Joint Sensing-Channel Selection and Power Control for Cognitive Radios

Xin Wang, *Senior Member, IEEE*

***Abstract*—We consider joint optimization for sensing-channel selection and ensuing power control problem with cognitive radios over time-varying fading channels. It is shown that this joint design can be judiciously formulated as a convex optimization problem. Optimal joint sensing-channel selection and power control scheme is then derived in closed-form under the constraints of average power budget and maximum allowable probability of collisions with the primary communications. In addition, we develop a stochastic optimization algorithm that can operate without a-priori knowledge of the fading channel statistics. It is rigourously established that the proposed stochastic scheme is capable of dynamically learning the intended wireless channels on-the-fly to approach the optimal strategy almost surely. Numerous results are also provided to evaluate the proposed schemes for cognitive transmissions over block fading channels.**

***Index Terms*—Cognitive radios, sensing-channel selection, power control, stochastic optimization.**

I. Introduction

THE exponentially increasing number of wireless communication systems and services has created a very crowded spectrum over the past decade. To accommodate the emerging wireless products and services, it is the Federal Communications Commission's (FCC) vision that cognitive radios will be a promising solution to the spectrum shortage [1]. For this reason cognitive radios have attracted growing research interests; see [2], [3] and a bulk of the references therein.

A critical application of cognitive radios is based on the idea of opportunistic communication, where the secondary cognitive users intelligently monitor the radio spectrum and opportunistically transmit over the "spectrum holes" that are not in use by the primary (licensed) users. To enable such cognitive radios, one key issue is to design channel-sensing schemes that can efficiently and accurately determine the presence/absence of the primary users. Owing to its importance, channel sensing is one of the better established fields in cognitive radios research. With the advanced sensing capabilities provided by the recently developed schemes [4], [5], [6], it is anticipated that sensing techniques are not a major barrier to the success of cognitive radios.

Manuscript received May 21, 2010; revised August 28, 2010; accepted November 16, 2010. The associate editor coordinating the review of this paper and approving it for publication was Y. Sanada.

The work in this paper was supported by the U.S. National Science Foundation grants CNS-0831671. The U. S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation thereon. The material in this paper was presented in part at IEEE Globecom, Miami, FL, Dec. 2010.

X. Wang is with the Dept. of Computer & Electrical Engineering and Computer Science, Florida Atlantic University, 777 Glades Road, Boca Raton, FL 33431 (e-mail: xin.wang@fau.edu).

Digital Object Identifier 10.1109/TWC.2011.121410.100874

With a given sensing scheme, a remaining problem is how to adaptively select the set of channels to be sensed provided that a cognitive radio capitalizes on opportunistic transmissions over a large spectrum band. This problem of sensing-channel selection was investigated in [7]. Another relevant problem is the optimal power allocation for the cognitive radios over the available channels that are sensed not in use by the primary users; the topic was explored in e.g., [8], [9]. For cognitive communications, these two problems are actually tightly coupled. For this reason, a recent work considered the joint design of sensing-channel selection and power control scheme [10]. Assuming static wireless channels and a perfect sensing scheme, this joint optimization was formulated as an integer program (that is NP-hard). Computationally tractable (yet suboptimal) algorithms were then developed to approximately solve the problem at hand.

Different from [10], this paper considers joint optimization of sensing-channel selection and power control scheme for cognitive radios over *time-varying* fading channels, and the sensing errors are also taken into account. For such a more general case, it is shown that the problem of interest can be formulated as a convex optimization. The optimal joint sensing selection and power control scheme is then derived under the constraints of average power budget and maximum allowable probability of collisions with the primary communications. When a-priori knowledge of fading cumulative distribution function (cdf) is available, it is shown that the optimal scheme can be obtained using Lagrange dual-based gradient iterations with fast convergence and linear complexity per iteration. More interestingly, we rely on the stochastic optimization tools to develop an on-line algorithm that can operate even without a-priori knowledge of the fading cdf. It is rigorously established that the proposed stochastic scheme is capable of dynamically learning the required channel statistics on-the-fly to approach the optimal strategy almost surely.

The rest of this paper is organized as follows. Section II describes the network model and problem formulation. The optimal joint sensing-channel selection and power control solution is derived in Section III, whereas the corresponding stochastic optimization scheme is developed in Section IV. Numerical results are provided in Section V, followed by the conclusions.

II. Network Model and Problem Formulation

For simplicity we consider only one cognitive user who performs opportunistic transmissions over M parallel frequency channels with equal bandwidth $B = 1$. These M channels are licensed to primary users and the secondary cognitive user can access a channel $m \in \{1, \ldots, M\}$ only when it is not utilized by the primary users. To this end, the cognitive user senses the channels every time interval (slot) of T seconds. Provided that M is a fairly large number, the cognitive user only senses a subset of $L < M$ channels per slot due to physical limitations. The selection of sensing-channels depends on primary user activities and the performance of the sensing scheme.

Let $P_{U,m}$ denote the probability that the channel m is in use by its primary users per slot, while $P_{D,m}$ and $P_{F,m}$ denote the detection probability and false alarm probability with the given sensing scheme for primary user detection. The knowledge of $P_{U,m}$ for the spectrum of interest can be obtained through the recent measurements and studies conducted by the FCC and industry; and $P_{D,m}$ and $P_{F,m}$ can be pre-determined for the selected sensing scheme given the sufficient statistics for the channels between primary users and cognitive user. Therefore, we assume that a-priori knowledge of $P_{U,m}$, $P_{D,m}$ and $P_{F,m}$ is available. Suppose neither primary nor cognitive users can receive packets upon collision of their transmissions. Clearly a successful cognitive transmission can be carried out at channel m if the channel is not in use by primary users and false alarm does not occur in this case; the probability for this event is:

$$q_m := (1 - P_{U,m})(1 - P_{F,m}), \qquad \forall m.$$

Taking into account that the cognitive user also transmits when the channel is used by primary users but a miss detection occurs, the total probability that it transmits at channel m is:

$$\check{q}_m := q_m + P_{U,m}(1 - P_{D,m}).$$

On the other hand, the probability that a cognitive transmission collides with the primary communication given that the latter occurs is clearly:

$$\tilde{q}_m := 1 - P_{D,m}, \qquad \forall m.$$

The cognitive transmissions at the channels $m = 1, \ldots, M$ are subject to random fading. Assume without loss of generality that the additive Gaussian noise at the cognitive receiver has unit variance. Let γ_m denote the fading channel power gain for cognitive transmissions over channel m. Suppose that the sensing period T is chosen to be greater than the channel coherence time. Then assuming a block fading channel model, we let $\boldsymbol{\gamma} := [\gamma_1, \ldots, \gamma_M]^T$ collect the channel gains which remain invariant over blocks (coherent time slots) but are allowed to vary across successive blocks according to a stationary and ergodic random process with a certain joint cdf.

To explore the fundamental limit of cognitive radios, we assume that the channel state information $\boldsymbol{\gamma}[n]$ is available at the cognitive user (e.g. through training) per slot n. Adapted to $\boldsymbol{\gamma}$, let $s_m(\boldsymbol{\gamma})$ denote the sensing-channel selection to optimize over. Given the maximum number L of the channels to be sensed, we enforce the constraints: $0 \leq s_m(\boldsymbol{\gamma}) \leq 1$, $\forall m$ and $\sum_{m=1}^{M} s_m(\boldsymbol{\gamma}) \leq L$, $\forall \boldsymbol{\gamma}$. Notice that different from [10] where $s_m(\boldsymbol{\gamma})$ can only take value 0 or 1, we allow $s_m(\boldsymbol{\gamma})$ to be a real number in $[0, 1]$. In other words, here $s_m(\boldsymbol{\gamma})$ stands for the probability that the channel m is selected to be sensed, with $s_m(\boldsymbol{\gamma}) = 0$ and $s_m(\boldsymbol{\gamma}) = 1$ as the special cases for which the channel is not sensed or sensed deterministically. Therefore, instead of limiting the sensing-channel selection to a deterministic one, we allow at the outset a probabilistic selection. Allowing $s_m(\boldsymbol{\gamma}) \in [0, 1]$ avoids the NP-hard integer program in [10]. In addition, as will be shown in the sequel, the resultant optimal selection policy can be easily implemented in a way that ensures the physical constraint of at most L channels being sensed per $\boldsymbol{\gamma}$.

Let $p_m(\boldsymbol{\gamma})$ denote the transmit-power for the opportunistic cognitive communication over channel m at fading realization $\boldsymbol{\gamma}$. With $p_{\max}$ being a nature power bound, we have $0 \leq p_m(\boldsymbol{\gamma}) \leq p_{\max}$ per $\boldsymbol{\gamma}$. Define the sensing selection policy $\boldsymbol{s}(\boldsymbol{\gamma}) := [s_1(\boldsymbol{\gamma}), \ldots, s_M(\boldsymbol{\gamma})]$ and power control policy $\boldsymbol{p}(\boldsymbol{\gamma}) := [p_1(\boldsymbol{\gamma}), \ldots, p_M(\boldsymbol{\gamma})]$. The joint policy $\{\boldsymbol{s}(\boldsymbol{\gamma}),\ \boldsymbol{p}(\boldsymbol{\gamma})\}$ then collects the optimization variables that we want to choose in order to maximize the throughput of the cognitive communications subject to (s. to) constraints; i.e., we wish to solve

$$\begin{aligned} \max_{(\boldsymbol{s}(\boldsymbol{\gamma}),\, \boldsymbol{p}(\boldsymbol{\gamma})) \in \mathcal{F}} \quad & \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M} s_m(\boldsymbol{\gamma}) q_m \log_2(1 + \gamma_m p_m(\boldsymbol{\gamma}))\Big] \\ \text{s. to} \quad & \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M} s_m(\boldsymbol{\gamma}) \check{q}_m p_m(\boldsymbol{\gamma})\Big] \leq \bar{P} \\ & \mathbb{E}_{\boldsymbol{\gamma}}\Big[s_m(\boldsymbol{\gamma}) \tilde{q}_m\Big] \leq \bar{\rho}_m, \quad \forall m \end{aligned} \tag{1}$$

where $\mathbb{E}_{\boldsymbol{\gamma}}$ denotes the expectation over all fading realizations, $\bar{P}$ is the average power budget for cognitive transmissions, $\bar{\rho}_m$ is the maximum allowable probability of collisions with the primary communications at channel m, and we define a set

$$\begin{aligned} \mathcal{F} := \{\boldsymbol{s}, \boldsymbol{p} \mid \sum_{m=1}^{M} s_m(\boldsymbol{\gamma}) \leq L,\ 0 \leq s_m(\boldsymbol{\gamma}) \leq 1, \\ 0 \leq p_m(\boldsymbol{\gamma}) \leq p_{\max},\ \forall m,\ \forall \boldsymbol{\gamma}\} \end{aligned}$$

to collect the extra constraints for $\{\boldsymbol{s}(\boldsymbol{\gamma}),\ \boldsymbol{p}(\boldsymbol{\gamma})\}$. Here we assume the cognitive user can support a transmission rate up to Shannon's limit, and the primary user occupancy per channel m is independently and identically distributed (i.i.d.) every slot.

Since its objective function is not a concave function and the average power constraint is not a convex constraint, (1) is not a convex optimization problem. However, we next show that it can be reformulated as a convex optimization with a variable change. Define a new variable $\tilde{p}_m(\boldsymbol{\gamma}) := s_m(\boldsymbol{\gamma}) p_m(\boldsymbol{\gamma})$, and a function

$$r_m(s_m, \tilde{p}_m) := \begin{cases} s_m \log_2(1 + \gamma_m \tilde{p}_m / s_m), & s_m > 0, \\ 0, & s_m = 0. \end{cases} \tag{2}$$

We can reformulate (1) into

$$\begin{aligned} \max_{(\boldsymbol{s}(\boldsymbol{\gamma}),\, \tilde{\boldsymbol{p}}(\boldsymbol{\gamma})) \in \mathcal{F}'} \quad & \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M} q_m r_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma}))\Big] \\ \text{s. to} \quad & \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M} \check{q}_m \tilde{p}_m(\boldsymbol{\gamma})\Big] \leq \bar{P} \\ & \mathbb{E}_{\boldsymbol{\gamma}}\Big[\tilde{q}_m s_m(\boldsymbol{\gamma})\Big] \leq \bar{\rho}_m, \quad \forall m \end{aligned} \tag{3}$$

where the set

$$\mathcal{F}' := \{\boldsymbol{s}, \tilde{\boldsymbol{p}} \mid \sum_{m=1}^{M} s_m(\boldsymbol{\gamma}) \le L,\ 0 \le s_m(\boldsymbol{\gamma}) \le 1,\ 0 \le \tilde{p}_m(\boldsymbol{\gamma}) \le s_m(\boldsymbol{\gamma}) p_{\max},\ \forall m,\ \forall \boldsymbol{\gamma}\}.$$

It can be shown that $r_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma}))$ is a jointly concave function of $s_m(\boldsymbol{\gamma})$ and $\tilde{p}_m(\boldsymbol{\gamma})$ [11], [12]. This together with all the constraints in (3) becoming linear constraints implies that (3) is a convex optimization problem.

Notice that our formulation is significantly different from [10] since we consider joint optimization of sensing-channel selection and power control scheme for cognitive radios over *time-varying* fading channels and the sensing errors are taken into account through the constraints on allowable probability of collisions with the primary communications. Furthermore, we judiciously reformulate the problem into a convex optimization that admits efficient solvers.

We remake that our formulation can be modified/generelized to address other cognitive radio scenarios. For instance, instead of collision probability, in some cases primary users may more care about the average interference levels it receives. It is then required that the average aggregated interference for a primary user cannot exceed an "interference temperature" [2]. Such a requirement imposes the constraints: $\mathbb{E}_{\boldsymbol{\gamma}}[\tilde{q}_m s_m(\boldsymbol{\gamma}) p_m(\boldsymbol{\gamma})] \le \bar{v}_m,\ \forall m$, where $\bar{v}_m$ denotes the required interference temperature at channel m. Using these constraints to replace the maximum allowable probability constraints actually does not prevent reformulating the problem (1) into a convex optimization. Upon defining $\tilde{p}_m(\boldsymbol{\gamma}) := s_m(\boldsymbol{\gamma}) p_m(\boldsymbol{\gamma})$, the new constraints become the linear ones $\mathbb{E}_{\boldsymbol{\gamma}}[\tilde{q}_m \tilde{p}_m(\boldsymbol{\gamma})] \le \bar{v}_m$ after change of variables. Again a similar convex program is obtained as with (3). The ensuing proposed approach then readily applies to derive the optimal solution in this scenario.

III. Joint Optimization of Sensing-Channel Selection and Power Control

We next solve the optimization (1) using a Lagrange dual based approach. Let λ denote the Lagrange multiplier associated with the constraint $\mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{m=1}^{M} \check{q}_m \tilde{p}_m(\boldsymbol{\gamma})\right] \le \bar{P}$, and $\boldsymbol{\mu} := [\mu_1, ..., \mu_M]^T$ collect the Lagrange multipliers associated with the constraints $\mathbb{E}_{\boldsymbol{\gamma}}\left[\tilde{q}_m s_m(\boldsymbol{\gamma})\right] \le \bar{\rho}_m,\ \forall m$. Using the convenient notations $\boldsymbol{X} := \{\boldsymbol{s}(\boldsymbol{\gamma}),\ \tilde{\boldsymbol{p}}(\boldsymbol{\gamma}),\ \forall \boldsymbol{\gamma}\}$ and $\boldsymbol{\Lambda} := \{\lambda,\ \boldsymbol{\mu}\}$, the Lagrangian of (3) is

$$\begin{aligned} L(\boldsymbol{X}, \boldsymbol{\Lambda}) &= \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{m=1}^{M} q_m r_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma}))\right] \\ &\quad -\lambda\left(\mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{m=1}^{M} \check{q}_m \tilde{p}_m(\boldsymbol{\gamma})\right] - \bar{P}\right) \\ &\quad - \sum_{m=1}^{M} \mu_m \left(\mathbb{E}_{\boldsymbol{\gamma}}\left[\tilde{q}_m s_m(\boldsymbol{\gamma})\right] - \bar{\rho}_m\right) \\ &= \lambda \bar{P} + \sum_{m=1}^{M} \mu_m \bar{\rho}_m + \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{m=1}^{M} \{q_m r_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma})) \right. \\ &\quad \left. -\lambda \check{q}_m \tilde{p}_m(\boldsymbol{\gamma}) - \mu_m \tilde{q}_m s_m(\boldsymbol{\gamma})\}\right] \end{aligned} \tag{4}$$

The dual problem of (3) is then

$$\min_{\boldsymbol{\Lambda} \ge 0} D(\boldsymbol{\Lambda}), \quad \text{where} \quad D(\boldsymbol{\Lambda}) = \max_{\boldsymbol{X} \in \mathcal{F}'} L(\boldsymbol{X}, \boldsymbol{\Lambda}). \tag{5}$$

Due to the convexity of (3), there is no duality gap between primal (3) and its dual problem (5). Therefore the solution of (3) can be obtained by solving (5) [13].

As with almost all the practical wireless propagation (e.g., Rayleigh, Rice and Nakagami) models, we assume at the moment that the fading $\boldsymbol{\gamma}$ has a continuous joint cdf. To solve (5), we first need to find the optimal $\boldsymbol{X}^*(\boldsymbol{\Lambda})$ that maximizes $L(\boldsymbol{X}, \boldsymbol{\Lambda})$ through solving [cf. (4)]:

$$\max_{\boldsymbol{X} \in \mathcal{F}'} \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M} \{q_m r_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma})) - \lambda \check{q}_m \tilde{p}_m(\boldsymbol{\gamma}) - \mu_m \tilde{q}_m s_m(\boldsymbol{\gamma})\}\Big] \tag{6}$$

It is easy to see that solving (6) is equivalent to solving: $\forall \boldsymbol{\gamma}$,

$$\max_{(\boldsymbol{s}(\boldsymbol{\gamma}), \tilde{\boldsymbol{p}}(\boldsymbol{\gamma})) \in \mathcal{F}'} \sum_{m=1}^{M} \{q_m r_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma})) - \lambda \check{q}_m \tilde{p}_m(\boldsymbol{\gamma}) - \mu_m \tilde{q}_m s_m(\boldsymbol{\gamma})\}$$

For a given $\boldsymbol{\Lambda}$, define

$$p_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) := \left[\frac{q_m}{\lambda \check{q}_m \ln 2} - \frac{1}{\gamma_m}\right]_0^{p_{\max}} \tag{7}$$

where $[\cdot]_0^{p_{\max}}$ denotes the projection into the interval $[0, p_{\max}]$. Based on $p_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$, define subsequently

$$\phi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) := q_m \log_2(1 + \gamma_m p_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})) - \lambda \check{q}_m p_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}), \tag{8}$$

$$\varphi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) := \phi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) - \mu_m \tilde{q}_m. \tag{9}$$

Sorting all the channel indices in the decreasing order of $\varphi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$, we can then obtain a permutation $\boldsymbol{\pi} := [\pi(1), \ldots, \pi(M)]^T$ with $\varphi_{\pi(k)}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) \ge \varphi_{\pi(k+1)}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}),\ \forall k$.

Let $L' := \arg\ \max_k\ \{\varphi_{\pi(k)}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) > 0\}$, i.e., it is the maximum index in $\boldsymbol{\pi}$ with a positive $\varphi_{\pi(k)}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ ($L' = 0$ if the set $\{\varphi_{\pi(k)}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) > 0\}$ is empty); and let $l = \min\{L', L\}$. For convenience, we omit the dependence of $\boldsymbol{\pi}$, L' and l on $\boldsymbol{\gamma}$ and $\boldsymbol{\Lambda}$. We can then show the following lemma (see the proof in Appendix A):

Lemma 1: *For ergodic fading channels with continuous cdf, the almost surely unique solution of (6) is given by:* $\forall \boldsymbol{\gamma}$,

$$\begin{cases} s_{\pi(k)}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) = 1, \quad \tilde{p}_{\pi(k)}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) = p_{\pi(k)}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}), & 1 \le k \le l; \\ s_{\pi(k)}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) = \tilde{p}_{\pi(k)}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) = 0, & k > l. \end{cases} \tag{10}$$

Lemma 1 asserts that a greedy sensing-channel selection per fading state $\boldsymbol{\gamma}$ along with a water-filling power allocation across $\boldsymbol{\gamma}$ realizations constitutes with probability one (w.p. 1) the optimal solution of (6), provided that the distribution function of the random fading channel is continuous. Indeed $\varphi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ in (9) can be interpreted as a *net-reward* (rate reward minus power and collision costs) for cognitive transmission at channel m over $\boldsymbol{\gamma}$. In order to maximize the total net-reward, the optimal policy amounts to a two-step greedy solution. In the first step, transmit-power p_m^* is allocated per channel across γ_m following a water-filling principle (7). The value $\varphi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ then represents the largest potential of net-reward for cognitive transmission over channel m at the fading realization $\boldsymbol{\gamma}$. In the second step, the l channels with the largest $\varphi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ are chosen to be sensed for maximum net-reward per $\boldsymbol{\gamma}$. Notice that a channel m is worth being sensed only when its net-reward $\varphi_m^*(\boldsymbol{\gamma};\boldsymbol{\Lambda})$ exceeds zero – the natural net-reward for no sensing and no transmission (i.e., $s_m = \tilde{p}_m = 0$). Therefore the cognitive user may choose to sense $l < L$ channels when there do not exist enough channels worthy of sensing at a $\boldsymbol{\gamma}$.

Using the notation $\boldsymbol{X}$ and $\boldsymbol{\Lambda}$, arrange the constraints in (3) into a compact form: $\boldsymbol{h}(\boldsymbol{X}) \geq \boldsymbol{0}$. Then with $\boldsymbol{X}^*(\boldsymbol{\Lambda}) := \{\boldsymbol{s}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}),\ \tilde{\boldsymbol{p}}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}),\ \forall\boldsymbol{\gamma}\}$ provided by Lemma 1 for a given $\boldsymbol{\Lambda}$, it can be shown that $\boldsymbol{h}(\boldsymbol{X}^*(\boldsymbol{\Lambda}))$ is a (sub-)gradient of the dual function $D(\boldsymbol{\Lambda})$ [14]. Therefore, the dual problem (5) can be solved through the following (sub-)gradient descent iteration

$$\boldsymbol{\Lambda}(t+1) = \Big[\boldsymbol{\Lambda}(t) - \beta\boldsymbol{h}(\boldsymbol{X}^*(\boldsymbol{\Lambda}(t)))\Big]^+; \tag{11}$$

specifically, we have

$$\lambda(t+1) = \Big[\lambda(t) + \beta\Big(\mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M}\check{q}_m\tilde{p}_m^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}(t))\Big] - \bar{P}\Big)\Big]^+$$
$$\mu_m(t+1) = [\mu_m(t) + \beta(\mathbb{E}_{\boldsymbol{\gamma}}[\tilde{q}_m s_m^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}(t))] - \bar{\rho}_m)]^+,\ \forall m \tag{12}$$

where β is a small stepsize, t is the iteration index, and $[x]^+ := \max(0,x)$. Convergence of the iteration (12) to the optimal $\boldsymbol{\Lambda}^* := \{\lambda^*,\ \boldsymbol{\mu}^*\}$ for (5) is guaranteed from any initial $\boldsymbol{\Lambda}(0) \geq \boldsymbol{0}$ [14].

Having obtained the optimal $\boldsymbol{\Lambda}^*$ for (5), the zero duality gap between the primal (3) and the dual (5) implies that $\boldsymbol{X}(\boldsymbol{\Lambda}^*)$, namely, $\{\boldsymbol{s}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*),\ \tilde{\boldsymbol{p}}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*),\ \forall\boldsymbol{\gamma}\}$, yields also the optimal solution to (3). Equivalently, upon defining

$$\boldsymbol{p}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*) := \{s_m^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*)\tilde{p}_m^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*),\ \forall m\},$$

we have $\{\boldsymbol{s}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*),\ \boldsymbol{p}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*),\ \forall\boldsymbol{\gamma}\}$ as the solution to the original problem (1) of interest.

Summarizing, we have established the following result:

Proposition 1: *For ergodic fading channels with continuous cdf, the almost surely optimal solution for (1) is given by* $\{\boldsymbol{s}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*),\ \boldsymbol{p}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*),\ \forall\boldsymbol{\gamma}\}$ *specified in (10), where* $\boldsymbol{\Lambda}^*$ *can be obtained from (12) with any initial* $\boldsymbol{\Lambda}(0) \geq \boldsymbol{0}$.

Interestingly, although a probabilistic sensing-channel selection was allowed at the outset, the almost surely optimal solution in Proposition 1 dictates a deterministic selection. This is because the event of $\{\varphi_m^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*) = \varphi_{m'}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*) > 0,\ m \neq m'\}$ has zero Lebesgue measure provided that $\boldsymbol{\gamma}$ has a continuous cdf. In other words, almost surely we do not have any ties in $\{\varphi_m^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*),\ \forall m\}$ and the joint sensing selection and power control policy $\{\boldsymbol{s}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*),\ \boldsymbol{p}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*)\}$ is unique; hence, it is almost surely optimal for all wireless channel models including Rayleigh, Rice and Nakagami that indeed have continuous cdfs. Also note that the optimal policy indicates that the cognitive radio can sense less than L channels or even not sense any channels at all when the quality of channels are low and/or the power and collision prices are very high. For instance, when all channels experience deep fades, i.e., $\gamma_m \leq (\check{q}_m/q_m)\lambda^*\ln 2,\ \forall m$, we have $\varphi_m^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*) = -\mu_m^*\tilde{q}_m \leq 0,\ \forall m$; hence no channel should be sensed at this fading state.

It is worth mentioning that other convex programming methods e.g., the interior point methods, can be also employed to solve the formulated problem (3). Here we adopt a Lagrange dual based approach since it allows derivation of the jointly optimal sensing selection and power control policy in closed-form as shown in Lemma 1. Such a solution reveals the specific structure of the optimal sensing and power control for cognitive radios, and thus contributes useful insight to the field.

A. Fading with Discontinuous CDF

Proposition 1 establishes that a deterministic sensing-channel selection is almost surely optimal for cognitive transmissions when fading channels have continuous distributions. This does not hold for random channels with discrete distributions because now the event of having ties in $\{\varphi_m^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*),\ \ \forall m\}$, if exists, will have non-zero measure. Note that the discretely distributed random channels subsume the static (i.e., deterministic) channels in [10] as a special case.

In these cases suppose first that the optimal $\boldsymbol{\Lambda}^*$ is known. Then we can define a fading region

$$\Gamma := \{\boldsymbol{\gamma} \mid \varphi_{\pi(L-l_1)}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*) = \cdots = \varphi_{\pi(L)}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*) = \cdots = \varphi_{\pi(L+l_2)}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*) > 0,\quad \text{with}\quad l_2 > 0\}$$

and Γ^c is the complementary set of Γ. It is easy to see that: (i) $\forall\boldsymbol{\gamma} \in \Gamma^c$, we have unique optimal policy specified by (10); and (ii) $\forall\boldsymbol{\gamma} \in \Gamma$, we may need to consider the optimal probabilistic sensing selection among the $l_1 + l_2 + 1$ channels $\{\pi(L - l_1), \cdots, \pi(L+l_2)\}$. Notice that the optimal transmit-power is always $p_m^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*)$ once a channel m is utilized for cognitive communications.

Recall that except for the possibly probabilistic sensing selection among the l_1+l_2+1 channels $\{\pi(L-l_1), \cdots, \pi(L+l_2)\}$ for $\boldsymbol{\gamma} \in \Gamma$, all other channel selections are deterministic. We can then define the average power and average collision probabilities incurred by the deterministic parts as

$$\begin{aligned}\bar{P}^{det} \ &:= \ \mathbb{E}_{\boldsymbol{\gamma}\in\Gamma^c}\left[\sum_{k=1}^{l}\check{q}_{\pi(k)}p_{\pi(k)}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*)\right] \\ &\quad +\mathbb{E}_{\boldsymbol{\gamma}\in\Gamma}\left[\sum_{k=1}^{L-l_1-1}\check{q}_{\pi(k)}p_{\pi(k)}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*)\right] \\ \bar{\rho}_m^{det} \ &:= \ \mathbb{E}_{\boldsymbol{\gamma}\in\Gamma^c}\left[\tilde{q}_m I_{m\in\{\pi(1),\ldots,\pi(l)\}}\right] \\ &\quad +\mathbb{E}_{\boldsymbol{\gamma}\in\Gamma}\left[\tilde{q}_m I_{m\in\{\pi(1),\ldots,\pi(L-l_1-1)\}}\right],\quad \forall m\end{aligned}$$

where I_x denotes the indicator function, i.e., $I_x = 1$ if x is true and $I_x = 0$ otherwise.

From the Karush-Kuhn-Tucker (KKT) optimality condition [13], it follows that the optimal probabilistic channel selection policy must satisfy:

$$\lambda^*(\mathbb{E}_{\boldsymbol{\gamma}\in\Gamma}[\sum_{k=L-l_1}^{L+l_2}\check{q}_{\pi(k)}s_{\pi(k)}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*)p_{\pi(k)}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*)] + \bar{P}^{det} - \bar{P}) = 0,$$
$$\mu_m^*(\mathbb{E}_{\boldsymbol{\gamma}\in\Gamma}[\tilde{q}_m I_{m\in\{\pi(L-l_1),\ldots,\pi(L+l_2)\}}s_m^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*)] + \bar{\rho}_m^{det} - \bar{\rho}_m) = 0,$$
$$0 \leq s_{\pi(k)}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*) \leq 1, \qquad \sum_{k=L-l_1}^{L+l_2}s_{\pi(k)}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*) = l_1 + 1,\quad \forall\boldsymbol{\gamma} \in \Gamma.$$

The unknowns in these linear equations are $\{s_{\pi(k)}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*),\quad k \in [L - l_1, L + l_2],\ \ \forall\boldsymbol{\gamma} \in \Gamma\}$ (note that the permutation $\boldsymbol{\pi}$ and the numbers l_1 and l_2 depend on $\boldsymbol{\gamma}$). For a discrete distribution, the random fading $\boldsymbol{\gamma}$

takes on a finite number of values; hence we have a finite number of unknowns. In fact typically the set Γ is empty or contains a small number of fading realizations, and thus the number of unknowns, if any, is small. Therefore the optimal probabilistic channel selection at $\gamma \in \Gamma$ can be easily found by solving a linear program with a constant objective function and the foregoing linear constraints via existing efficient algorithms (e.g., Matlab function linprog($\cdot$)). Once this is found, the optimal sensing selection and power control strategy is determined. We note that a similar approach is also adopted for optimum orthogonal multiple access over fading channels using quantized channel state information in an independently developed work [15].

To specify the optimal policy, a remaining problem is to obtain the optimal $\boldsymbol{\Lambda}^*$. Interestingly, this can still be done using the (sub-)gradient iteration (12). For a given $\boldsymbol{\Lambda}$, the permutation $\boldsymbol{\pi}$ is possibly not unique (due to the ties) for some $\boldsymbol{\gamma}$, and thus the solution to (6) is not unique provided that the random fading has a discontinuous cdf. However, it can be shown that $\boldsymbol{h}(\boldsymbol{X}^*(\boldsymbol{\Lambda}))$ for any given solution $\boldsymbol{X}^*(\boldsymbol{\Lambda})$ of (6) is a sub-gradient of the dual function $D(\boldsymbol{\Lambda})$. Therefore, the iteration (12) can be still run to obtain $\boldsymbol{\Lambda}^*$ if a vanishing stepsize e.g., $\beta_t = 1/(t+1)$ is adopted [14].

Another problem is practical implementation of the probabilistic channel selection. Recall that the physical limitation of cognitive user allows at most L channel to be sensed per time slot. To enforce this limitation, we can implement the following probabilistic selection among $l_1 + l_2 + 1$ channels $\{\pi(L-l_1), \ldots, \pi(L+l_2)\}$. Suppose that the derived probabilities of choosing individual channels are $\{\Pr(\pi(L-l_1)), \ldots, \Pr(\pi(L+l_2))\}$ and a set of l_1+1 channels needs to be selected per slot. Clearly we have $N_c := \frac{(l_1+l_2+1)!}{(l_1+1)!l_2!}$ possible sets of l_1+1 channels selected from a total of l_1+l_2+1 channels. Let x_i, $i = 1, \ldots, N_c$, denote the probability of selecting a specific set $\mathcal{S}_i$. Then we set the following l_1+l_2+1 equations for the N_c unknowns: $\sum_{i:\, \pi(k)\in\mathcal{S}_i} x_i = \Pr(\pi(k))$, $k = L-l_1, \ldots, L+l_2$. Again, solving a linear program with a constant objective function and the foregoing linear constraints can yield the desirable probabilities x_i, $i = 1, \ldots, N_c$; then in accordance with these probabilities a set of l_1+1 channels is randomly selected for sensing.[1] With channels $\{\pi(1), \ldots, \pi(L-l_1-1)\}$ deterministically selected and additional l_1+1 channels selected in the probabilistic manner, the number of sensing-channels obey the physical limitation of at most L channels being sensed per slot.

IV. Stochastic Optimization Scheme

In the previous section we showed that the optimal joint sensing selection and power control strategy can be efficiently obtained using the dual gradient iteration (12). To this end, we would need the knowledge of fading channel cdf since only with the latter can we evaluate the related expected values $\mathbb{E}_{\boldsymbol{\gamma}}$ per iteration. This is meaningful, especially for theoretic studies in e.g., [11]. However, practical mobile applications also motivate on-line schemes that can operate even without knowing fading cdf since this knowledge is seldom available in closed-form a priori in practice.

Interestingly, this can be achieved through a stochastic optimization paradigm [16], [17], [18]. Relying on this approach, a *stochastic gradient* iteration can be developed based on (12) to solve (3) without the fading cdf a priori. To this end, we consider dropping $\mathbb{E}_{\boldsymbol{\gamma}}$ from (12), and replacing the iteration index t with the (coherent time) slot index n, to devise the on-line iterations based on per slot fading realization $\boldsymbol{\gamma}[n]$:

$$\hat{\lambda}[n+1] = \Big[\hat{\lambda}[n] + \beta\Big(\sum_{m=1}^{M} \check{q}_m p_m^*(\boldsymbol{\gamma}[n]; \hat{\boldsymbol{\Lambda}}[n]) - \bar{P}\Big)\Big]^+$$
$$\hat{\mu}_m[n+1] = \Big[\hat{\mu}_m[n] + \beta\Big(\tilde{q}_m s_m^*(\boldsymbol{\gamma}[n]; \hat{\boldsymbol{\Lambda}}[n]) - \bar{\rho}_m\Big)\Big]^+, \ \forall m \quad (13)$$

where hats are to stress that these iterations are stochastic estimates of those in (12), based on *instantaneous* (instead of average) powers and collisions. As with (11), we can re-write (13) into:

$$\hat{\boldsymbol{\Lambda}}[n+1] = \Big[\hat{\boldsymbol{\Lambda}}[n] - \beta\hat{\boldsymbol{h}}\Big(\boldsymbol{\gamma}[n]; \boldsymbol{X}^*(\hat{\boldsymbol{\Lambda}}[n])\Big)\Big]^+ \quad (14)$$

where $\hat{\boldsymbol{h}}(\boldsymbol{\gamma}[n]; \boldsymbol{X}^*(\hat{\boldsymbol{\Lambda}}[n]))$ is the stochastic gradient depending on current fading realization $\boldsymbol{\gamma}[n]$. Provided that the random fading process is stationary and ergodic, it is clear that $\mathbb{E}[\hat{\boldsymbol{h}}(\boldsymbol{\gamma}; \boldsymbol{X}^*(\boldsymbol{\Lambda}))] = \boldsymbol{h}(\boldsymbol{X}^*(\boldsymbol{\Lambda}))$; i.e., stochastic gradient $\hat{\boldsymbol{h}}$ is a random realization (or an unbiased estimate based on a single realization) of the "ensemble" gradient $\boldsymbol{h}$. To show the near-optimality of the joint sensing-channel selection and power control scheme $\{\boldsymbol{s}^*(\boldsymbol{\gamma}[n]; \hat{\boldsymbol{\Lambda}}[n]), \boldsymbol{p}^*(\boldsymbol{\gamma}[n]; \hat{\boldsymbol{\Lambda}}[n])\}$ that results from the stochastic iteration (13), we define the sequence

$$g[n] := \sum_{m=1}^{M} q_m \log_2(1 + \gamma_m[n] p_m^*(\boldsymbol{\gamma}[n]; \hat{\boldsymbol{\Lambda}}[n])).$$

As a benchmark, we denote the optimal throughput for (1) as

$$g^* := \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M} s_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}^*) q_m \log_2(1 + \gamma_m p_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}^*))\Big].$$

Upon defining a constant

$$B := \bar{P}^2 + (L p_{\max})^2 + \sum_{m=1}^{M} (\tilde{q}_m^2 + \bar{\rho}_m^2),$$

we can then show the following lemma (see the proof in Appendix B):

Lemma 2: *For ergodic fading channels, we have*

$$\lim_{N\to\infty} \mathbb{E}\Big[\frac{1}{N}\sum_{n=0}^{N-1} g[n]\Big] \geq g^* - \beta\frac{B}{2} \quad (15)$$

from any finite initial $\hat{\boldsymbol{\Lambda}}[0]$.

Lemma 2 asserts that the expected value of the time average of $g[n]$ resulting from the proposed stochastic scheme converges to a point with optimality gap smaller than $\beta B/2$, through "learning" the required channel statistics on-the-fly. The proof of this lemma modifies from the Lyapunov optimization method in [18]. Yet different from [18] where a

[1] For instance, suppose we need to select two channels from channels 1–3 with the probabilities of choosing a particular channel being $\{1/2, 2/3, 5/6\}$. Assuming the desirable probabilities of selecting $\{1,2\}$, $\{1,3\}$ and $\{2,3\}$ are x, y and z, respectively. Then we set $x+y = 1/2$, $x+z = 2/3$ and $y+z = 5/6$. Solving these equations yield $x = 1/6$, $y = 1/3$ and $z = 1/2$. These probability values are then used for channel selection.

"virtual queue" argument was used to regulate the related constraints, here we rely on the Lagrange dual approach to deal with constraints and thus simplify the proof.

It is meaningful to characterize the mean of a stochastic process across different realizations as in Lemma 2. However, since in practice a single sequence is observed, it is thus more interesting to show the convergence of the individual sample paths. To this end, we can define $\bar{g}[N] := (1/N)\sum_{n=0}^{N-1} g[n]$, and prove the following proposition:

Proposition 2: *For ergodic fading channels, the time average*

$$\lim_{N\to\infty} \bar{g}[N] = \lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N-1} g[n] \geq g^* - \beta\frac{B}{2}, \quad a.s. \quad (16)$$

for any sample path $\{\hat{\mathbf{\Lambda}}[n],\ n = 0, 1, \ldots\}$.

Stemming from Lemma 2, Proposition 2 is implied by the Markov property of the dual vector $\hat{\mathbf{\Lambda}}[n]$ and ergodicity of the fading process. The proof follows similar lines with that of [19, Theorem 1 (ii)], and it is omitted for conciseness. Associated with different sample paths $\{\hat{\mathbf{\Lambda}}[n],\ n = 0, 1, \ldots\}$, the time average limit $\lim_{N\to\infty} \bar{g}[N]$ is a random variable. Proposition 2 shows that all realizations of this random variable have an optimality gap smaller than $\beta B/2$ almost surely. Clearly we can make this gap arbitrarily small by selecting a sufficiently small stepsize β.

Note that convergence of the stochastic scheme (14) could be proved in different senses using other methods. For instance, convergence of the stochastic iteration (14) to $\mathbf{\Lambda}^*$ in probability as $\beta \to 0$ can be established through the fluid limit and/or Lyapunov drift techniques when the traffic and fading process confines to a finite-state Markov process [16], [17]. Such a convergence can be also proved by drawing from a stochastic locking theorem, which justifies that the trajectory of the primary system (14) is always "locked", i.e. stays close to that of its averaged system (11) in probability under some regularity conditions; see our recent work [20]. Different from these approaches, here we provide a simpler and (arguably) stronger proof on almost sure optimality of the stochastic optimization schemes.

Based on Proposition 2, we put forth a stochastic sensing selection and power control algorithm:

- **Initialize** with any $\hat{\mathbf{\Lambda}}[0]$.
- **Repeat per slot** n**:** with $\hat{\mathbf{\Lambda}}[n]$ and $\boldsymbol{\gamma}[n]$ available, perform joint sensing selection and power control policy $\{\boldsymbol{s}^*(\boldsymbol{\gamma}[n]; \hat{\mathbf{\Lambda}}[n]),\ \ \boldsymbol{p}^*(\boldsymbol{\gamma}[n]; \hat{\mathbf{\Lambda}}[n])\}$ in (10), and update $\hat{\mathbf{\Lambda}}[n+1]$ using (14).

Note that in the proof of Proposition 2 we do not have assumptions on continuity of the fading cdf and/or uniqueness of $\{\boldsymbol{s}^*(\boldsymbol{\gamma}[n]; \hat{\mathbf{\Lambda}}[n]),\ \ \boldsymbol{p}^*(\boldsymbol{\gamma}[n]; \hat{\mathbf{\Lambda}}[n])\}$. The proposition thus holds for both continuous and discontinuous fading cdf cases. For both cases, the cognitive user needs only to obtain a (possibly non-unique) permutation $\boldsymbol{\pi}$ for the given $\boldsymbol{\gamma}[n]$ and $\hat{\mathbf{\Lambda}}[n]$, and then sense first l channels in $\boldsymbol{\pi}$ as specified in (10) per slot n. It is easy to see that this operation has a linear computational complexity $\mathcal{O}(M)$ in the number of channels.

Near-optimality of the proposed algorithm is confirmed by Proposition 2 when fading channels are stationary and ergodic. Due to its "stochastic learning" capability, the proposed stochastic iteration can actually track even non-stationary channels, if dynamic changes of the channel statistics are slow with respect to the convergence speed of the proposed scheme. In this case, the algorithm may "re-learn" the changed channel statistics and converge to the new optimum.

V. Numerical Results

In this section, we present numerical tests of the proposed stochastic scheme for cognitive communications. Suppose that the cognitive user can perform opportunistic transmissions over $M = 32$ frequency channels, each of which has bandwidth $B = 1$ KHz and the channel coherence time $T = 1$ ms. Due to physical limitations, the cognitive user can only sense $L = 8$ channels per coherence time slot T. The fading processes for the channels are independent and are generated from a Nakagami distribution with corresponding average channel power gain $\bar{\gamma}_m$ and Nakagami parameter a_m. The value of $\bar{\gamma}_m$ in dBW is a uniformly distributed random integer between 5 and 10, whereas the value of a_m is a uniformly distributed random number within $[1, 2]$, $\forall m$. The probabilities $P_{U,m}$ of channel m in use by primary users are independently generated from a uniform distribution over $[0.2, 0.6]$. Assume that the channel gains from the primary users to the cognitive user over all M channels are Rayleigh distributed with variance SNR_p. Then for a pre-determined false alarm probability $P_{F,m} = \alpha$, it is easy to show that the corresponding detection probability $P_{D,m} = \alpha^{\frac{1}{1+SNR_p}}$ given that the additive Gaussian noise at the cognitive user has a unit variance.

Setting $\alpha = 0.05$, $SNR_p = 15$ dB, $\forall m$ and using a stepsize $\beta = 0.001$, we run the proposed stochastic algorithm when the average power budget $\bar{P} = 10$ Watts and the maximum allowable probability of collision $\bar{\rho}_m = 0.06$, $\forall m$. Fig. 1 depicts the evolutions of the achieved average throughput $g[n]$ as well as the average power consumption and the probabilities of collisions with the primary users. Fast convergence is clearly seen and all constraints are met upon convergence. Notice that the fading cdf is assumed unknown a priori in the simulation, and the proposed stochastic scheme is supposed to learn this knowledge on-the-fly. For this reason, here only a "stochastic" convergence is achieved. In other words, $g[n]$ only converges to a small neighborhood (with a size proportional to β) around the optimal value g^* per Proposition 2.

To gauge the performance of the proposed scheme, we compare it with three (suboptimal) heuristic schemes. For all these three schemes, the cognitive user senses a fixed set of $L = 8$ channels with the smallest $P_{U,m}$. (Another fixed sensing option could consists of $L = 8$ channels with the highest $\bar{\gamma}_m$; however, this option is not feasible here since channel statistics are assumed not available a priori.) Based on such a fixed sensing selection, we implement the following three schemes: 1) a fixed-sensing/optimal-power scheme where the transmit-powers are optimally allocated across fading realizations over the given set of $L = 8$ channels and all time slots subjective to the average power budget $\bar{P}$ (it can be easily shown that this amounts to a "water-filling" type power allocation); 2) a fixed-sensing/equal-power scheme where $\bar{P}$ are equally divided among the active communications per slot, i.e., if after sensing $l \neq 0$ channels are found not in

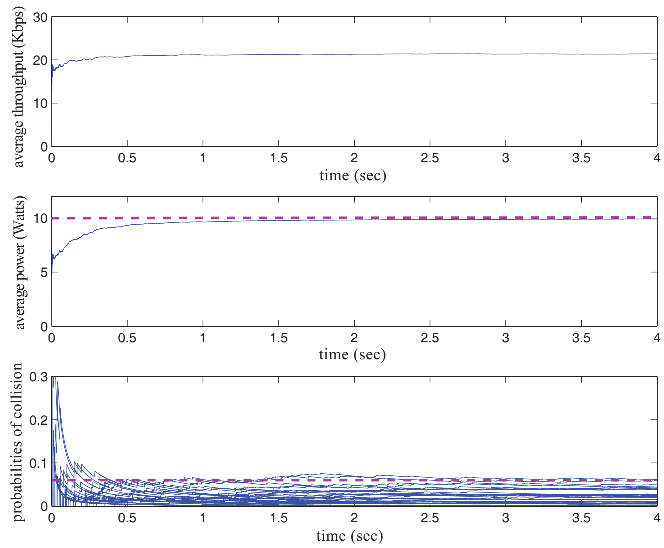

Fig. 1. Independent fading case: evolutions of the achieved average throughput, average power consumption and probabilities of collisions (dashed lines indicate the constraints $\bar{P} = 10$ Watts and $\bar{\rho}_m = 0.06$).

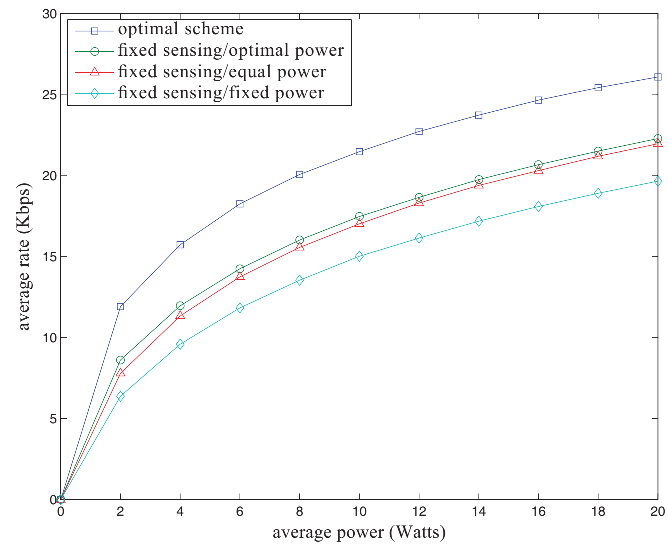

Fig. 2. Independent fading case: comparison with suboptimal schemes.

use by primary users at a given slot, the power for transmission over each channel is $\bar{P}/l$; and 3) a fixed-sensing/fixed-power scheme where transmit-power is fixed at $\bar{P}/L$ per transmission per slot. Fig. 2 compares the achieved throughput of cognitive transmissions with the proposed scheme and the three heuristic schemes when different average power budgets $\bar{P}$ are adopted. Notice that the requirements on maximum allowable probabilities of collisions are not guaranteed for the three heuristic schemes. Despite this, the derived optimal scheme significantly outperforms the heuristic schemes for all $\bar{P}$ since it is capable of fully exploiting all the spectral and temporal diversity on channel fading and primary user activity. The fixed-sensing schemes ignore part of spectral diversity and thus have performance loss. Compared with the fixed-sensing/optimal-power scheme, the fixed-sensing/equal-power scheme ignores more spectral and temporal diversity. However, the resultant performance loss is small. This is consistent with the prior finding that the gain from optimal (water-filling type) channel-adaptive power allocation is small in medium and large SNR cases. The fixed-sensing/fixed-power scheme causes noticeable extra performance loss due to under-utilization of the power budget. When only a small number $l < L$ of channels are sensed idle and then utilized for transmissions, the total transmit-power for the cognitive user at this given slot is $(l/L)\bar{P} < \bar{P}$ as the power per transmission is fixed at $\bar{P}/L$. This clearly incurs loss in achieved throughput. Overall Fig. 2 clearly demonstrates that the derived optimal joint sensing selection and power control scheme can result in large performance gains over heuristic schemes.

To show the performance of the proposed scheme under different channel fading scenario, we next consider the cognitive radio transmits based on orthogonal frequency division multiplexing (OFDM), where the frequency channels are provided by the 32 subcarriers, each with sub-bandwidth 1 KHz. For cognitive user's wireless link, assume a light-of-sight channel gain of 5 dBW. A profile of 20 μs exponentially and independently decaying tap gains is then generated independently across coherent time slots. Different from the previous case, fading coefficients over the subcarriers determined by the Fourier transform of the time-domain impulse responses are correlated. Again setting $\alpha = 0.05$, $SNR_p = 15$ dB, $\forall m$ and using a stepsize $\beta = 0.001$, we run the proposed stochastic algorithm when $\bar{P} = 10$ Watts and $\bar{\rho}_m = 0.06$, $\forall m$. Fig. 3 depicts the evolutions of the achieved average throughput $g[n]$ as well as the average power consumption and the probabilities of collisions with the primary users, where fast convergence is also seen and all constraints are met upon convergence. Throughput comparison between the proposed scheme and the aforementioned three heuristic schemes is shown in Fig. 4, when different average power budgets $\bar{P}$ are adopted. Again it is corroborated that the derived optimal joint sensing selection and power control scheme has large performance gains over heuristic schemes.

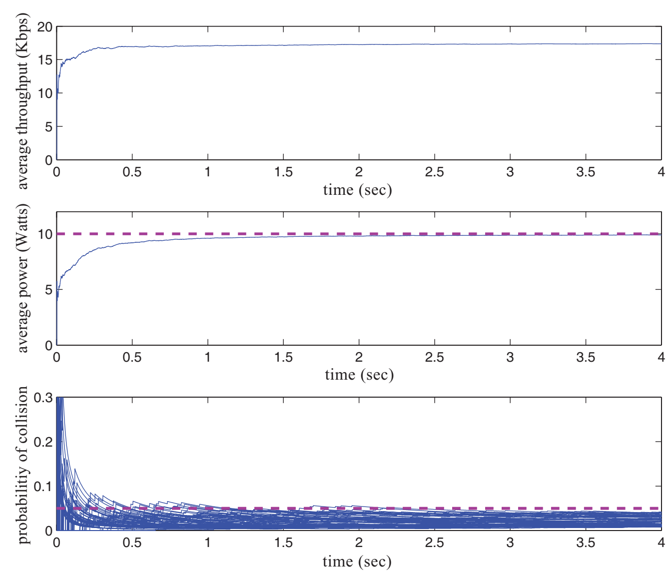

Fig. 3. Correlated fading case: evolutions of the achieved average throughput, average power consumption and probabilities of collisions (dashed lines indicate the constraints $\bar{P} = 10$ Watts and $\bar{\rho}_m = 0.06$).

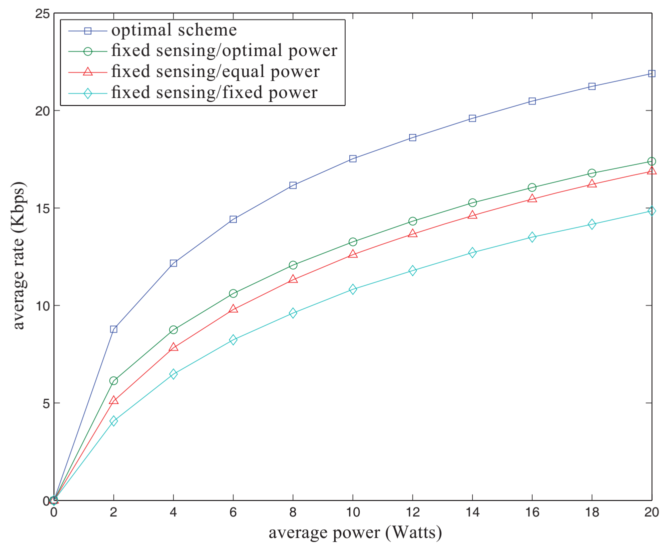

Fig. 4. Correlated fading case: comparison with suboptimal schemes.

VI. Concluding Remarks

We have investigated the joint optimization of sensing-channel selection and power control scheme with the cognitive radios over time-varying fading channels. The optimal strategy was derived under the constraints of average power budget and maximum allowable probability of collisions with the primary communications, for both cases that the fading channels have continuous and discontinuous cdfs. In addition, we developed a stochastic optimization algorithm that can operate without a-priori knowledge of the fading channel statistics. It was rigorously established that the proposed stochastic scheme approaches the optimal strategy almost surely for any stationary and ergodic fading channels.

For simplicity the current work considers only a single cognitive user. Generalization of our results to the cognitive networks with multiple users is under investigation. It is also interesting to incorporate the temporal/spectral correlation (if known) of fading process and that of the primary user activity (e.g., a Markov modulated ON-OFF model for primary user channel occupancy process in [21]) into the proposed framework for joint optimization of sensing selection and power control. Furthermore, practical channel estimation/feedback schemes for cognitive radios and their impact on the proposed sensing and power control strategy are worth further investigations. These directions will be explored in future work.

Appendix

A. Proof of Lemma 1

Let us define a convenient notation

$$\phi_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma}); \lambda) := q_m r_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma})) - \lambda \check{q}_m \tilde{p}_m(\boldsymbol{\gamma}). \tag{17}$$

To prove the lemma, we will need the following two claims.

Claim A.1: *For any* $\boldsymbol{\Lambda} \geq 0$, *it holds that* $\phi_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma}); \lambda) \leq s_m(\boldsymbol{\gamma}) \phi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$, $\forall s_m(\boldsymbol{\gamma}) \in [0, 1]$ *and* $\forall p_m(\boldsymbol{\gamma}) \in [0, p_{\max}]$.

Proof: Consider the following two cases:

1) If $s_m(\boldsymbol{\gamma}) > 0$, substituting (2) into (17) yields

$$\begin{aligned}\phi_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma}); \lambda) &= s_m(\boldsymbol{\gamma}) q_m \log_2\left(1 + \gamma_m \frac{\tilde{p}_m(\boldsymbol{\gamma})}{s_m(\boldsymbol{\gamma})}\right) - \lambda \check{q}_m \tilde{p}_m(\boldsymbol{\gamma}) \\ &= s_m(\boldsymbol{\gamma}) \left[q_m \log_2(1 + \gamma_m p_m(\boldsymbol{\gamma})) - \lambda \check{q}_m p_m(\boldsymbol{\gamma})\right]\end{aligned}$$

where we use $\tilde{p}_m(\boldsymbol{\gamma}) := s_m(\boldsymbol{\gamma}) p_m(\boldsymbol{\gamma})$. Due to the concavity of $\log_2$ function, it is easy to see that $q_m \log_2(1 + \gamma_m p_m(\boldsymbol{\gamma})) - \lambda \check{q}_m p_m(\boldsymbol{\gamma})$ is maximized by $p_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ defined in (7); i.e., $\forall p_m(\boldsymbol{\gamma}) \in [0, p_{\max}]$,

$$\begin{aligned}q_m \log_2(1 + \gamma_m p_m(\boldsymbol{\gamma})) - \lambda \check{q}_m p_m(\boldsymbol{\gamma}) &\leq q_m \log_2(1 + \gamma_m p_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})) - \lambda \check{q}_m p_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) \\ &= \phi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})\end{aligned}$$

where the equality is due to the definition of $\phi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ in (8). It thus readily follows that $\phi_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma}); \lambda) \leq s_m(\boldsymbol{\gamma}) \phi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$, $\forall p_m(\boldsymbol{\gamma}) \in [0, p_{\max}]$.

2) If $s_m(\boldsymbol{\gamma}) = 0$, it clearly holds that $s_m(\boldsymbol{\gamma}) \phi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) = 0$. On the other hand, (17) and (2) imply that $\phi_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma}); \lambda) = -\lambda \check{q}_m \tilde{p}_m(\boldsymbol{\gamma}) \leq 0$, $\forall \tilde{p}_m(\boldsymbol{\gamma}) \geq 0$. Therefore, again $\phi_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma}); \lambda) \leq s_m(\boldsymbol{\gamma}) \phi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$, $\forall p_m(\boldsymbol{\gamma}) \in [0, p_{\max}]$.

Cases 1) and 2) imply that $\phi_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma}); \lambda) \leq s_m(\boldsymbol{\gamma}) \phi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$, $\forall s_m(\boldsymbol{\gamma}), p_m(\boldsymbol{\gamma})$. □

Claim A.2: *For any* $\boldsymbol{\Lambda} \geq 0$, *it holds that*

$$\sum_{m=1}^{M} s_m(\boldsymbol{\gamma}) \varphi_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) \leq \sum_{k=1}^{l} \varphi_{\pi(k)}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}),$$

$\forall s_m(\boldsymbol{\gamma}) \in [0, 1]$ *with* $\sum_{m=1}^{M} s_m(\boldsymbol{\gamma}) \leq L$.

Proof: Consider the following two cases:

1) If $l = \min\{L', L\} = L \leq L'$, then it follows from the definition of L' that $\varphi_{\pi(L)}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) > 0$. In turn we have

$$\begin{aligned}\sum_{k=1}^{l} \varphi_{\pi(k)}^* - \sum_{m=1}^{M} s_m \varphi_m^* &= \sum_{k=1}^{L} [(1 - s_{\pi(k)}) \varphi_{\pi(k)}^*] - \sum_{k=L+1}^{M} s_{\pi(k)} \varphi_{\pi(k)}^* \\ &\geq \sum_{k=1}^{L} [(1 - s_{\pi(k)}) \varphi_{\pi(L)}^*] - \sum_{k=L+1}^{M} s_{\pi(k)} \varphi_{\pi(L)}^* \\ &= \varphi_{\pi(L)}^* \left(L - \sum_{k=1}^{M} s_{\pi(k)}\right) \geq 0\end{aligned}$$

where the first inequality is due to the definition of the permutation $\boldsymbol{\pi}$ and $s_{\pi(k)} \in [0, 1]$, $\forall k$; whereas the second inequality is due to $\varphi_{\pi(L)}^* > 0$ and $\sum_{k=1}^{M} s_{\pi(k)} \leq L$.

2) If $l = \min\{L', L\} = L' < L$, then the definition of L'

implies that $\varphi^*_{\pi(k)}(\boldsymbol{\gamma};\boldsymbol{\Lambda}) \le 0$, $\forall k > l$. We then deduce

$$\begin{aligned}\sum_{k=1}^{l}\varphi^*_{\pi(k)} - \sum_{m=1}^{M} s_m\varphi^*_m &= \sum_{k=1}^{l}[(1-s_{\pi(k)})\varphi^*_{\pi(k)}] - \sum_{k=l+1}^{M} s_{\pi(k)}\varphi^*_{\pi(k)}\\ &\ge \sum_{k=1}^{l}[(1-s_{\pi(k)})\varphi^*_{\pi(k)}] \ge 0\end{aligned}$$

where the first inequality is due to $s_{\pi(k)} \ge 0$ and $\varphi^*_{\pi(k)} \le 0$, $\forall k > l$; whereas second inequality is due to $s_{\pi(k)} \le 1$ and $\varphi^*_{\pi(k)} > 0$, $\forall k \le l$.

Cases 1) and 2) readily prove the claim. □

Based on Claims A.1 and A.2, we are ready to prove the lemma. Using $\phi_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma}); \lambda)$ defined in (17), we find that the objective per $\boldsymbol{\gamma}$ in (6) is

$$\begin{aligned}&\sum_{m=1}^{M}[\phi_m(s_m(\boldsymbol{\gamma}), \tilde{p}_m(\boldsymbol{\gamma}); \lambda) - \mu_m\tilde{q}_m s_m(\boldsymbol{\gamma})]\\ &\quad\le \sum_{m=1}^{M}\{s_m(\boldsymbol{\gamma})[\phi^*_m(\boldsymbol{\gamma};\boldsymbol{\Lambda}) - \mu_m\tilde{q}_m]\}\\ &\quad= \sum_{m=1}^{M} s_m(\boldsymbol{\gamma})\varphi^*_m(\boldsymbol{\gamma};\boldsymbol{\Lambda}) \le \sum_{k=1}^{l}\varphi^*_{\pi(k)}(\boldsymbol{\gamma};\boldsymbol{\Lambda})\end{aligned}$$

where the first inequality is due to Claim A.1, the second equality is due to the definition of $\varphi^*_m(\boldsymbol{\gamma};\boldsymbol{\Lambda})$ in (9), and the third inequality is due to Claim A.2. Furthermore, the equalities can be achieved using the joint sensing selection and power control scheme $\{\boldsymbol{s}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}),\ \tilde{\boldsymbol{p}}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda})\}$ specified in (10), which is thus optimal for (6).

To show the almost sure uniqueness of (10), we can substitute (7) into (9) and then differentiate $\varphi^*_m(\boldsymbol{\gamma};\boldsymbol{\Lambda})$ to prove that the first derivative $\frac{\partial\varphi^*_m(\boldsymbol{\gamma};\boldsymbol{\Lambda})}{\partial\gamma_m}$ is positive when $\varphi^*_m(\boldsymbol{\gamma};\boldsymbol{\Lambda}) > 0$. This implies that $\varphi^*_m(\boldsymbol{\gamma};\boldsymbol{\Lambda})$ is a strictly increasing function of γ_m when it is greater than zero. Therefore, the event of $\{\varphi^*_m(\boldsymbol{\gamma};\boldsymbol{\Lambda}) = \varphi^*_{m'}(\boldsymbol{\gamma};\boldsymbol{\Lambda}) > 0, m \ne m'\}$ is an event of Lebesgue measure zero provided that $\boldsymbol{\gamma}$ has a continuous cdf. This in turn implies that almost surely we do not have any ties for $\{\varphi^*_m(\boldsymbol{\gamma};\boldsymbol{\Lambda}),\ \forall m\}$ and the permutation $\boldsymbol{\pi}$ is unique. Under the latter case, the joint sensing selection and power control policy (10) is clearly unique. The desired almost sure uniqueness readily follows.

B. *Proof of Lemma 2*

Using the fact that projections reduce length and the shorthand notations $p^*_m[n] := p^*_m(\boldsymbol{\gamma}[n];\hat{\boldsymbol{\Lambda}}[n])$ and $s^*_m[n] := s^*_m(\boldsymbol{\gamma}[n];\hat{\boldsymbol{\Lambda}}[n])$, we have from (13) that

$$\begin{aligned}(\hat{\lambda}[n+1])^2 &\le [\hat{\lambda}[n] + \beta(\sum_{m=1}^{M}\check{q}_m p^*_m[n] - \bar{P})]^2\\ &\le (\hat{\lambda}[n])^2 + 2\beta\hat{\lambda}[n](\sum_{m=1}^{M}\check{q}_m p^*_m[n] - \bar{P})\\ &\quad + \beta^2(\sum_{m=1}^{M}\check{q}_m p^*_m[n] - \bar{P})^2\\ &\le (\hat{\lambda}[n])^2 + 2\beta\hat{\lambda}[n](\sum_{m=1}^{M}\check{q}_m p^*_m[n] - \bar{P})\\ &\quad + \beta^2((Lp_{\max})^2 + \bar{P}^2).\end{aligned}$$

Similarly, we have: $\forall m$,

$$\begin{aligned}(\hat{\mu}_m[n+1])^2 \le &(\hat{\mu}_m[n])^2 + 2\beta\hat{\mu}_m[n](\tilde{q}_m s^*_m[n] - \bar{\rho}_m)\\ &+ \beta^2(\tilde{q}_m^2 + \bar{\rho}_m^2).\end{aligned}$$

Define the functions

$$\begin{aligned}V(\hat{\boldsymbol{\Lambda}}) &:= \hat{\lambda}^2 + \sum_{m=1}^{M}\hat{\mu}_m^2,\\ \triangle V(\hat{\boldsymbol{\Lambda}}[n]) &:= V(\hat{\boldsymbol{\Lambda}}[n+1]) - V(\hat{\boldsymbol{\Lambda}}[n]).\end{aligned}$$

Then we readily have

$$\begin{aligned}\triangle V(\hat{\boldsymbol{\Lambda}}[n]) \le \beta^2 B + 2\beta\Big[&\hat{\lambda}[n](\sum_{m=1}^{M}\check{q}_m p^*_m[n] - \bar{P})\\ &+ \sum_{m=1}^{M}\hat{\mu}_m[n](\tilde{q}_m s^*_m[n] - \bar{\rho}_m)\Big]\end{aligned}$$

Taking expectation over all fading realizations at slot n, we have:

$$\begin{aligned}\mathbb{E}[\triangle V(\hat{\boldsymbol{\Lambda}}[n])] &\le \beta^2 B\\ &+ 2\beta\Big[\hat{\lambda}[n](\sum_{m=1}^{M}\mathbb{E}_{\boldsymbol{\gamma}}[\check{q}_m s^*_m(\boldsymbol{\gamma};\hat{\boldsymbol{\Lambda}}[n])p^*_m(\boldsymbol{\gamma};\hat{\boldsymbol{\Lambda}}[n])] - \bar{P})\\ &+ \sum_{m=1}^{M}\hat{\mu}_m[n](\mathbb{E}_{\boldsymbol{\gamma}}[\tilde{q}_m s^*_m(\boldsymbol{\gamma};\hat{\boldsymbol{\Lambda}}[n])] - \bar{\rho}_m)\Big]\end{aligned}$$

Subtracting $2\beta\mathbb{E}[g[n]]$ from the both sides, we would have:

$$\begin{aligned}\mathbb{E}[\triangle V(\hat{\boldsymbol{\Lambda}}[n])] - 2\beta\mathbb{E}[g[n]] &\le \beta^2 B - 2\beta\mathbb{E}[g[n]]\\ &+ 2\beta\Big[\hat{\lambda}[n](\sum_{m=1}^{M}\mathbb{E}_{\boldsymbol{\gamma}}[\check{q}_m s^*_m(\boldsymbol{\gamma};\hat{\boldsymbol{\Lambda}}[n])p^*_m(\boldsymbol{\gamma};\hat{\boldsymbol{\Lambda}}[n])] - \bar{P})\\ &+ \sum_{m=1}^{M}\hat{\mu}_m[n](\mathbb{E}_{\boldsymbol{\gamma}}[\tilde{q}_m s^*_m(\boldsymbol{\gamma};\hat{\boldsymbol{\Lambda}}[n])] - \bar{\rho}_m)\Big]\\ &= \beta^2 B - 2\beta L(\boldsymbol{X}^*(\hat{\boldsymbol{\Lambda}}[n]), \hat{\boldsymbol{\Lambda}}[n]),\end{aligned} \tag{18}$$

where $L(\boldsymbol{X}, \boldsymbol{\Lambda})$ is the Lagrangian function defined in (4).

Since $\boldsymbol{X}^*(\hat{\boldsymbol{\Lambda}}[n])$ maximizes $L(\boldsymbol{X}, \hat{\boldsymbol{\Lambda}}[n])$ for all $\boldsymbol{X} \in \mathcal{F}'$, we have:

$$\begin{aligned} L(\boldsymbol{X}^*(\hat{\boldsymbol{\Lambda}}[n]), \hat{\boldsymbol{\Lambda}}[n]) &\geq L(\boldsymbol{X}^*(\boldsymbol{\Lambda}^*), \hat{\boldsymbol{\Lambda}}[n]) \\ &= g^* - \hat{\lambda}[n]\Big(\sum_{m=1}^{M} \mathbb{E}_{\boldsymbol{\gamma}}[\tilde{q}_m s_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}^*) p_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}^*)] - \bar{P}\Big) \\ &\quad - \sum_{m=1}^{M} \hat{\mu}_m[n](\mathbb{E}_{\boldsymbol{\gamma}}[\tilde{q}_m s_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}^*)] - \bar{\rho}_m) \geq g^*, \end{aligned}$$

where the first inequality is due to $\boldsymbol{X}(\boldsymbol{\Lambda}^*) \in \mathcal{F}'$, and the second inequality is due to $\hat{\lambda}[n] \geq 0$, $\hat{\mu}_m[n] \geq 0$ and $\boldsymbol{X}(\boldsymbol{\Lambda}^*)$ must satisfy the constraints: $\mathbb{E}_{\boldsymbol{\gamma}}[\tilde{q}_m s_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}^*) p_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}^*)] \leq \bar{P}$ and $\mathbb{E}_{\boldsymbol{\gamma}}[\tilde{q}_m s_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}^*)] \leq \bar{\rho}_m$, $\forall m$.

Therefore, it follows from (18) that

$$\mathbb{E}[\triangle V(\hat{\boldsymbol{\Lambda}}[n])] - 2\beta \mathbb{E}[g[n]] \leq \beta^2 B - 2\beta g^*.$$

This implies that

$$\begin{aligned} &\sum_{n=1}^{N-1} \mathbb{E}[\triangle V(\hat{\boldsymbol{\Lambda}}[n])] - 2\beta \sum_{n=0}^{N-1} \mathbb{E}[g[n]] \\ &\quad = \mathbb{E}[V(\hat{\boldsymbol{\Lambda}}[N])] - V(\hat{\boldsymbol{\Lambda}}[0]) - 2\beta \sum_{n=0}^{N-1} \mathbb{E}[g[n]] \\ &\quad \leq N(\beta^2 B - 2\beta g^*). \end{aligned}$$

Since $\mathbb{E}[V(\hat{\boldsymbol{\Lambda}}[N])] \geq 0$, we further have:

$$\frac{1}{N} \sum_{n=0}^{N-1} \mathbb{E}[g[n]] \geq g^* - \beta \frac{B}{2} - \frac{V(\hat{\boldsymbol{\Lambda}}[0])}{2\beta N}.$$

For any finite $\hat{\boldsymbol{\Lambda}}[0]$, it then clearly follows that

$$\lim_{N \to \infty} \mathbb{E}\Big[\frac{1}{N} \sum_{n=0}^{N-1} g[n]\Big] \geq g^* - \beta \frac{B}{2}.$$

REFERENCES

[1] FCC Spectrum Policy Task Force, "Report of the spectrum efficiency working group," Technical Report 02-135, Nov. 2002.

[2] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[3] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: an information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.

[4] F. Digham, M. Alouini, and M. Simon, "On the energy detection of unknown signals over fading channels," *IEEE Trans. Commun.*, vol. 55, no. 1, pp. 21–24, Jan. 2007.

[5] R. Tandra and A. Sahai, "SNR walls for signal detection," *IEEE J. Sel. Top. Signal Process.*, vol. 2, no. 1, pp. 4–17, Feb. 2008.

[6] G. Ganesan and Y. Li, "Cooperative spectrum sensing in cognitive radio–part I and part II," *IEEE Trans. Wireless Commun.*, vol. 6, no. 6, pp. 2204–2222, June 2007.

[7] Y. Song, Y. Fang, and Y. Zhang, "Stochastic channel selection for cognitive radio networks," in *Proc. IEEE Globecom*, pp. 4878–4882, Washington DC, Nov. 2007.

[8] A. Marques, X. Wang, and G. Giannakis, "Dynamic resource management for cognitive radios using limited-rate feedback," *IEEE Trans. Signal Process.*, vol. 57, no. 9, pp. 3651–3666, Sep. 2009.

[9] R. Urgaonkar and M. Neely, "Opportunistic scheduling with reliability guarantees in cognitive radio networks," *IEEE Trans. Mobile Comp.*, vol. 8, no. 6, pp. 766–777, June 2009.

[10] G. Chung, S. Vishwanath, and C. Hwang, "On the fundamental limits of interweaved cognitive radios," preprint available at http://arxiv.org/PS_cache/arxiv/pdf/0910/0910.1639v1.pdf, Oct. 2009.

[11] X. Wang and G. Giannakis, "Power-efficient resource allocation for time-division multiple access over fading channels," *IEEE Trans. Inf. Theory*, vol. 54, no. 3, pp. 1225–1240, Mar. 2008.

[12] C. Y. Wong, R. Cheng, K. Lataief, and R. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 10, pp. 1747–1758, Oct. 1999.

[13] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge Uiversity Press, 2004.

[14] D. Bertsekas, *Nonlinear Programming*, 2nd edition. Athena Scientific, 1999.

[15] A. Marques, G. Giannakis, and J. Ramos, "Optimum scheduling for orthogonal multiple access over fading channels using quantized channel state information," in *Proc. SPAWC*, July 2008 (full version at arXiv:0909.0760v1 [cs.IT]).

[16] A. Stolyar, "Maximizing queueing network utility subject to stability: greedy primal-dual algorithm," *Queueing Sys.*, vol. 50, pp. 401–457, 2005.

[17] A. Eryilmaz and R. Srikant, "Fair resource allocation in wireless networks using queue-length-based scheduling and congestion control," in *Proc. INFOCOM Conf.*, vol. 3, pp. 1794–1803, Mar. 2005.

[18] M. Neely, E. Modiano, and C. Li, "Fairness and optimal stochastic control for heterogenous networks," *IEEE/ACM Trans. Networking*, vol. 16, no. 2, pp. 396–409, Apr. 2008.

[19] A. Ribeiro, "Ergodic stochastic optimization algorithms for wireless communication and networking," *IEEE Trans. Signal Process.*, vol. 58, no. 12, Dec. 2010 (to appear).

[20] X. Wang, G. Giannakis, and A. Marques, "A unified approach to QoS-guaranteed scheduling for channel-adaptive wireless networks," *Proc. IEEE*, vol. 95, no. 12, pp. 2410–2431, Dec. 2007.

[21] Y. Chen, Q. Zhao, and A. Swami, "Joint design and separation principle for opportunistic spectrum access in the presence of sensing errors," *IEEE Trans. Inf. Theory*, vol. 54, no. 5, pp. 2053–2071, May 2008.

Xin Wang (SM'09) received the B.Sc. degree and the M.Sc. degree from Fudan University, Shanghai, China, in 1997 and 2000, respectively, and the Ph.D. degree from Auburn University, Auburn, AL, in 2004, all in electrical engineering. From September 2004 to August 2006, he was a Postdoctoral Research Associate with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis. Since September 2006, he has been with the Department of Computer & Electrical Engineering and Computer Science, Florida Atlantic University, Boca Raton, FL, where he is now an associate professor. His research interests include medium access control, cross-layer design, stochastic resource allocation, and signal processing for communication networks.