Stochastic Resource Allocation Over Fading Multiple Access and Broadcast Channels

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***Abstract*—We consider the optimal rate and power allocation that maximizes a general utility function of average user rates in a fading multiple-access or broadcast channel. By exploiting the greedy structure of the capacity-achieving resource allocation for both multiple-access and broadcast channels, it is established that a utility-maximizing allocation policy can be obtained through dual-based gradient descent iterations with fast convergence and low complexity per iteration. Relying on stochastic averaging tools, we further develop a class of stochastic gradient iterations which are capable of asymptotically converging to the optimal benchmark with guarantees on the minimum average user rates, even when the fading channel distribution is unknown *a priori*.**

***Index Terms*—Fading channels, resource allocation, ergodic capacity, stochastic optimization.**

I. Introduction

In wireless cellular networks, the communications channels between the access point and user terminals are analyzed as fading Gaussian multiple-access (uplink) and broadcast (downlink) channels in information-theoretic research. For these fading channels, the ergodic capacity regions have been studied in [1]–[4]. To determine the boundary points of the capacity regions, these works derived the optimal rate and power allocation schemes that maximize a linear utility function, i.e., weighted sum, of average user rates. It was shown that the optimal resource allocation schemes for both Gaussian multiple access and broadcast channels rely on successive decoding and adopt a similar greedy structure.

Instead of maximization of weighted sum-rate, recently there have also been efforts to consider maximizing a suitable utility function of average user rates as well as introducing minimum rate constraints per user to address the fairness and quality-of-service (QoS) guarantees in resource allocation among the users. To cope with user mobility and network dynamics which induce uncertainty in the wireless channels, a class of "opportunistic" schemes were developed to carry out the network utility maximization through essentially learning the underlying channel distribution on-the-fly [5]–[12]. However, these "opportunistic" schemes are developed for specific multiplexing or multiple-access systems, e.g., TDM/A (time-division multiplexing/multiaccess), rather than the general broadcast or multiple-access channels. Furthermore, development and analysis of these approaches are based on a key assumption that the random channel fading is confined to follow a finite-state Markov chain model [6]–[8], which is simply not the case of wireless channels with continuous (not necessarily Markov) fading distributions, considered by the information theorists in [1]–[3].

To overcome the limitations of the existing approaches, this paper considers optimal rate and power allocation that maximizes a general utility function over the ergodic rate capacity region of the fading multiple-access or broadcast channels. In this direction, a recent work [13] has achieved some success. Specifically, an approximation projection approach was put forth to iteratively obtain the utility-maximizing resource allocation in fading multiple access channels. Relying on this approach, optimal rate and power allocation was developed for the case when channel statistics are available, whereas a suboptimal scheme was proposed for the case when transmission powers are constant (across fading realizations) and channel statistics are unknown. Different from [13], we propose a Lagrange dual-based stochastic approach to obtain the optimal (utility-maximizing) rate and power allocation for both multiple-access and broadcast channels. And different from the fluid limit and Lyapunov drift arguments employed in [6]–[8], the proposed approach draws from stochastic averaging tools pioneered by Solo in development of adaptive signal processing algorithms [14]. This entails a more systematic and powerful framework for design and analysis of the stochastic resource allocation schemes in wireless networks. Compared with the existing alternatives, the proposed schemes entail provably optimal dynamic power control (across fading states) and thus fully exploit ergodic capacity of the fading channels with continuous distributions, even when the fading statistics are unknown *a priori*.

The rest of the paper is organized as follows. Section II describes the channel models. Sections III and IV present the proposed stochastic approaches to utility-maximizing resource allocation in fading multiple-access and broadcast channels, respectively. Section V evaluates the proposed schemes with numerical examples, followed by conclusions.

II. System Models

We consider two different discrete-time systems. The first system is a many-to-one multiple-access channel, where J independent user terminals each transmit a signal X_j to an access point. The received signal Y is the sum of the J transmitted signal and additive Gaussian noise Z, i.e.,

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$Y = \sum_{j=1}^{J} \sqrt{h_j} X_j + Z$ where h_j denotes the channel gain of user j and the noise variance of Z is equal to $\sigma^2 = 1$ without loss of generality (w.l.o.g).

The second system to consider is a one-to-many broadcast channel where the access point sends independent information to each user by broadcasting a signal X to J terminals. The received signal at the jth user is $Y_j = \sqrt{h_j} X + Z_j$, where h_j denotes the channel gain of receiver j and we assume w.l.o.g. that the noise variance of Z_j is equal to $\sigma_j^2 = 1, \forall j$.

Let $\boldsymbol{h} := [h_1, \ldots, h_J]^T$ denote the vector of channel gains. In the fading multiple-access or broadcast channel, the channel gains in $\boldsymbol{h}$ follow a jointly stationary and ergodic random process with a cumulative distribution function (cdf) $F(\boldsymbol{h})$; and we assume the access point and users have full information about $\boldsymbol{h}$.

Notation: Using boldface letters to denote column vectors and with inequalities for vectors defined element-wise, we let $\mathbb{E}_{\boldsymbol{h}}[\cdot]$ denote the expectation operator over fading states $\boldsymbol{h}$, $\boldsymbol{0}$ the all-zero vector, ^{T} the transposition, $\mathbb{R}_+^J$ the J-dimensional nonnegative orthant, $\|\boldsymbol{x}\|$ the vector norm, $\mathbf{int}(S)$ the interior of a set S, and $\nabla f(\boldsymbol{x})$ the gradient of $f(\boldsymbol{x})$.

III. Fading Multiple Access Channel

As a stepping stone, we first review the ergodic capacity region for fading multiple access channel in [1]. The novel utility-maximizing stochastic resource allocation schemes are then developed and analyzed afterwards.

A. *Ergodic Capacity Region*

Given the channel gain vector $\boldsymbol{h}$ and transmit-power vector $\boldsymbol{p} := [p_1, \ldots, p_J]^T$, the Shannon capacity region of the Gaussian multiaccess channel is [15]

$$\mathcal{C}_g(\boldsymbol{h}, \boldsymbol{p}) := \left\{ \boldsymbol{r} : \sum_{j \in S} r_j \le \log\left(1 + \sum_{j \in S} h_j p_j\right) \; \forall S \subset \mathcal{J} \right\}$$

where $\boldsymbol{r} := [r_1, \ldots, r_J]^T \in \mathbb{R}_+^J$ denotes the vector of achievable user rates in nats/s/Hz, and $\mathcal{J} := \{1, \ldots, J\}$.

For a fading multiaccess channel, consider a power control policy $\boldsymbol{p}(\cdot)$ as a mapping from the fading state space to $\mathbb{R}_+^J$, which allocates transmit-power $\boldsymbol{p}(\boldsymbol{h}) := [p_1(\boldsymbol{h}), \ldots, p_J(\boldsymbol{h})]^T$ for the given fading state $\boldsymbol{h}$. Then for a $\boldsymbol{p}(\cdot)$, the achievable average rate vector $\bar{\boldsymbol{r}}$ is contained by the set

$$\mathcal{C}_f(\boldsymbol{p}(\cdot)) := \Bigg\{ \bar{\boldsymbol{r}} : \sum_{j \in S} \bar{r}_j \le \mathbb{E}_{\boldsymbol{h}}\left[\log\left(1 + \sum_{j \in S} h_j p_j(\boldsymbol{h})\right)\right], \forall S \subset \mathcal{J} \Bigg\} \quad (1)$$

Let $\bar{\boldsymbol{P}} := [\bar{P}_1, \ldots, \bar{P}_J]^T$ collect the power budget of the transmitters, and define the set of all feasible power control policies satisfying the average power constraints as: $\mathcal{F} := \{\boldsymbol{p}(\cdot) : \mathbb{E}_{\boldsymbol{h}}[\boldsymbol{p}(\boldsymbol{h})] \le \bar{\boldsymbol{P}}\}$. Assuming that the transmitters have side-information of the current fading state, the ergodic capacity region for multiaccess fading Gaussian channel is then [1]

$$\mathcal{C}(\bar{\boldsymbol{P}}) := \bigcup_{\boldsymbol{p}(\cdot) \in \mathcal{F}} \mathcal{C}_f(\boldsymbol{p}(\cdot)). \quad (2)$$

Since $\mathcal{C}(\bar{\boldsymbol{P}})$ is a convex set of $\bar{\boldsymbol{r}}$, each of its boundary points maximizes a weighted sum of average rates; i.e., it solves

$$\max_{\bar{\boldsymbol{r}}} \boldsymbol{w}^T \bar{\boldsymbol{r}}, \quad \text{subject to (s. to)} \quad \bar{\boldsymbol{r}} \in \mathcal{C}(\bar{\boldsymbol{P}}) \quad (3)$$

where the weight vector $\boldsymbol{w} := [w_1, \ldots, w_J]^T \ge \boldsymbol{0}$. For a given $\boldsymbol{w}$, there exists a (Lagrange multiplier vector) $\boldsymbol{\lambda} > \boldsymbol{0}$ such that the optimal rate and power allocation achieving the corresponding boundary point, i.e., $(\boldsymbol{r}^*(\boldsymbol{h}), \boldsymbol{p}^*(\boldsymbol{h}))$ for every joint fading state $\boldsymbol{h}$, is the solution to the problem

$$\max_{(\boldsymbol{r}(\boldsymbol{h}), \boldsymbol{p}(\boldsymbol{h}))} \boldsymbol{w}^T \boldsymbol{r}(\boldsymbol{h}) - \boldsymbol{\lambda}^T \boldsymbol{p}(\boldsymbol{h}) \quad \text{s. to} \quad \boldsymbol{r}(\boldsymbol{h}) \in \mathcal{C}_g(\boldsymbol{h}, \boldsymbol{p}(\boldsymbol{h})) \quad (4)$$

and $\boldsymbol{\lambda}$ is chosen to satisfy $\mathbb{E}_{\boldsymbol{h}}[p_j^*(\boldsymbol{h})] = \bar{P}_j, \forall j$.

Define marginal "utility" functions (which are first derivatives of $w_j \log(1+z) - \lambda_j z / h_j$)

$$u_j(z) := \frac{w_j}{1+z} - \frac{\lambda_j}{h_j}, \quad j = 1, \ldots, J$$

and the sets

$$\mathcal{A}_j := \{z \in [0, \infty) : u_j(z) > u_i(z), \forall i \neq j, \text{and } u_j(z) > 0\}.$$

Then it is shown in [1] that the optimal rate and power allocation is a greedy one given by

$$r_j^*(\boldsymbol{h}) = \int_{\mathcal{A}_j} \frac{1}{1+z} dz, \qquad p_j^*(\boldsymbol{h}) = \frac{1}{h_j} \int_{\mathcal{A}_j} dz. \quad (5)$$

Due to the underlying polymatroid structure of the capacity region, the optimal rates must be achieved by successive decoding in an increasing order of the components of $\boldsymbol{w}$, regardless of $\boldsymbol{h}$.

With the optimal power and rate allocation in (5), the optimal solution to (3) is $\bar{\boldsymbol{r}}^* = \mathbb{E}_{\boldsymbol{h}}[\boldsymbol{r}^*(\boldsymbol{h})]$, which is also a boundary point of $\mathcal{C}(\bar{\boldsymbol{P}})$. After obtaining all the boundary points by varying $\boldsymbol{w}$, the ergodic capacity can be determined.

B. *Utility Maximization With Average Rate Guarantees*

Instead of the linear utility (weighted sum) in (3), let us now consider maximizing a general utility function under constraints on prescribed minimum average rates $\bar{\boldsymbol{R}} := [\bar{R}_1, \ldots, \bar{R}_J]^T \ge \boldsymbol{0}$, i.e.,

$$\max_{\bar{\boldsymbol{r}}} U(\bar{\boldsymbol{r}}), \quad \text{s. to} \quad \bar{\boldsymbol{r}} \ge \bar{\boldsymbol{R}}, \quad \bar{\boldsymbol{r}} \in \mathcal{C}(\bar{\boldsymbol{P}}) \quad (6)$$

where $U : \mathbb{R}_+^J \to \mathbb{R}$ is a chosen utility function of average user rate vector $\bar{\boldsymbol{r}}$. The utility maximization in (6) is of great interest recently since it provides a quantitative manner to address the fairness among the user terminals. For instance, it was shown that a maximizer of a class of the *concave and increasing* utility functions satisfies the definition of α-fair allocation, with the notion of α-fairness including max-min fairness (with $\alpha \to \infty$), proportional fairness (with $\alpha = 1$), and throughput maximization (with $\alpha = 0$) as special cases (larger α means more fairness) [12]. On the other hand, the minimum average rates $\bar{\boldsymbol{R}}$ are imposed here to guarantee the QoS of the connections, as requested by the practical applications [9], [10].

To ensure (6) a well-defined problem, it is assumed:

(A1) Function $U(\bar{\boldsymbol{r}})$ is concave and uniformly bounded, $\forall \bar{\boldsymbol{r}} \in \mathcal{C}(\bar{\boldsymbol{P}})$; and the prescribed minimum rates $\bar{\boldsymbol{R}} \in \mathbf{int}(\mathcal{C}(\bar{\boldsymbol{P}}))$.

The conditions in (A1) assure that (6) is a strictly feasible convex optimization problem,[1] provided that $\mathcal{C}(\bar{\boldsymbol{P}})$ is a closed convex set of $\bar{\boldsymbol{r}}$. Notice that U should be also an increasing function of $\bar{r}_j$ when all other user rates are fixed, since the benefit of the jth user naturally increases as $\bar{r}_j$ increases.

We next solve (6) using a Lagrange dual method. To this end, we introduce an auxiliary vector $\boldsymbol{x} := [x_1, \ldots, x_J]^T$, and rewrite (6) as

$$\max_{\boldsymbol{x},\bar{\boldsymbol{r}}} U(\boldsymbol{x}), \quad \text{s. to} \quad \boldsymbol{x} = \bar{\boldsymbol{r}}, \quad \boldsymbol{x} \geq \bar{\boldsymbol{R}}, \bar{\boldsymbol{r}} \in \mathcal{C}(\bar{\boldsymbol{P}}). \tag{7}$$

It is easy to see that the solution $\boldsymbol{x}^* = \bar{\boldsymbol{r}}^*$ for (7) is also the solution for (6).

From the definition of $\mathcal{C}(\bar{\boldsymbol{P}})$ in (2), we have $\bar{\boldsymbol{r}} \in \mathcal{C}(\bar{\boldsymbol{P}})$ only if there exists a power control policy $\boldsymbol{p}(\cdot)$ such that $\mathbb{E}_{\boldsymbol{h}}[\boldsymbol{p}(\boldsymbol{h})] \leq \bar{\boldsymbol{P}}$, and $\bar{\boldsymbol{r}} \in \mathcal{C}_f(\boldsymbol{p}(\cdot))$. It is also not difficult to show that for an optimal power control policy $\boldsymbol{p}^*(\cdot)$ for (6), which yields a boundary point in the capacity region, we must have $\mathbb{E}_{\boldsymbol{h}}[p_j^*(\boldsymbol{h})] = \bar{P}_j, \forall j$. This implies we can reduce the feasible set $\mathcal{F}$ and find the optimal power allocation $\boldsymbol{p}^*(\cdot)$ for (6) only over all $\boldsymbol{p}(\cdot)$ satisfying $\mathbb{E}_{\boldsymbol{h}}[\boldsymbol{p}(\boldsymbol{h})] = \bar{\boldsymbol{P}}$. For these reasons, we can further reformulate (7) as

$$\begin{aligned} &\max_{\boldsymbol{x},\bar{\boldsymbol{r}},\boldsymbol{p}(\cdot)} U(\boldsymbol{x}), \quad \text{s. to} \quad \boldsymbol{x} = \bar{\boldsymbol{r}}, \quad \boldsymbol{x} \geq \bar{\boldsymbol{R}} \\ &\bar{\boldsymbol{r}} \in \mathcal{C}_f(\boldsymbol{p}(\cdot)), \quad \mathbb{E}_{\boldsymbol{h}}[\boldsymbol{p}(\boldsymbol{h})] = \bar{\boldsymbol{P}}. \end{aligned} \tag{8}$$

Again, it is clear the optimal solution for (8) will yield as a byproduct the optimal $\boldsymbol{x}^* = \bar{\boldsymbol{r}}^*$ for (7) and (6).

To further proceed, we first show the following lemma (see the proof in Appendix A).

Lemma 1: Under (A1), the problem in (8) is a strictly feasible convex optimization problem.

Under (A1), it is clear that (6) is a convex optimization problem since $\mathcal{C}(\bar{\boldsymbol{P}})$ is a convex set of $\bar{\boldsymbol{r}}$. Here, Lemma 1 establishes that the reformulated problem (8) is still a convex optimization problem under the same condition. Let $\boldsymbol{\mu} := [\mu_1, \ldots, \mu_J]^T$ collect the Lagrange multipliers associated with the constraints $\boldsymbol{x} = \bar{\boldsymbol{r}}$, and $\boldsymbol{\lambda} := [\lambda_1, \ldots, \lambda_J]^T$ the multipliers associated with the average power constraints $\mathbb{E}_{\boldsymbol{h}}[\boldsymbol{p}(\boldsymbol{h})] = \bar{\boldsymbol{P}}$. The (partial) Lagrange for (8) is then

$$\begin{aligned} &L(\boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{x}, \bar{\boldsymbol{r}}, \boldsymbol{p}(\cdot)) \\ &\quad = U(\boldsymbol{x}) - \boldsymbol{\mu}^T(\boldsymbol{x} - \bar{\boldsymbol{r}}) - \boldsymbol{\lambda}^T(\mathbb{E}_{\boldsymbol{h}}[\boldsymbol{p}(\boldsymbol{h})] - \bar{\boldsymbol{P}}) \\ &\quad = U(\boldsymbol{x}) - \boldsymbol{\mu}^T\boldsymbol{x} + \boldsymbol{\mu}^T\bar{\boldsymbol{r}} - \boldsymbol{\lambda}^T\mathbb{E}_{\boldsymbol{h}}[\boldsymbol{p}(\boldsymbol{h})] + \boldsymbol{\lambda}^T\bar{\boldsymbol{P}}. \end{aligned} \tag{9}$$

[1]The problem is strictly feasible since there must $\exists \bar{\boldsymbol{r}} \in \mathcal{C}(\bar{\boldsymbol{P}})$ such that $\bar{\boldsymbol{r}} > \bar{\boldsymbol{R}}$, when $\bar{\boldsymbol{R}} \in \mathbf{int}(\mathcal{C}(\bar{\boldsymbol{P}}))$.

A corresponding Lagrange dual function can be found as

$$D(\boldsymbol{\mu}, \boldsymbol{\lambda}) := \max_{\boldsymbol{x},\bar{\boldsymbol{r}},\boldsymbol{p}(\cdot)} L(\boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{x}, \bar{\boldsymbol{r}}, \boldsymbol{p}(\cdot)) \quad \text{s. to} \quad \boldsymbol{x} \geq \bar{\boldsymbol{R}}, \quad \bar{\boldsymbol{r}} \in \mathcal{C}_f(\boldsymbol{p}(\cdot)) \tag{10}$$

and the dual problem of (8) is

$$\min_{\boldsymbol{\mu},\boldsymbol{\lambda}} D(\boldsymbol{\mu}, \boldsymbol{\lambda}). \tag{11}$$

Since (8) is a strictly feasible convex optimization problem by Lemma 1, it follows that there is no duality gap between the primal problem (8) and its dual problem (11), and the solution of (8) can be obtained via solving its dual [16].

To this end, we need first to find the solution of (10). Interesting, solving (10) amounts to solving two decoupled subproblems [cf. (9)]

$$\max_{\boldsymbol{x}} U(\boldsymbol{x}) - \boldsymbol{\mu}^T\boldsymbol{x}, \quad \text{s. to} \quad \boldsymbol{x} \geq \bar{\boldsymbol{R}} \tag{12}$$

$$\max_{\bar{\boldsymbol{r}},\boldsymbol{p}(\cdot)} \boldsymbol{\mu}^T\bar{\boldsymbol{r}} - \boldsymbol{\lambda}^T\mathbb{E}_{\boldsymbol{h}}[\boldsymbol{p}(\boldsymbol{h})], \quad \text{s. to} \quad \bar{\boldsymbol{r}} \in \mathcal{C}_f(\boldsymbol{p}(\cdot)). \tag{13}$$

For any concave U, the first subproblem (12) is a simple convex optimization problem with respect to $\boldsymbol{x}$, for which efficient algorithms are available to obtain the optimal $\boldsymbol{x}^*(\boldsymbol{\mu})$. In fact, if $U(\cdot)$ is differentiable and its gradient ∇U has a well-defined inverse ∇U^{-1}, we can readily derive a closed-form solution

$$\boldsymbol{x}^*(\boldsymbol{\mu}) = \max\left(\bar{\boldsymbol{R}}, \nabla U^{-1}(\boldsymbol{\mu})\right) \tag{14}$$

where $\max(\cdot,\cdot)$ is a component-wise maximum operator.

By the definition of $\mathcal{C}_f(\boldsymbol{p}(\cdot))$ and $\bar{\boldsymbol{r}} = \mathbb{E}_{\boldsymbol{h}}[\boldsymbol{r}(\boldsymbol{h})]$, solving the second subproblem (13) is equivalent to finding the optimal rate and power allocation per $\boldsymbol{h}$ via

$$\max_{(\boldsymbol{r}(\boldsymbol{h}),\boldsymbol{p}(\boldsymbol{h}))} \boldsymbol{\mu}^T\boldsymbol{r}(\boldsymbol{h}) - \boldsymbol{\lambda}^T\boldsymbol{p}(\boldsymbol{h}) \quad \text{s. to} \quad \boldsymbol{r}(\boldsymbol{h}) \in \mathcal{C}_g(\boldsymbol{h}, \boldsymbol{p}(\boldsymbol{h})). \tag{15}$$

But this problem is the same as (4) with $\boldsymbol{w} \equiv \boldsymbol{\mu}$, and its solution is clearly specified by (5). With the optimal $\boldsymbol{r}^*(\boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{h})$ and $\boldsymbol{p}^*(\boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{h})$ obtained for (15), the optimal rate vector for (13) is given by $\bar{\boldsymbol{r}}^*(\boldsymbol{\mu}, \boldsymbol{\lambda}) = \mathbb{E}_{\boldsymbol{h}}[\boldsymbol{r}^*(\boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{h})]$.

Notice that with introduction of the auxiliary $\boldsymbol{x}$ in (8), we can solve the desired utility maximization using a dual-based approach, where U is only required to be a (possibly nondifferentiable) concave function. This is different from the primal or primal-dual-based schemes in [5]–[10], [13], where a differentiable U is needed since the gradient $\nabla U(\bar{\boldsymbol{r}})$ is employed as the user weights [i.e., playing the role of $\boldsymbol{u}$ in (13)] for each iteration.

Summarizing, we have the following lemma.

Lemma 2: For a given $(\boldsymbol{\mu}, \boldsymbol{\lambda})$ in (10), the optimal $\boldsymbol{x}^*(\boldsymbol{\mu})$ is provided by solution to (12), the optimal rate and power allocation, $(\boldsymbol{r}^*(\boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{h}), \boldsymbol{p}^*(\boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{h}))$, is provided by a greedy policy in (5) with $\boldsymbol{w} \equiv \boldsymbol{\mu}$, and the optimal average rate vector is $\bar{\boldsymbol{r}}^*(\boldsymbol{\mu}, \boldsymbol{\lambda}) = \mathbb{E}_{\boldsymbol{h}}[\boldsymbol{r}^*(\boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{h})]$.

With the optimal power allocation $\boldsymbol{p}^*(\boldsymbol{\mu},\boldsymbol{\lambda},\boldsymbol{h})$ in Lemma 2, we define an average power vector $\bar{\boldsymbol{p}}^*(\boldsymbol{\mu},\boldsymbol{\lambda}) = \mathbb{E}_{\boldsymbol{h}}[\boldsymbol{p}^*(\boldsymbol{\mu},\boldsymbol{\lambda},\boldsymbol{h})]$. Then it can be shown that

$$\bar{\boldsymbol{g}}(\boldsymbol{\mu},\boldsymbol{\lambda}) := [(\bar{\boldsymbol{r}}^*(\boldsymbol{\mu},\boldsymbol{\lambda}) - \boldsymbol{x}^*(\boldsymbol{\mu}))^T, (\bar{\boldsymbol{P}} - \bar{\boldsymbol{p}}^*(\boldsymbol{\mu},\boldsymbol{\lambda}))^T]^T \quad (16)$$

is a subgradient of $D(\boldsymbol{\mu},\boldsymbol{\lambda})$ such that [17, p. 604]

$$D(\boldsymbol{\mu}',\boldsymbol{\lambda}') - D(\boldsymbol{\mu},\boldsymbol{\lambda}) \geq \bar{\boldsymbol{g}}^T(\boldsymbol{\mu},\boldsymbol{\lambda})\left(\begin{bmatrix}\boldsymbol{\mu}'\\ \boldsymbol{\lambda}'\end{bmatrix} - \begin{bmatrix}\boldsymbol{\mu}\\ \boldsymbol{\lambda}\end{bmatrix}\right) \forall(\boldsymbol{\mu}',\boldsymbol{\lambda}').$$

In fact, the optimal rate and power allocation, $\boldsymbol{r}^*(\boldsymbol{\mu},\boldsymbol{\lambda},\boldsymbol{h})$ and $\boldsymbol{p}^*(\boldsymbol{\mu},\boldsymbol{\lambda},\boldsymbol{h})$ in Lemma 2, is almost surely unique per $\boldsymbol{h}$, provided that the fading process has a continuous joint cdf [1]. This implies that the values of average rate and power vectors $\bar{\boldsymbol{r}}^*(\boldsymbol{\mu},\boldsymbol{\lambda})$ and $\bar{\boldsymbol{p}}^*(\boldsymbol{\mu},\boldsymbol{\lambda})$ are unique, and thus the subgradient in (16) is unique. In such a case, the dual function $D(\boldsymbol{\mu},\boldsymbol{\lambda})$ is differentiable and $\bar{\boldsymbol{g}}(\boldsymbol{\mu},\boldsymbol{\lambda})$ is indeed its gradient $\nabla D(\boldsymbol{\mu},\boldsymbol{\lambda})$.

With the gradient $\bar{\boldsymbol{g}}(\boldsymbol{\mu},\boldsymbol{\lambda})$, we can then find the optimal multipliers $\boldsymbol{\mu}^*$ and $\boldsymbol{\lambda}^*$, i.e., solve the dual problem (11), via the following gradient descent iterations:

$$\begin{bmatrix}\boldsymbol{\mu}[n+1]\\ \boldsymbol{\lambda}[n+1]\end{bmatrix} = \begin{bmatrix}\boldsymbol{\mu}[n]\\ \boldsymbol{\lambda}[n]\end{bmatrix} - \beta\bar{\boldsymbol{g}}(\boldsymbol{\mu}[n],\boldsymbol{\lambda}[n])$$

where β is a chosen small stepsize, n denotes the iteration index, and the iterations can be initialized with arbitrary $(\boldsymbol{\mu}[0],\boldsymbol{\lambda}[0])$. Using (16), these gradient descent iterations can be rewritten more specifically as

$$\begin{aligned}\boldsymbol{\mu}[n+1] &= \boldsymbol{\mu}[n] + \beta(\boldsymbol{x}^*(\boldsymbol{\mu}[n]) - \bar{\boldsymbol{r}}^*(\boldsymbol{\mu}[n],\boldsymbol{\lambda}[n]))\\ \boldsymbol{\lambda}[n+1] &= \boldsymbol{\lambda}[n] + \beta(\bar{\boldsymbol{p}}^*(\boldsymbol{\mu}[n],\boldsymbol{\lambda}[n]) - \bar{\boldsymbol{P}}).\end{aligned} \quad (17)$$

It follows from [17, p. 60] that the iterations in (17) are guaranteed to converge to the optimal $(\boldsymbol{\mu}^*,\boldsymbol{\lambda}^*)$ from any initial $(\boldsymbol{\mu}[0],\boldsymbol{\lambda}[0])$, and the convergence can be geometrically fast.

Having obtained $(\boldsymbol{\mu}^*,\boldsymbol{\lambda}^*)$, strong duality between the primal (8) and dual problems (11) inferred by Lemma 1 implies that replacing $(\boldsymbol{\mu},\boldsymbol{\lambda})$ with $(\boldsymbol{\mu}^*,\boldsymbol{\lambda}^*)$ in Lemma 2 provides (almost surely) optimal rate and power allocation $(\boldsymbol{r}^*(\boldsymbol{\mu}^*,\boldsymbol{\lambda}^*,\boldsymbol{h}),\boldsymbol{p}^*(\boldsymbol{\mu}^*,\boldsymbol{\lambda}^*,\boldsymbol{h}))$, as well as the optimal achievable rate vectors $\boldsymbol{x}^*(\boldsymbol{\mu}^*) = \bar{\boldsymbol{r}}^*(\boldsymbol{\mu}^*,\boldsymbol{\lambda}^*) = \mathbb{E}_{\boldsymbol{h}}[\boldsymbol{r}^*(\boldsymbol{\mu}^*,\boldsymbol{\lambda}^*,\boldsymbol{h})]$ for (8). By the equivalence between (6) and (8), these solutions also provide the optimal rate and power allocation as well as optimal average user rate vector for the original utility maximization in (6). Formally stated, we have the following.

Proposition 1: Under (A1), the iterates in (17) converge to $(\boldsymbol{\mu}^*,\boldsymbol{\lambda}^*)$ from any initial $(\boldsymbol{\mu}[0],\boldsymbol{\lambda}[0])$, and the optimal solution of (6) is given by $\boldsymbol{x}^*(\boldsymbol{\mu}^*)$, while the corresponding optimal rate and power allocation are $\boldsymbol{r}^*(\boldsymbol{\mu}^*,\boldsymbol{\lambda}^*,\boldsymbol{h})$ and $\boldsymbol{p}^*(\boldsymbol{\mu}^*,\boldsymbol{\lambda}^*,\boldsymbol{h})$ per $\boldsymbol{h}$ specified in Lemma 2.

It is clear from Proposition 1 that the globally optimal resource allocation maximizing the general utility coincides with the optimal one maximizing a linear utility in (3) when $\boldsymbol{w} = \boldsymbol{\mu}^*$; i.e., the resultant optimal $\bar{\boldsymbol{r}}^*$ for (6) resides on a boundary point of the ergodic capacity region when the weight vector is $\boldsymbol{\mu}^*$. The latter is, however, unknown prior to convergence of the gradient descent iterations.

It is also interesting to see the relationship between (6) and (3) from a different view. Supposing that there are no minimum average rate constraints, i.e., $\bar{\boldsymbol{R}} = \boldsymbol{0}$, and the utility function is differentiable in (6), we must have from (14) that $\boldsymbol{\mu}^* = \nabla U(\boldsymbol{x}^*) = \nabla U(\bar{\boldsymbol{r}}^*)$ where $\boldsymbol{x}^* = \bar{\boldsymbol{r}}^*$ is the optimal solution for (6). Now if the utility function $U(\bar{\boldsymbol{r}}) = \boldsymbol{w}^T\bar{\boldsymbol{r}}$, then we have $\boldsymbol{\mu}^* = \nabla U(\bar{\boldsymbol{r}}^*) = \boldsymbol{w}$. This implies that the proposed schemes can also solve (3) as a special case of (6) with $U(\bar{\boldsymbol{r}}) = \boldsymbol{w}^T\bar{\boldsymbol{r}}$ and $\bar{\boldsymbol{R}} = \boldsymbol{0}$ (note that the constraints $\bar{\boldsymbol{r}} \geq \boldsymbol{0}$ are redundant and can be removed).

The gradient descent iterations in (17) provide an efficient way to find the optimal resource allocation policy for (6). However, this method requires knowledge of the cdf $F(\boldsymbol{h})$ of the fading channels; only then can we obtain the average rate and power vectors $\bar{\boldsymbol{r}}^*(\boldsymbol{\mu}[n],\boldsymbol{\lambda}[n]) = \mathbb{E}_{\boldsymbol{h}}[\boldsymbol{r}^*(\boldsymbol{\mu}[n],\boldsymbol{\lambda}[n],\boldsymbol{h})]$ and $\bar{\boldsymbol{p}}^*(\boldsymbol{\mu}[n],\boldsymbol{\lambda}[n]) = \mathbb{E}_{\boldsymbol{h}}[\boldsymbol{p}^*(\boldsymbol{\mu}[n],\boldsymbol{\lambda}[n],\boldsymbol{h})]$ encountered in evaluating the gradient vector $\bar{\boldsymbol{g}}(\boldsymbol{\mu}[n],\boldsymbol{\lambda}[n])$ per iteration n. Interestingly, based on (17), it is also possible to develop the corresponding *stochastic* gradient descent iterations which are capable of solving (6) without the channel cdf *a priori*. In fact, to solve the convex optimization problem in (6), other efficient algorithms may be also available from the rich convex programming toolbox [16], [17]; such as the primal-based conditional gradient method in [13] when the utility function is differentiable and there are no minimum rate constraints. (Notice that the approach in [13] is not capable of adaptive power control when the channel cdf is not available.) But here we adopt (8) and the dual-based gradient iterations (17), because they facilitate development of a stochastic solution which can essentially learn the unknown channel cdf $F(\boldsymbol{h})$ on-the-fly to approach the optimal rate and power allocation, as we show next.

C. Stochastic Resource Allocation Without Channel CDF

To bypass the need of $F(\boldsymbol{h})$ in (17), we consider dropping the expectation operators $\mathbb{E}_{\boldsymbol{h}}$ from $\bar{\boldsymbol{r}}^*(\boldsymbol{\mu}[n],\boldsymbol{\lambda}[n]) = \mathbb{E}_{\boldsymbol{h}}[\boldsymbol{r}^*(\boldsymbol{\mu}[n],\boldsymbol{\lambda}[n],\boldsymbol{h})]$ and $\bar{\boldsymbol{p}}^*(\boldsymbol{\mu}[n],\boldsymbol{\lambda}[n]) = \mathbb{E}_{\boldsymbol{h}}[\boldsymbol{p}^*(\boldsymbol{\mu}[n],\boldsymbol{\lambda}[n],\boldsymbol{h})]$, to put forth the following stochastic iterations based only on fading realization $\boldsymbol{h}[n]$ per block n

$$\begin{aligned}\hat{\boldsymbol{\mu}}[n+1] &= \hat{\boldsymbol{\mu}}[n] + \beta(\boldsymbol{x}^*(\hat{\boldsymbol{\mu}}[n]) - \boldsymbol{r}^*(\hat{\boldsymbol{\mu}}[n],\hat{\boldsymbol{\lambda}}[n],\boldsymbol{h}[n]))\\ \hat{\boldsymbol{\lambda}}[n+1] &= \hat{\boldsymbol{\lambda}}[n] + \beta(\boldsymbol{p}^*(\hat{\boldsymbol{\mu}}[n],\hat{\boldsymbol{\lambda}}[n],\boldsymbol{h}[n]) - \bar{\boldsymbol{P}})\end{aligned} \quad (18)$$

where hats are to stress these iterations are stochastic estimates of those in (17), based on *instantaneous* (instead of average) power and rates specified by Lemma 2 with $\hat{\boldsymbol{\mu}}[n],\hat{\boldsymbol{\lambda}}[n]$ and random fading realization $\boldsymbol{h}[n]$. Notice that with a given initial $(\boldsymbol{\mu}[0],\boldsymbol{\lambda}[0])$, the iterates in (17) consist of a *deterministic* sequence. However, the stochastic iterates in (18) are in fact a *random* sequence since the "stochastic" gradient used per iteration depends on a fading realization $\boldsymbol{h}[n]$ randomly drawn from the channel cdf $F(\boldsymbol{h})$. In a block fading channel, it is reasonable that the current channel state information $\boldsymbol{h}[n]$ is available (e.g., through training) per block n, although the channel cdf $F(\boldsymbol{h})$ is unknown. In such a case, each iteration of (18) can be implemented per block without *a priori* $F(\boldsymbol{h})$.

The stochastic gradient descent iterations in fact belong to the same class of well-documented adaptive signal processing algorithms, e.g., the least-mean-square (LMS) algorithm that is a result of dropping the expectation operators from the steepest descent iterations [14, p. 77]. As the convergence (to the optimum) of the latter has been well established, it is possible to show the convergence of the proposed iterations in (18) through a similar stochastic averaging argument.

To see it, define the notations $\boldsymbol{\Lambda}[n] := [\boldsymbol{\mu}^T[n], \boldsymbol{\lambda}^T[n]]^T, \hat{\boldsymbol{\Lambda}}[n] := [\hat{\boldsymbol{\mu}}^T[n], \hat{\boldsymbol{\lambda}}^T[n]]^T$, and stochastic gradient

$$\boldsymbol{g}(\hat{\boldsymbol{\Lambda}}[n], \boldsymbol{h}[n]) := \begin{bmatrix} \boldsymbol{r}^*(\hat{\boldsymbol{\mu}}[n], \hat{\boldsymbol{\lambda}}[n], \boldsymbol{h}[n]) - \boldsymbol{x}^*(\hat{\boldsymbol{\mu}}[n]) \\ \bar{\boldsymbol{P}} - \boldsymbol{p}^*(\hat{\boldsymbol{\mu}}[n], \hat{\boldsymbol{\lambda}}[n], \boldsymbol{h}[n]) \end{bmatrix}.$$

Then we can rewrite (18) and (17) into compact forms

$$\hat{\boldsymbol{\Lambda}}[n+1] = \hat{\boldsymbol{\Lambda}}[n] - \beta \boldsymbol{g}(\hat{\boldsymbol{\Lambda}}[n], \boldsymbol{h}[n]) \tag{19}$$

$$\boldsymbol{\Lambda}[n+1] = \boldsymbol{\Lambda}[n] - \beta \bar{\boldsymbol{g}}(\boldsymbol{\Lambda}[n]) \tag{20}$$

where the gradient $\bar{\boldsymbol{g}}(\boldsymbol{\Lambda}[n])$ is defined in (16).

Since it clearly follows that $\bar{\boldsymbol{g}}(\boldsymbol{\Lambda}) = \mathbb{E}_{\boldsymbol{h}[n]}[\boldsymbol{g}(\boldsymbol{\Lambda}, \boldsymbol{h}[n])] = \mathbb{E}_{\boldsymbol{h}}[\boldsymbol{g}(\boldsymbol{\Lambda}, \boldsymbol{h})]$ for a stationary fading process, the stochastic gradient iterations in (19) and "ensemble" gradient iterations in (20) consist of a pair of *primary* and *averaged* systems [14, p. 232]. For these two systems, we can further employ the stochastic locking theorem in [14, p. 234] to prove the following.

Lemma 3: For ergodic fading channels with continuous cdf, if the primary system (19) and its averaged system (20) have the identical initialization $\hat{\boldsymbol{\Lambda}}[0] \equiv \boldsymbol{\Lambda}[0]$, then it holds for any time interval T that

$$\max_{1 \le n \le T/\beta} \|\hat{\boldsymbol{\Lambda}}[n] - \boldsymbol{\Lambda}[n]\| \le c_T(\beta) \quad \text{w.p.1}$$

with $c_T(\beta) \to 0$, as $\beta \to 0$.

The proof of the Lemma 3 can be found in Appendix B. This lemma shows that for a given stepsize β, the distance between "trajectories" of the primary (19) and averaged systems (20) is always bounded by a constant $c_T(\beta)$ in probability, and this constant $c_T(\beta)$ vanishes as $\beta \to 0$. (In fact, it is shown in [14] that the constant $c_T(\beta)$ is proportional to β.) This implies that the random sequence $\hat{\boldsymbol{\Lambda}}[n]$ with (19) can always stay "close" to the deterministic sequence $\boldsymbol{\Lambda}[n]$ with (20) over any time interval T, when a sufficiently small β is adopted.

With the trajectory locking proved in Lemma 3 and the convergence (geometrically fast under conditions) of the averaged system (20) shown in Proposition 1, we can then apply the stochastic hovering theorem to further have [14, p. 252].

Theorem 1: Under (A1), the iterates in (19), or equivalently (18), converge to $\boldsymbol{\Lambda}^* := [\boldsymbol{\mu}^{*T}, \boldsymbol{\lambda}^{*T}]^T$ in probability, i.e., $\sup_{n \to \infty} \Pr\{\|\hat{\boldsymbol{\Lambda}}[n] - \boldsymbol{\Lambda}^*\| > \epsilon\} \to 0$, from any initial $\hat{\boldsymbol{\Lambda}}[0]$ as $\beta \to 0$; and, thus, the corresponding rate and power allocation converges to the globally optimal one for (6).

Theorem 1 establishes the convergence as well as stability of the stochastic gradient descent iterations in (18) in the following sense. Given any $\epsilon > 0$, there exist (controllably small) constants $\delta(\epsilon)$ and $\beta(\epsilon, \delta)$ such that the probability of the stochastic iterates $\hat{\boldsymbol{\Lambda}}[n]$ to escape from a ball of radius ϵ around the optimal $\boldsymbol{\Lambda}^*$ is less than $\delta(\epsilon)$ when using a (sufficiently small) stepsize $\beta \le \beta(\epsilon, \delta)$. This convergence does not rely on Makov property of the random fading channels, and only ergodicity of the fading process is required.

It is now evident that the proposed novel dual-based stochastic approach is capable of providing optimal rate and power allocation that maximizes a utility of user rates with guarantees on prescribed minimum rates $\bar{\boldsymbol{R}}$ (if feasible) over the capacity region of ergodic fading multiple-access channels even when the channel cdf is not available. This approach works for both differentiable and nondifferentiable utility functions, and its convergence is established for typical wireless channels with continuous fading distributions; i.e., it overcomes limitations of the existing alternatives in e.g., [13], [5]–[10].

We have so far established the convergence of the stochastic iterations (18) when the problem (6) is strictly feasible. It is worth commenting that these iterations diverge, i.e., $\hat{\boldsymbol{\mu}}[n], \hat{\boldsymbol{\lambda}}[n] \to \infty$, if the convex optimization problem (6) is infeasible [16]. Note that (6) is infeasible iff $\bar{\boldsymbol{R}} \notin \mathbf{int}(\mathcal{C}(\bar{\boldsymbol{P}}))$, i.e., the minimum average rate requests from the users cannot be handled by the intended fading multiple access channel. With the proposed stochastic iterations (18), these "over-requests" can be detected by the divergence of the iterations, and an external scheme, e.g., connection admission control, can be then invoked to address this infeasibility.

D. Discussions

We next discuss significance of the proposed stochastic resource allocation in practice and theory.

1) Practical Applications: As an immediate application, the proposed stochastic iterations (18) can be employed to devise an on-line stochastic resource scheduling scheme at the access point in practical multiple-access networks. Suppose a block-fading channel, where the fading state $\boldsymbol{h}[n]$ remains the same per block but is allowed to vary from block to block. Assuming that the access point has available the current channel state information $\boldsymbol{h}[n]$ through training, then an on-line resource scheduling algorithm can be operated as follows.

Algorithm 1 On-line stochastic dual-gradient iterations

1) **initialize** with any $\hat{\boldsymbol{\mu}}[0]$ and $\hat{\boldsymbol{\lambda}}[0]$;
2) **repeat on-line:** with $\hat{\boldsymbol{\mu}}[n]$, $\hat{\boldsymbol{\lambda}}[n]$, and current channel state information $\boldsymbol{h}[n]$ available per block n, the access point allocates resources according to the policy $\boldsymbol{r}^*(\hat{\boldsymbol{\mu}}[n], \hat{\boldsymbol{\lambda}}[n], \boldsymbol{h}[n])$ and $\boldsymbol{p}^*(\hat{\boldsymbol{\mu}}[n], \hat{\boldsymbol{\lambda}}[n], \boldsymbol{h}[n])$, and then updates $\hat{\boldsymbol{\mu}}[n+1]$ and $\hat{\boldsymbol{\lambda}}[n+1]$ using (18).

The proposed Algorithm 1 has a very low complexity. In each iteration of this algorithm, the access point needs to find out the rate and power allocation policy $\boldsymbol{r}^*(\hat{\boldsymbol{\mu}}[n], \hat{\boldsymbol{\lambda}}[n], \boldsymbol{h}[n])$ and $\boldsymbol{p}^*(\hat{\boldsymbol{\mu}}[n], \hat{\boldsymbol{\lambda}}[n], \boldsymbol{h}[n])$ for the given $\hat{\boldsymbol{\mu}}[n], \hat{\boldsymbol{\lambda}}[n]$, and $\boldsymbol{h}[n]$, which only requires a computational complexity on the order of $\mathcal{O}(J \log J)$ [1]. On the other hand, the iterations can converge to the globally optimal rate and power allocation with minimum average rate guarantees geometrically fast when the fading process is stationary and ergodic. As a comparison, the approximate projection method in [13] requires a computational complexity $\mathcal{O}(J^2 \log J)$ per iteration, and it cannot yield the optimal resource allocation for general utility maximization when the fading cdf is not available *a priori*.

Although the convergence of the proposed stochastic iterations is only established for the stationary fading channels, it is worth mentioning that these iterations are robust to nonstationarity. This is because such a scheme is capable of dynamically learning the channel statistics on-the-fly. Therefore, if the channel statistics are changed due to significant network dynamics, e.g., sudden bandwidth and/or topology variations, the proposed stochastic scheme in Algorithm 1 can relearn the intended channels and converge to the new optimum from any given state. Indeed it is this "learning" capability that makes the stochastic schemes so attractive.

2) Theoretical Applications: The stochastic iterations in (18) are also meaningful for theoretical studies where the fading channel cdf is assumed available. With the channel cdf, the gradient descent iterations (17) need to evaluate the average rate and power vectors per iteration. However, since $\boldsymbol{r}^*(\boldsymbol{\mu}[n], \boldsymbol{\lambda}[n], \boldsymbol{h})$ and $\boldsymbol{p}^*(\boldsymbol{\mu}[n], \boldsymbol{\lambda}[n], \boldsymbol{h})$ are not rational functions of $\boldsymbol{h}$, their ensemble averages $\mathbb{E}_{\boldsymbol{h}}$ may have to be obtained through a Monte Carlo method when joint fading distribution is not so simple as, e.g., uncorrelated Gaussian. In this method, provided the cdf $F(\boldsymbol{h})$, we draw sufficiently many (say $N_t \to \infty$) realizations $\boldsymbol{h}(t)$ from $F(\boldsymbol{h})$ per iteration, and form the sample averages, which thanks to the ergodicity of $\boldsymbol{h}$ approaches the ensemble averages. Using such averages, the iterations (17) can be carried out to find the optimal rate and power allocation.

In the stochastic gradient iterations (18), on the other hand, only one ($N_t = 1$) sample $\boldsymbol{h}[n]$ is drawn, and "rough" (stochastic) estimates of the averages rates and powers, i.e., instantaneous rates and powers $\boldsymbol{r}^*(\hat{\boldsymbol{\mu}}[n], \hat{\boldsymbol{\lambda}}[n], \boldsymbol{h}[n])$ and $\boldsymbol{p}^*(\hat{\boldsymbol{\mu}}[n], \hat{\boldsymbol{\lambda}}[n], \boldsymbol{h}[n])$, are used in the iterations. And it is proved that using such "rough estimates" suffices to approach the optimal solution of (6) with controllable error and convergence speed. Therefore, Algorithm 1 provides a simple alternative for solving (6), when the ensemble averages are not easy to evaluate. Note that the ergodic capacity region can be also determined in a stochastic manner, by obtaining the boundary points via using (18) to solve (3) (as a special case of (6) with $U(\bar{\boldsymbol{r}}) = \boldsymbol{w}^T\bar{\boldsymbol{r}}$ and $\bar{\boldsymbol{R}} = \boldsymbol{0}$).

3) Stochastic Scheme Versus Ensemble Scheme: To run the "ensemble" scheme (17), the fading channel cdf is required to be known *a priori*. However, this knowledge is seldom available in closed form for practical applications. Hence, we may need to "learn" this through observations on the fading realizations. After a sufficiently large number of the fading samples are collected, the required cdf could be accurately estimated, and (17) can be then run off-line to obtain the optimal power and rate allocation policy for the subsequent time blocks. Clearly, here we have a separate "learning" process before implementation of the "ensemble" algorithm. With this approach, although the bandwidth can be optimally utilized once the optimal resource allocation is obtained, the system may have to stay idle during "learning" the fading channels. By contrast, the stochastic scheme (18) facilitates learning the intended channels on-the-fly to approach the optimal resource allocation. It is true that the scheme is operating under suboptimal strategies before convergence. However, compared with the foregoing separate learning process, system bandwidth is not wasted for all time blocks.

It is worth mentioning that the stochastic scheme (18) can converge to the exact optimal $\boldsymbol{\Lambda}^*$ if a vanishing stepsize $\beta[n] \to 0$ with $\sum_{n=1}^{\infty} \beta[n] = \infty$, is adopted [14], [7]. Therefore, with the fading cdf available (beforehand or after learning), (18) with such a $\beta[n]$ (e.g., $\beta[n] = 1/n$) can be run off-line or on-line to obtain the optimal resource allocation policy. As a drawback, however, using a vanishing stepsize will make (18) lose its learning capability. As a result, a change in channel cdf, e.g., due to user mobility, will fail this scheme. Clearly this failure occurs with the ensemble scheme (17) as well. Different from these two options, a stochastic scheme (18) using a small but constant β is robust to the nonstationary channels. With a constant β, only a "stochastic convergence" can be achieved for (18). Upon such a convergence, the Lagrange multipliers only hover within a small neighborhood (proportional to stepsize β) around the optimal values, and thus, the scheme performs only "near-optimal" resource allocation policy while it keeps learning the channels. As such, the nonstationary channels can be tracked. This is clearly critical for mobile applications.

IV. Fading Broadcast Channel

In this section, we generalize the proposed stochastic resource allocation approach to fading broadcast channels. To this end, again we review the ergodic capacity of the fading Gaussian broadcast channel. For a given power vector $\boldsymbol{p}$ and a fading state $\boldsymbol{h}$, the Shannon capacity region for a degraded Gaussian broadcast channel is

$$\mathcal{C}_b(\boldsymbol{p}(\cdot)) := \left\{ \bar{\boldsymbol{r}} : \bar{r}_j \leq \mathbb{E}_{\boldsymbol{h}}\left[\log\left(1 + \frac{p_j(\boldsymbol{h})}{1/h_j + \sum_{i=1}^{J} p_i(\boldsymbol{h})\mathbb{I}(h_j < h_i)}\right)\right]\right\}$$

where $\mathbb{I}(x) = 1$ if x is true and zero otherwise.

Let $\bar{P}$ denote the power budget of the single transmitter, and define the set of all feasible power control policies satisfying the average power constraint: $\mathcal{F}' := \{\boldsymbol{p}(\cdot) : \mathbb{E}_{\boldsymbol{h}}[\sum_{j=1}^{J} p_j(\boldsymbol{h})] \leq \bar{P}\}$. Assuming that the transmitter has channel state information, the ergodic capacity region for the broadcast fading Gaussian channel is then given by [3]: $\mathcal{C}(\bar{P}) := \bigcup_{\boldsymbol{p}(\cdot)\in\mathcal{F}'} \mathcal{C}_b(\boldsymbol{p}(\cdot))$.

For the convex $\mathcal{C}(\bar{P})$, each boundary point maximizes a weighted sum of average rates, similarly as in (3). For a given $\boldsymbol{w}$, there exists a single Lagrange multiplier $\lambda > 0$ (associated with the sum-power constraint) such that the optimal rate and power allocation achieving the corresponding boundary point, $(\boldsymbol{r}^*(\boldsymbol{h}), \boldsymbol{p}^*(\boldsymbol{h}))$ per $\boldsymbol{h}$, is determined as follows.

Define marginal "utility" functions:

$$u_j(z) := \frac{w_j}{1/h_j + z} - \lambda, \quad j = 1, \ldots, J$$

and the sets

$$\mathcal{A}_j := \{z \in [0, \infty) : u_j(z) > u_i(z) \forall i \neq j, \text{ and } u_j(z) > 0\}.$$

It is shown in [3] that the optimal rate and power allocation for a given $\boldsymbol{w}$ is a greedy one specified by $\forall j = 1, \ldots J$

$$r_j^*(\boldsymbol{h}) = \int_{\mathcal{A}_j} \frac{1}{1/h_j + z} dz, \qquad p_j^*(\boldsymbol{h}) = \int_{\mathcal{A}_j} dz. \tag{21}$$

In addition, λ is chosen to satisfy $\mathbb{E}_{\boldsymbol{h}}[\sum_{j=1}^{J} p_j^*(\boldsymbol{h})] = \bar{P}$.

Note that for broadcast channel, the optimal user rates are achieved by successive decoding per receiver in an increasing order of the channel gains in $\boldsymbol{h}$ (rather than the increasing order of user weights in $\boldsymbol{w}$ for multiple-access case).

A. Stochastic Resource Allocation for Utility Maximization

Consider the utility maximization in (6) with the capacity region $\mathcal{C}(\bar{\boldsymbol{P}})$ replaced by $\mathcal{C}(\bar{P})$. As with (8), we can reformulate the problem as

$$\max_{\boldsymbol{x}, \bar{\boldsymbol{r}}, \boldsymbol{p}(\cdot)} U(\boldsymbol{x}), \qquad \text{s. to} \boldsymbol{x} = \bar{\boldsymbol{r}}, \boldsymbol{x} \geq \bar{\boldsymbol{R}},$$
$$\bar{\boldsymbol{r}} \in \mathcal{C}_b(\boldsymbol{p}(\cdot)), \mathbb{E}_{\boldsymbol{h}}[\sum_{j=1}^{J} p_j(\boldsymbol{h})] = \bar{P}. \tag{22}$$

Let $\boldsymbol{\mu}$ collect the Lagrange multipliers associated with $\boldsymbol{x} = \bar{\boldsymbol{r}}$ and λ the multiplier with the average sum-power constraint $\mathbb{E}_{\boldsymbol{h}}[\sum_{j=1}^{J} p_j(\boldsymbol{h})] = \bar{P}$. Then the problem in (22) can be solved by the dual-based gradient descent iterations [cf. (17)]

$$\boldsymbol{\mu}[n+1] = \boldsymbol{\mu}[n] + \beta(\boldsymbol{x}^*(\boldsymbol{\mu}[n]) - \mathbb{E}_{\boldsymbol{h}}[\boldsymbol{r}^*(\boldsymbol{\mu}[n], \lambda[n], \boldsymbol{h})])$$
$$\lambda[n+1] = \lambda[n] + \beta \left(\mathbb{E}_{\boldsymbol{h}} \left[\sum_{j=1}^{J} p_j^*(\boldsymbol{\mu}[n], \lambda[n], \boldsymbol{h}) \right] - \bar{P} \right) \tag{23}$$

where $\boldsymbol{x}^*(\boldsymbol{\mu}[n])$ is the solution to the simple problem

$$\max_{\boldsymbol{x}} U(\boldsymbol{x}) - \boldsymbol{\mu}^T[n]\boldsymbol{x}, \quad \text{s. to} \quad \boldsymbol{x} \geq \bar{\boldsymbol{R}}$$

and the rate and power allocation, i.e., $\boldsymbol{r}^*(\boldsymbol{\mu}[n], \lambda[n], \boldsymbol{h})$ and $(\boldsymbol{p}^*(\boldsymbol{\mu}[n], \lambda[n], \boldsymbol{h})$ per $\boldsymbol{h}$, are given by (21) with $\boldsymbol{w} \equiv \boldsymbol{\mu}[n]$ and $\lambda \equiv \lambda[n]$.

From (23), the stochastic gradient descent iterations can be further developed as

$$\hat{\boldsymbol{\mu}}[n+1] = \hat{\boldsymbol{\mu}}[n] + \beta(\boldsymbol{x}^*(\hat{\boldsymbol{\mu}}[n]) - \boldsymbol{r}^*(\hat{\boldsymbol{\mu}}[n], \hat{\lambda}[n], \boldsymbol{h}[n]))$$
$$\hat{\lambda}[n+1] = \hat{\lambda}[n] + \beta \left(\sum_{j=1}^{J} p_j^*(\hat{\boldsymbol{\mu}}[n], \hat{\lambda}[n], \boldsymbol{h}[n]) - \bar{P} \right).$$

Again, it can be shown that the proposed stochastic scheme is capable of dynamically learning the channel statistics and approaching the optimally rate-guaranteed and fair (i.e., utility-maximizing) resource allocation. As with Section III, the proposed approach has significant practical and theoretical applications for fading broadcast channels.

V. Numerical Results

In this section, we provide numerical examples to evaluate the proposed stochastic resource allocation schemes. We consider two-user fading Gaussian multiple-access or broadcast channels. The system bandwidth is 10 KHz. The fading processes between the access point and the two users are generated independently from Nakagami-m distributions with different signal-to-noise ratio (SNR) values.

Suppose that average normalized SNRs for users 1 and 2 are $\bar{h}_1 = 10$ dBW, and $\bar{h}_2 = 8$ dBW. The Nakagami parameters for the two users' channels are $m_1 = 1$ (corresponding to Rayleigh fading), and $m_2 = 1.2$. Relying on the proposed stochastic scheme, Fig. 1 shows the ergodic capacity regions of the resultant fading broadcast or multiple-access channels for: i) sum-power budget $\bar{P} = 1$ W or individual power budgets; ii) $\bar{P}_1 = \bar{P}_2 = 0.5$ W; iii) $\bar{P}_1 = 0.8$ W; $\bar{P}_2 = 0.2$ W; and iv) $\bar{P}_1 = 0.2$ Watt, $\bar{P}_2 = 0.8$ Watt. (The receive SNR is $\bar{h}_j$ dBW multiplied by the transmit-power measured in Watts.) Clearly, the individual average power constraints can be seen as realizations of the average sum-power constraint, i.e., $\bar{P}_1 + \bar{P}_2 = \bar{P}$. Hence, the broadcast capacity region contains the multiple-access capacity regions, and each multiple-access region touches the broadcast region at one point, as dictated by the duality between broadcast and multiaccess channels [4].

We next test the proposed utility-based stochastic resource allocation scheme in the aforementioned fading broadcast channel with power budget $\bar{P} = 1$. Here the utility function is selected as $U(\bar{\boldsymbol{r}}) = \log(\bar{r}_1) + \log(\bar{r}_2)$ (maximizing this logarithmic utility results a proportional fair scheme [5]). And the stepsize is chosen as $\beta = 0.001$. We first assume that the two users have no rate (QoS) requirements, i.e., $\boldsymbol{R} = \boldsymbol{0}$. Fig. 2 shows the resultant average rate vector after 10000 iterations with a square marker. It is clear that the stochastic resource allocation scheme can optimally exploit the ergodic capacity of the fading channel, since the resultant rate vector resides on a boundary point of the capacity region.

We next assume that the two users request minimum average rates $\bar{R}_1 = 10$ and $\bar{R}_2 = 18$ Kbps. Fig. 2 shows also the resultant average user rates after 10000 iterations in this case, with a triangle marker. It can be seen that the achieved rate vector settles at a boundary point of the capacity region with both minimum rate requirements satisfied. Notice that except for the minimum rate constraints, the stochastic dual-gradient iterations here solve the same utility maximization problem as the previous case without QoS requests [cf. (6)]. Therefore, taking into account the minimum rate constraints, the QoS-guaranteed solution should be the projection of the non-QoS-guaranteed solution on the feasible set dictated by the area between the boundaries of the capacity region and the two dashed lines $\bar{R}_1 = 10$ and $\bar{R}_2 = 18$ Kbps. This is clearly shown in Fig. 2, which confirms the optimality of the proposed stochastic dual-gradient scheme with minimum average rate guarantees.

Finally, Fig. 3 depicts the Lagrange multiplier evolutions of the proposed iterations with and without minimum average rate constraints. It is clear that the iterations converge fast. Note that here only "stochastic" convergence can be achieved; i.e., the Lagrange multipliers only hover within a small neighborhood (proportional to stepsize β) around the optimal values.

VI. Conclusion

We have derived the optimal rate and power allocation policy that maximizes a general utility function of average user rates

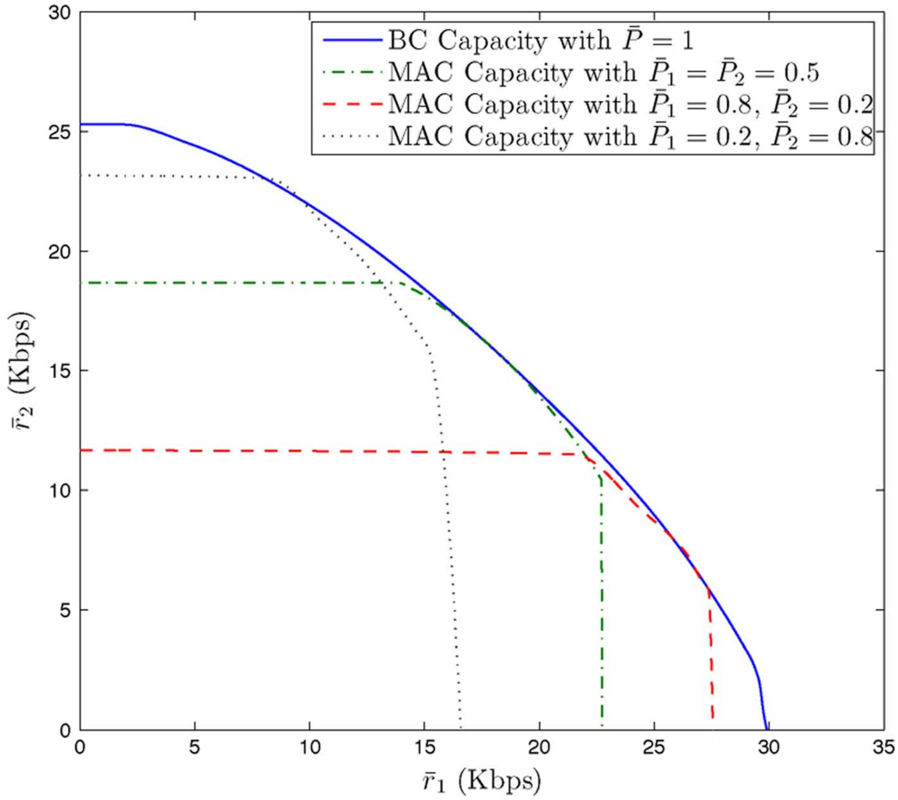

Fig. 1. Ergodic capacity regions for two-user fading broadcast and multiple-access channels.

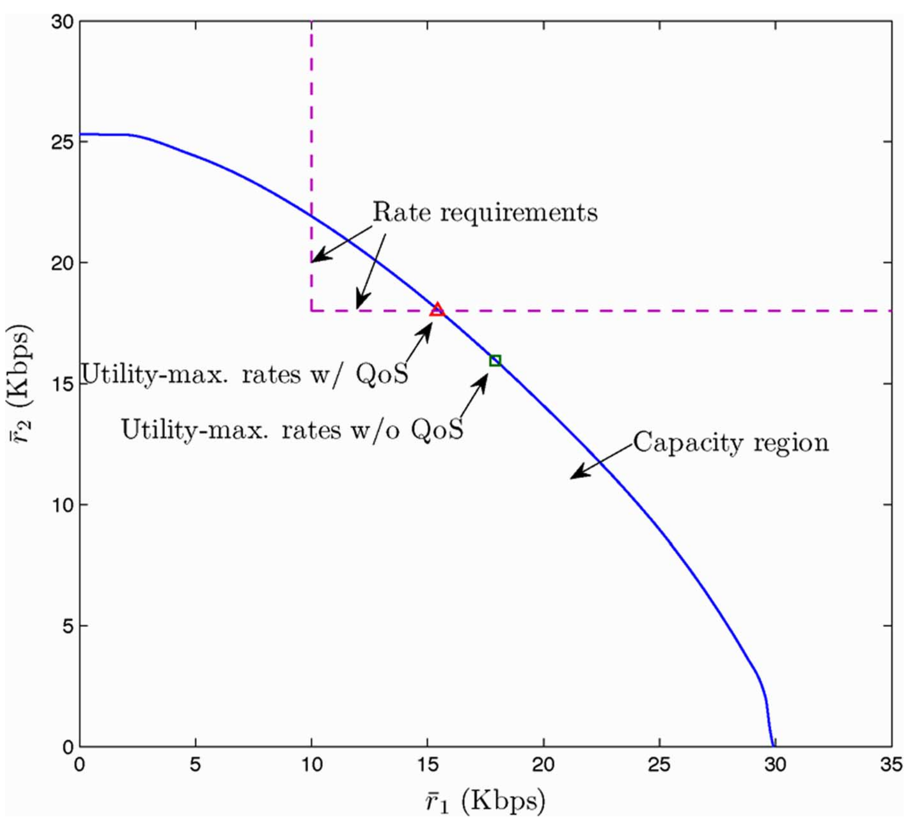

Fig. 2. Capacity region (benchmark) and achieved rates with stochastic dual-gradient decent schemes with or without QoS guarantees.

with guarantees on minimum average rates in a fading multiple-access or broadcast channel. Specifically, we developed a powerful dual-based stochastic gradient descent approach with fast convergence and low complexity per iteration. Relying on

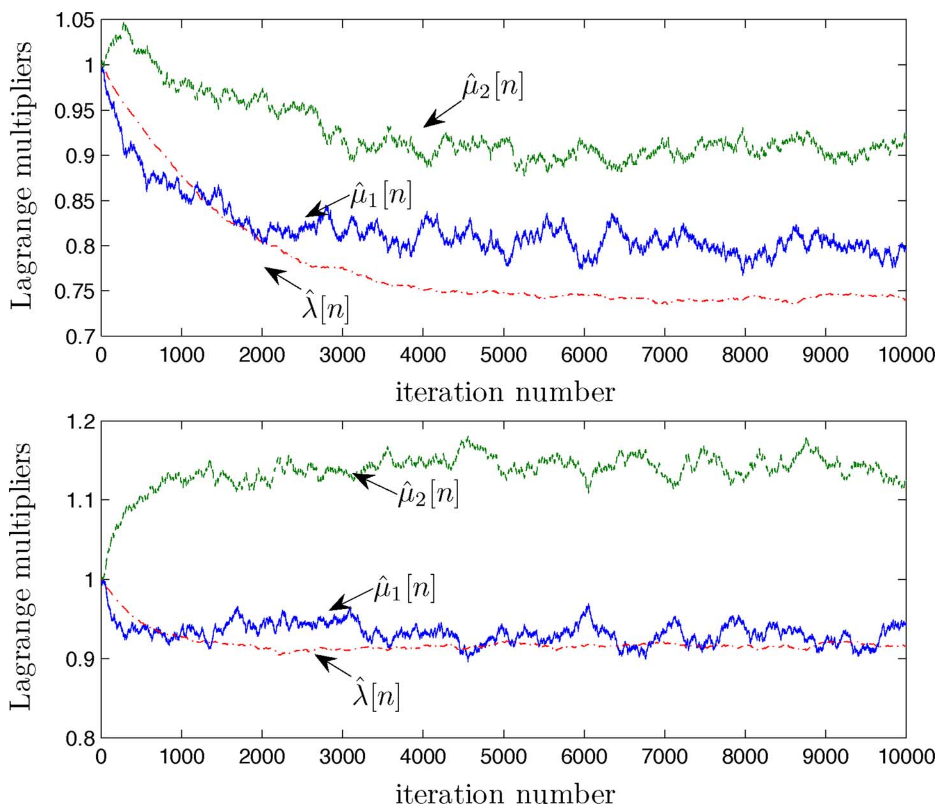

Fig. 3. Lagrange multiplier evolutions of the proposed stochastic scheme, (top) when there are no minimum average rate requirements and (bottom) when there are minimum average rate requirements $\bar{R}_1 = 10$ Kbps and $\bar{R}_2 = 18$ Kbps.

the stochastic averaging tools, it is shown that the proposed approach is capable of iteratively finding the optimal resource allocation with guarantees on the minimum average user rates, even when the fading channel distribution is unknown *a priori*. The proposed stochastic resource allocation schemes have valuable applications for theoretic studies and practical network designs.

Appendix

A. Proof of Lemma 1

Since the utility function U is concave and $\bar{\boldsymbol{R}} \in \mathbf{int}(\mathcal{C}(\bar{\boldsymbol{P}}))$ by (A1), the problem in (8) is a strictly feasible convex optimization problem if the feasible set defined by the constraints $\boldsymbol{x} = \bar{\boldsymbol{r}}, \boldsymbol{x} \geq \bar{\boldsymbol{R}}$, $\mathbb{E}_{\boldsymbol{h}}[\boldsymbol{p}(\boldsymbol{h})] = \bar{\boldsymbol{P}}$, and $\bar{\boldsymbol{r}} \in \mathcal{C}_f(\boldsymbol{p}(\cdot))$ is a convex set of the optimization variables $(\boldsymbol{x}, \bar{\boldsymbol{r}}, \boldsymbol{p}(\cdot))$. Because the first three constraints are linear constraints, it is clear that the feasible set of (8) is convex if the set, say $\mathcal{A}$, defined by $\bar{\boldsymbol{r}} \in \mathcal{C}_f(\boldsymbol{p}(\cdot))$ is a convex set of $(\bar{\boldsymbol{r}}, \boldsymbol{p}(\cdot))$.

To see this, we assume that we have two doublets $(\bar{\boldsymbol{r}}^{(a)}, \boldsymbol{p}^{(a)}(\cdot)) \in \mathcal{A}$ and $(\bar{\boldsymbol{r}}^{(b)}, \boldsymbol{p}^{(b)}(\cdot)) \in \mathcal{A}$. From the definition of $\mathcal{C}_f(\boldsymbol{p}(\cdot))$ in (8), this simply implies that

$$\sum_{j\in S} \bar{r}_j^{(a)} \leq \mathbb{E}_{\boldsymbol{h}}\left[\log\left(1 + \sum_{j\in S} h_j p_j^{(a)}(\boldsymbol{h})\right)\right], \forall S \subset \mathcal{J}$$
$$\sum_{j\in S} \bar{r}_j^{(b)} \leq \mathbb{E}_{\boldsymbol{h}}\left[\log\left(1 + \sum_{j\in S} h_j p_j^{(b)}(\boldsymbol{h})\right)\right], \forall S \subset \mathcal{J}.$$

Now consider a convex combination $(\bar{\boldsymbol{r}}^{(c)}, \boldsymbol{p}^{(c)}(\cdot)) := (\alpha\bar{\boldsymbol{r}}^{(a)} + (1-\alpha)\bar{\boldsymbol{r}}^{(b)}, \alpha\boldsymbol{p}^{(a)}(\cdot) + (1-\alpha)\boldsymbol{p}^{(b)}(\cdot))$, for any $\alpha \in [0,1]$. It is clear that

$$\begin{aligned}\sum_{j\in S} \bar{r}_j^{(c)} &= \alpha \sum_{j\in S} \bar{r}_j^{(a)} + (1-\alpha)\sum_{j\in S} \bar{r}_j^{(b)} \\ &\leq \mathbb{E}_{\boldsymbol{h}}\left[\alpha \log\left(1 + \sum_{j\in S} h_j p_j^{(a)}(\boldsymbol{h})\right)\right] \\ &\quad + \mathbb{E}_{\boldsymbol{h}}\left[(1-\alpha)\log\left(1 + \sum_{j\in S} h_j p_j^{(b)}(\boldsymbol{h})\right)\right] \\ &\leq \mathbb{E}_{\boldsymbol{h}}\left[\log\left(1 + \sum_{j\in S} h_j(\alpha p_j^{(a)}(\boldsymbol{h}) \right.\right. \\ &\qquad \left.\left. + (1-\alpha)p_j^{(b)}(\boldsymbol{h}))\right)\right] \\ &= \mathbb{E}_{\boldsymbol{h}}\left[\log(1 + \sum_{j\in S} h_j p_j^{(c)}(\boldsymbol{h}))\right], \quad \forall S \subset \mathcal{J} \end{aligned} \tag{24}$$

where the last inequality is due to the concavity of log function. From (24), it follows that $\bar{\boldsymbol{r}}^{(c)} \in \mathcal{C}_f(\boldsymbol{p}^{(c)}(\cdot))$, and, thus, $(\bar{\boldsymbol{r}}^{(c)}, \boldsymbol{p}^{(c)}(\cdot)) \in \mathcal{A}$. This, in turn, implies that $\mathcal{A}$ is a convex set of $(\bar{\boldsymbol{r}}, \boldsymbol{p}(\cdot))$; and the proof is complete.

B. Proof of Lemma 3

To prove the desired locking between primary and averaged trajectories, it suffices to verify that [14, Theorem 9.1, eqns. (9.2A1)–(9.2A5)] are satisfied. These conditions with our notational conventions are as follows.

(**C1**) The expectation of $\boldsymbol{g}(\boldsymbol{\Lambda}, \boldsymbol{h}[n])$ in the primary system (19), i.e., $\mathbb{E}_{\boldsymbol{h}[n]}[\boldsymbol{g}(\boldsymbol{\Lambda}, \boldsymbol{h}[n])]$, is time-invariant.

(**C2**) For the averaged system (20), $\bar{\boldsymbol{g}}(\boldsymbol{\Lambda})$ is bounded in a ball; i.e., for $\|\boldsymbol{\Lambda}\| \leq B_\Lambda$, it holds that $\|\bar{\boldsymbol{g}}(\boldsymbol{\Lambda})\| \leq B_g$.

(**C3**) Initialization $\boldsymbol{\Lambda}[0]$ is small enough so that iterates of the averaged system remain bounded; i.e., $\|\boldsymbol{\Lambda}[n]\| \leq B_\Lambda$ for $n \in [1, T/\beta]$.

(**C4**) Function $\boldsymbol{g}(\boldsymbol{\Lambda}, \boldsymbol{h}[n])$ obeys a stochastic Lipschitz condition; i.e.,$\|\boldsymbol{g}(\boldsymbol{\Lambda}, \boldsymbol{h}[n]) - \boldsymbol{g}(\boldsymbol{\Lambda}', \boldsymbol{h}[n])\| \leq L_g[n]\|\boldsymbol{\Lambda} - \boldsymbol{\Lambda}'\|, \forall \|\boldsymbol{\Lambda}\|, \|\boldsymbol{\Lambda}'\|$, where $L_g[n]$ is a random sequence obeying $N^{-1}\sum_{n=1}^{N} L_g[n] \rightarrow L_g$ w.p. 1, as $N \rightarrow \infty$.

(**C5**) The deviation $\Delta(N, \boldsymbol{\Lambda}) := \sum_{n=1}^{N}(\boldsymbol{g}(\boldsymbol{\Lambda}, \boldsymbol{h}[n]) - \bar{\boldsymbol{g}}(\boldsymbol{\Lambda}))$, is also stochastic Lipschitz; i.e., (a) $\|\Delta(N, \boldsymbol{\Lambda})\| \leq B_\Delta[N]$; and (b) $\|\Delta(N, \boldsymbol{\Lambda}) - \Delta(N, \boldsymbol{\Lambda}')\| \leq L_\Delta[N]\|\boldsymbol{\Lambda} - \boldsymbol{\Lambda}'\|, \forall \|\boldsymbol{\Lambda}\|, \|\boldsymbol{\Lambda}'\|$, where as $N \rightarrow \infty, B_\Delta[N]/N \rightarrow 0$ and $L_\Delta[N]/N \rightarrow 0$ w.p. 1.

Conditions (C1)–(C3) clearly hold for the given stationary and ergodic fading process, whereas (C4) and (C5) can be shown using similar arguments in the proof of [18, Lemma 5], provided that the fading channel has a continuous cdf. With (C1)–(C5) verified, the lemma follows from [14, Theorem 9.1].

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