}\int_{-\infty}^{\infty}h_k^z(t-\tau)s_k(\tau)\,d\tau.$$

If the channel impulse response corresponding to the kth user is given by

$$h_k(t)=\sum_{p=1}^{L_k}\alpha_{k,p}\delta(t-\tau_{k,p})$$

and assuming the DOAs corresponding to L_k paths are $(\phi_{k,1}, \psi_{k,1}), \ldots, (\phi_{k,L_k}, \psi_{k,L_k})$, where $\phi_{k,p}$, $\psi_{k,p}$ are, respectively, the azimuth and elevation of the pth path of the kth user's signal (see Fig. 2), then

$$h_k^x(t)=\sum_{p=1}^{L_1}\alpha_{k,p}\cos\phi_{k,p}\cos\psi_{k,p}\delta(t-\tau_{k,p})$$
$$h_k^y(t)=\sum_{p=1}^{L_1}\alpha_{k,p}\sin\phi_{k,p}\cos\psi_{k,p}\delta(t-\tau_{k,p})$$
$$h_k^z(t)=\sum_{p=1}^{L_1}\alpha_{k,p}\sin\psi_{k,p}\delta(t-\tau_{k,p}) \qquad (12)$$

where $\phi_{k,p}\in(-\pi,\pi]$, $\psi_{k,p}\in[-\pi/2,\pi/2]$.

So we can first estimate the channel parameters corresponding to three sensors, then search the same signal replica through comparing the propagation delays, and, finally, estimate the DOA according to the associated amplitude weights.

Discrete-time signals $x[n]$, $y[n]$, and $z[n]$ are obtained by using the matched filter as (2), and observation vectors are given as (3)

$$\boldsymbol{y}_i^x=[x[iN]\quad x[iN+1]\quad\cdots\quad x[iN+N-1]]^{\mathrm{T}}\in\mathbb{C}^N$$
$$\boldsymbol{y}_i^y=[y[iN]\quad y[iN+1]\quad\cdots\quad y[iN+N-1]]^{\mathrm{T}}\in\mathbb{C}^N$$
$$\boldsymbol{y}_i^z=[z[iN]\quad z[iN+1]\quad\cdots\quad z[iN+N-1]]^{\mathrm{T}}\in\mathbb{C}^N.$$

By applying (5)–(11) to observation vectors $\boldsymbol{y}_i^x$, $\boldsymbol{y}_i^y$, and $\boldsymbol{y}_i^z$, we can obtain channel parameters corresponding to them respectively

$$\boldsymbol{A}^x=[\boldsymbol{A}_1^x,\boldsymbol{A}_2^x,\ldots,\boldsymbol{A}_K^x]$$
$$\boldsymbol{A}^y=[\boldsymbol{A}_1^y,\boldsymbol{A}_2^y,\ldots,\boldsymbol{A}_K^y]$$
$$\boldsymbol{A}^z=[\boldsymbol{A}_1^z,\boldsymbol{A}_2^z,\ldots,\boldsymbol{A}_K^z]$$
$$\boldsymbol{T}^x=[\boldsymbol{T}_1^x,\boldsymbol{T}_2^x,\ldots,\boldsymbol{T}_K^x]$$
$$\boldsymbol{T}^y=[\boldsymbol{T}_1^y,\boldsymbol{T}_2^y,\ldots,\boldsymbol{T}_K^y]$$
$$\boldsymbol{T}^z=[\boldsymbol{T}_1^z,\boldsymbol{T}_2^z,\ldots,\boldsymbol{T}_K^z]$$

where

$$\boldsymbol{A}_k^x=[\alpha_{k,1}^x,\alpha_{k,2}^x,\ldots,\alpha_{k,L_k}^x]$$
$$\boldsymbol{A}_k^y=[\alpha_{k,1}^y,\alpha_{k,2}^y,\ldots,\alpha_{k,L_k}^y]$$
$$\boldsymbol{A}_k^z=[\alpha_{k,1}^z,\alpha_{k,2}^z,\ldots,\alpha_{k,L_k}^z]$$
$$\boldsymbol{T}_k^x=[\tau_{k,1}^x,\tau_{k,2}^x,\ldots,\tau_{k,L_k}^x]$$
$$\boldsymbol{T}_k^y=[\tau_{k,1}^y,\tau_{k,2}^y,\ldots,\tau_{k,L_k}^y]$$
$$\boldsymbol{T}_k^z=[\tau_{k,1}^z,\tau_{k,2}^z,\ldots,\tau_{k,L_k}^z].$$

Comparing the elements in $\boldsymbol{T}_k^x$, $\boldsymbol{T}_k^y$, and $\boldsymbol{T}_k^z$, we can find the same signal replica by searching the equal or approximately equal delay values in three matrices. If $(\alpha_{k,p}^x, \tau_{k,p}^x)$, $(\alpha_{k,p}^y, \tau_{k,p}^y)$, and $(\alpha_{k,p}^z, \tau_{k,p}^z)$ is correctly judged as the parameters of the same signal replica, the DOA of it is given by

$$\phi_{k,p}=\begin{cases}\arccos\left(\dfrac{\alpha_{k,p}^x}{\alpha_{k,p}\cos\psi_{k,p}}\right), & \text{if } \alpha_{k,p}^y\ge 0\\ -\arccos\left(\dfrac{\alpha_{k,p}^x}{\alpha_{k,p}\cos\psi_{k,p}}\right), & \text{if } \alpha_{k,p}^y< 0\end{cases} \qquad (13)$$
$$\psi_{k,p}=\arcsin\left(\frac{\alpha_{k,p}^z}{\alpha_{k,p}}\right)$$

where

$$\alpha_{k,p}=\sqrt{(\alpha_{k,p}^x)^2+(\alpha_{k,p}^y)^2+(\alpha_{k,p}^z)^2}.$$

Shown as (12), when $\psi_{k,p}=0,\pm\pi/2$ or $\phi_{k,p}=0,\pi,\pm\pi/2$, this signal replica will only appear in one or two of the three sensors, namely, a certain delay value is only in the elements of one or two vectors among $\boldsymbol{T}_k^x$, $\boldsymbol{T}_k^y$, and $\boldsymbol{T}_k^z$. For example, assume there are two approximately equal delay values $\tau_{k,p}^x$, $\tau_{k,p}^y$ in, respectively, $\boldsymbol{T}_k^x$ and $\boldsymbol{T}_k^y$, but another approximately equal delay does not exist in $\boldsymbol{T}_k^z$, then we can conclude that $\alpha_{k,p}^z=0$, and substitute $\alpha_{k,p}^x$, $\alpha_{k,p}^y$ associated with $\tau_{k,p}^x$, $\tau_{k,p}^y$ and $\alpha_{k,p}^z=0$ into (12) to calculate the corresponding DOA.

IV. Simulation Results

For simplicity, here we just consider a two-user situation. Assume the PN codes of the users are m sequences, their cycle du-

TABLE I
THE PROPAGATION DELAYS AND AMPLITUDE WEIGHTS

User1	Mt1	Mt2	Mt3	Mt4	Mt5
τ (chips)	26.7621	37.4565	48.0185	61.8214	76.4447
α	4.8504	2.6935	3.8205	3.4579	4.6725
User2	Mt1	Mt2	Mt3	Mt4	Mt5
τ (chips)	33.4057	42.9355	53.9169	68.4103	83.8936
α	3.8463	4.3731	4.7655	4.2147	2.5288

TABLE II
TRUE AND ESTIMATED CHANNEL PARAMETERS OF SENSOR X

User1	Mt1	Mt2	Mt3	Mt4	Mt5
τ_{Real} (chips)	26.7621	37.4565	48.0185	61.8214	76.4447
τ_{Estim} (chips)	26.7483	37.4574	48.0216	61.8155	76.4446
α_{Real}	-0.2174	1.1586	-1.4549	-0.7337	-2.9850
α_{Estim}	-0.2242	1.1569	-1.4578	-0.7470	-2.9850
User2	Mt1	Mt2	Mt3	Mt4	Mt5
τ_{Real} (chips)	33.4057	42.9355	53.9169	68.4103	83.8936
τ_{Estim} (chips)	33.4040	42.9380	53.9178	68.4120	83.8837
α_{Real}	-1.0883	-0.6437	3.7757	0.3482	-0.6853
α_{Estim}	-1.0819	-0.6403	3.7739	0.3475	-0.6957

ration is $2^8 - 1 = 255\,T_c$, and their feedback coefficients are 453 and 435, respectively.

BPSK is used to modulate the transmitted data. Assume that each user's signal has five multipaths. And for simplicity, assume that the random carrier phase ϕ in (1) of each multipath equals zero. This means that the amplitude weights become real other than complex. For convenience, we assume that both user signal have five paths each. Let the amplitude weights and propagation delays of each multipath component of each user's signal be the values shown in Table I, where Mti stands for the ith multipath component. Also assume the DOAs of these 10 paths are

$$\phi = 2\pi * \mathrm{rand}\,(10) - \pi, \qquad \psi = \pi * \mathrm{rand}\,(10) - \pi/2$$

where $\mathrm{rand}\,(10)$ stands for 10 random values uniformly distributed in 0–1.

Assume that the values of the Gaussian white noises added to the three sensors are zero mean with variance $\sigma^2 = 0.01$. Then the results of channel parameters estimations corresponding to the three sensors are shown in Tables II–IV, where τ_{Real}, τ_{Estim} are, respectively, the true value and the estimated value of the propagation delay, and α_{Real}, α_{Estim} are, respectively the true value and the estimated value of the amplitude weight. As shown in Tables II–IV, the differences between the estimated values and the true values are very small. Substituting the estimated channel parameters into (13), we can obtain the DOA of each path. The results are shown in Table V, where Φ_{Real}, Φ_{Estim} are, respectively the true value and the estimated value of azimuths of incoming signals, and Ψ_{Real}, Ψ_{Estim} are, respectively, the true value and the estimated value of elevations of incoming signals. As shown in Table V, they are approximately equal.

V. COMPARISON WITH CRAMER–RAO LOWER BOUND (CRLB)

Cramer–Rao inequality [18] set a lower bound for the variance of any unbiased estimator. So we can evaluate the performance of the estimator through comparing with it.

TABLE III
TRUE AND ESTIMATED CHANNEL PARAMETERS OF SENSOR Y

User1	Mt1	Mt2	Mt3	Mt4	Mt5
τ_{Real} (chips)	26.7621	37.4565	48.0185	61.8214	76.4447
τ_{Estim} (chips)	26.8171	37.4595	48.0166	61.9196	76,4447
α_{Real}	-0.0828	-1.5357	3.4713	-0.0455	-3.5600
α_{Estim}	-0.0835	-1.5376	3.4685	-0.0437	-3.5608
User2	Mt1	Mt2	Mt3	Mt4	Mt5
τ_{Real} (chips)	33.4057	42.9355	53.9169	68.4103	83.8936
τ_{Estim} (chips)	33.4039	42.9365	53.9139	68.4070	83.8899
α_{Real}	-3.5587	-1.9219	2.8831	-2.4813	-2.0568
α_{Estim}	-3.5457	-1.9219	2.8777	-2.4631	-2.0566

TABLE IV
TRUE AND ESTIMATED CHANNEL PARAMETERS OF SENSOR Z

User1	Mt1	Mt2	Mt3	Mt4	Mt5
τ_{Real} (chips)	26.7621	37.4565	48.0185	61.8214	76.4447
τ_{Estim} (chips)	26.7610	37.4556	48.0231	61.8211	76.4441
α_{Real}	-4.8448	1.8852	-0.6557	3.3789	-0.4984
α_{Estim}	-4.9366	1.8881	-0.6637	3.3808	-0.4953
User2	Mt1	Mt2	Mt3	Mt4	Mt5
τ_{Real} (chips)	33.4057	42.9355	53.9169	68.4103	83.8936
τ_{Estim} (chips)	33.4057	42.9350	53.9185	68.4108	83.8899
α_{Real}	-0.9723	3.8750	0.3762	-3.3890	1.3019
α_{Estim}	-0.9702	3.8812	0.3776	-3.3947	1.3078

TABLE V
COMPARISON OF TRUE AND ESTIMATED DOA

User1	Mt1	Mt2	Mt3	Mt4	Mt5
ϕ_{Real} (rad)	-2.7779	-0.9245	1.9677	-3.0796	-2.2689
ϕ_{Estim} (rad)	-2.7850	-0.9258	1.9687	-3.0831	-2.2685
ψ_{Real} (rad)	-1.5228	0.7753	-0.1725	1.3566	-0.1068
ψ_{Estim} (rad)	-1.5214	0.7759	-0.1746	1.3530	-0.1062
User2	Mt1	Mt2	Mt3	Mt4	Mt5
ϕ_{Real} (rad)	-1.8676	-1.8930	0.6521	-1.4314	-1.8924
ϕ_{Estim} (rad)	-1.8669	-1.8924	0.6515	-1.4316	-1.8970
ψ_{Real} (rad)	-0.2556	1.0877	0.0790	-0.9342	0.5408
ψ_{Estim} (rad)	-0.2560	1.0898	0.0794	-0.9384	0.5422

Let

$$\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{y}_i^x \\ \boldsymbol{y}_i^y \\ \boldsymbol{y}_i^z \end{bmatrix}, \qquad \boldsymbol{\theta} = [\varphi \;\; \psi]^{\mathrm{T}}.$$

Then the conditional probability density function of Y is

$$p(\boldsymbol{Y}|\boldsymbol{\theta}) = \frac{1}{(2\pi)^{2K/2}|\boldsymbol{Q}|^{1/2}} \exp\left\{-\frac{1}{2}(\boldsymbol{Y}-\boldsymbol{Gh})^{\mathrm{T}}\boldsymbol{Q}^{-1}(\boldsymbol{Y}-\boldsymbol{Gh})\right\}$$

where K is the number of estimated signals. So the CRLB with respect to $\boldsymbol{\theta}$ is given by

$$\boldsymbol{\Phi}^0 = \left\{ \mathrm{E}\left[\left(\frac{\partial}{\partial\boldsymbol{\theta}}\ln p(\boldsymbol{Y}|\boldsymbol{\theta})\right)\left(\frac{\partial}{\partial\boldsymbol{\theta}}\ln p(\boldsymbol{Y}|\boldsymbol{\theta})\right)^{\mathrm{T}}\right]\Bigg|_{\boldsymbol{\theta}=\boldsymbol{\theta}^0}\right\}^{-1}$$

where $\boldsymbol{\theta}^0$ is the true vale of $\boldsymbol{\theta}$. Then the partial derivative of $\ln p(\boldsymbol{Y}|\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$ is

$$\frac{\partial}{\partial\boldsymbol{\theta}}\ln p(\boldsymbol{Y}|\boldsymbol{\theta}) = \frac{\partial\boldsymbol{h}^{\mathrm{T}}}{\partial\boldsymbol{\theta}}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{Q}^{-1}(\boldsymbol{Y}-\boldsymbol{Gh}).$$

Hence

$$\mathbf{\Phi}^0 = \left(\frac{\partial \boldsymbol{h}^{\mathrm{T}}}{\partial \boldsymbol{\theta}} \boldsymbol{G}^{\mathrm{T}} \boldsymbol{Q}^{-1} \boldsymbol{G} \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{\theta}} \right)^{-1} \Bigg|_{\boldsymbol{\theta}=\boldsymbol{\theta}^0} = \left(\frac{\partial \boldsymbol{\theta}^{\mathrm{T}}}{\partial \boldsymbol{h}} (\boldsymbol{G}^{\mathrm{T}} \boldsymbol{Q}^{-1} \boldsymbol{G})^{-1} \frac{\partial \boldsymbol{\theta}}{\partial \boldsymbol{h}} \right) \Bigg|_{\boldsymbol{h}=\boldsymbol{h}^0} \tag{14}$$

and the covariance matrix of the ML estimate of $\boldsymbol{h}$ is given by [16]

$$\mathrm{cov}(\boldsymbol{h}) = \mathrm{E}\left[\Delta \boldsymbol{h} \Delta \boldsymbol{h}^{\mathrm{T}}\right] = \left(\boldsymbol{G}^{\mathrm{T}} \boldsymbol{Q}^{-1} \boldsymbol{G}\right)^{-1}.$$

So, the covariance matrix of $\boldsymbol{\theta}$ is

$$\mathrm{cov}(\boldsymbol{\theta}) = \mathrm{E}\left[\Delta \boldsymbol{\theta} \Delta \boldsymbol{\theta}^{\mathrm{T}}\right] = \mathrm{E}\left[\frac{\partial \boldsymbol{\theta}^{\mathrm{T}}}{\partial \boldsymbol{h}} \Delta \boldsymbol{h} \Delta \boldsymbol{h}^{\mathrm{T}} \frac{\partial \boldsymbol{\theta}}{\partial \boldsymbol{h}}\right] = \left(\frac{\partial \boldsymbol{\theta}^{\mathrm{T}}}{\partial \boldsymbol{h}} \left(\boldsymbol{G}^{\mathrm{T}} \boldsymbol{Q}^{-1} \boldsymbol{G}\right)^{-1} \frac{\partial \boldsymbol{\theta}}{\partial \boldsymbol{h}} \right) \Bigg|_{\boldsymbol{h}=\boldsymbol{h}^0}. \tag{15}$$

Comparing (14) with (15), we can say that the DOA estimator proposed here is nearly an optimal estimator.

Assuming that the same signal replicas can be correctly judged by finding the equal or approximately equal delay values as described in Section III, the $\partial \boldsymbol{\theta}^{\mathrm{T}}/\partial \boldsymbol{h}$ in (15) is given by

$$\frac{\partial \boldsymbol{\theta}^{\mathrm{T}}}{\partial \boldsymbol{h}} = \frac{\partial \boldsymbol{\theta}^{\mathrm{T}}}{\partial \boldsymbol{A}} \cdot \frac{\partial \boldsymbol{A}^{\mathrm{T}}}{\partial \boldsymbol{h}}$$

where $\boldsymbol{A} = [\alpha_{1,1}, \ldots, \alpha_{1,L_1}, \ldots, \alpha_{K,1}, \ldots, \alpha_{K,L_K}]^T$. So, $\partial \boldsymbol{\theta}^{\mathrm{T}}/\partial \boldsymbol{A}$ can be calculated according to (13), and $\partial \boldsymbol{A}^{\mathrm{T}}/\partial \boldsymbol{h}$ can be calculated according to (9) then $\partial \boldsymbol{\theta}^{\mathrm{T}}/\partial \boldsymbol{h}$ is subsequently obtained.

VI. Conclusion

A new DOA estimation method using three orthogonal sensors is proposed. First, the channel parameters are estimated in three sensors, respectively; then the same signal replica is found through searching the equal or approximately equal delay values in those three groups of channel parameters; finally, the DOA is calculated based on the three associated amplitude weights. As shown in simulation results and analysis, this method has fairly high precision. Compared with other techniques, it only requires three sensors, without the constraint between the number of sensors and the number of signals.

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