Optimal Subcarrier-Chunk Scheduling for Wireless OFDMA Systems

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***Abstract*—In practical orthogonal frequency division multiple-access (OFDMA) systems, subcarriers are grouped into chunks and a chunk of subcarriers is regarded as the minimum unit for subcarrier allocation. Given that the number of chunks and the number of subcarriers in each chunk are predefined, we consider the optimal chunk allocation that maximizes a utility function of average user rates for a wireless OFDMA system under different power control policies. The relevant optimization problems are formulated as non-convex mixed-integer programs; yet it is shown that the optimal schemes can be obtained through Lagrange dual-based gradient iterations with fast convergence and low computational complexity under conditions. Furthermore, novel low-complexity algorithms are developed to approach the optimal strategies without a-priori knowledge of statistics of the intended wireless channels. Numerical results are provided to gauge the performance of the proposed schemes.**

***Index Terms*—Resource allocation, wireless OFDMA networks, subcarrier chunk, stochastic optimization.**

I. INTRODUCTION

THE emerging demand for diverse data applications entails both high data rate wireless connections and intelligent multiuser scheduling designs in next-generation wireless networks. Orthogonal frequency division multiple-access (OFDMA) is capable of delivering high-speed connections in a multi-path environment by dividing an entire channel into many orthogonal narrowband subcarriers and thus largely eliminating inter-symbol interferences, which limits the available data rates. Furthermore, those subcarriers can be allocated dynamically among different users, providing a new degree of freedom in multiuser scheduling [1]. For these reasons, OFDMA becomes the workhorse for broadband wireless applications/standards.

Resource allocation for OFDMA networks has attracted a lot of interest, where the goal is to jointly allocate subcarriers and rate/power in order to maximize (respectively minimize) the weighted sum of user rates (powers) under a prescribed power (rate) budget. In this context, [2], [3], [4], [5] reported suboptimal algorithms which tradeoff complexity for (sub)optimality. Instead of maximizing the weighted sum of user rates, recently there was interest to consider maximizing a suitable utility function of average rates to ensure fairness among users [6], [7], [8]. In addition, stochastic optimization tools were employed to develop on-line adaptive scheduling schemes for time-division multiplexing networks, which are capable of learning the intended channel distribution on-the-fly [9], [10], [11]. Generalizing these approaches, a unifying framework was proposed for design and analysis of stochastic resource allocation schemes for OFDMA systems in [12].

Most of the prior works on OFDMA resource allocation assumed that individual subcarriers can be assigned to a user. In practical OFDMA systems, however, the single-subcarrier based allocation schemes incur significant signaling overhead and entail complicated implementation, since they must simultaneously coordinate a large number (e.g. 1024) of subcarriers for broadband applications. To mitigate the overhead and implementation complexity, the correlation between adjacent subcarriers can be utilized by properly grouping a set of subcarriers into one chunk, and making a chunk of subcarriers as the minimum unit for subcarrier allocation. Such chunk-based OFDMA has actually been adopted as the air-interface for emerging wireless systems including IEEE WiMax, European next generation and 3GPP systems [13], [14]. Resource allocation for the chunk-based OFDMA systems was only addressed in a few works. The performance analysis of the chunk-based allocation was provided to guide the multiuser scheduling design for downlink OFDMA systems in [15]. Some heuristic algorithms were proposed in [16], [17] for chunk-based scheduling of single-carrier frequency multiple-access (SC-FDMA) – a modified form of OFDMA for uplink transmission considered in the 3GPP-LTE. These algorithms aimed to maximize the (weighted) sum of the user rates, where constant power allocation across chunks was considered and/or a suboptimal chunk scheduling was employed.

In this paper, we investigate the optimal chunk allocation that maximizes a general utility function of average user rates for wireless OFDMA systems, provided that the number of chunks and the number of subcarriers in each chunk are predefined. To balance the system throughput and transceiver complexity, we consider three different power control policies adopted in the system design: i) different transmit-powers can be allocated across individual subcarriers, ii) different powers can be allocated across chunks (but the same power is employed for all subcarriers in a chunk); and iii) a constant transmit-power is used. When different transmit-powers are allowed across individual subcarriers, we show that a Lagrange dual based "greedy water-filling" approach can be applied to obtain the optimal subcarrier and power allocation with or without the predefined subcarrier-chunks [12]; yet, the power allocation values and subcarrier scheduling strategy can be sig-

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nificantly different for the optimal schemes with and without predefined chunks. This approach is also generalized to find the optimal subcarrier-chunk scheduling under the other two power control policies. Under different power control policies, it is shown that the optimal chunk assignment adopts a similar greedy structure. Although the relevant optimization problems are formulated as non-convex mixed-integer programs, we prove that the optimal strategies can be obtained through Lagrange dual-based gradient iterations with fast convergence and low computational complexity per iteration. Furthermore, we rely on stochastic optimization tools to develop a class of on-line algorithms capable of approaching the optimal strategies without knowing statistics of the intended wireless channels a priori.

The rest of this paper is organized as follows. The system model is described in Section II. The optimal chunk scheduling schemes are derived under the three different power control policies in Sections III, IV, and V, respectively. Numerical results are provided in Section VI to demonstrate the merit of the proposed schemes, followed by conclusions.

II. Modeling Preliminaries

For specificity, we consider a downlink air interface between an access point (AP) and wireless users $k = 1, \ldots, K$ connected to it; but the results readily carry over to the uplink operation along the similar lines of our recent work [10], [12]. The overall bandwidth B is divided into $M \times J$ orthogonal narrowband subcarriers, each with sub-bandwidth $\Delta = B/(MJ)$ small enough for each subcarrier to experience only flat fading. As pre-determined by the given system or standard, the subcarriers are grouped into M chunks, each consisting of J subcarriers. Let $\gamma_{k,m}^{(j)}$ denote the frequency-domain power gain for the kth user at the jth subcarrier of chunk m. Assume a block fading model where the fading coefficients $\boldsymbol{\gamma} = \{\gamma_{k,m}^{(j)},\ k = 1, \ldots, K,\ m = 1, \ldots, M,\ j = 1, \ldots, J\}$ are fixed per (coherent time) slot n but are allowed to change randomly from slot to slot according to a stationary and ergodic random process with a cumulative distribution function (cdf). Given time-domain complex channel taps $h_{k,l}[n]$ with delays τ_l, $l = 1, \ldots, L_k$, at slot n, we obtain the discrete-time Fourier transform amplitude square:

$$\gamma_{k,m}^{(j)}[n] = \left|\sum_{l=1}^{L_k} h_{k,l}[n] e^{-i2\pi\tau_l((m-1)J+j)\Delta}\right|^2. \quad (1)$$

Assume that the AP has full information about $\boldsymbol{\gamma}[n]$ per slot n through e.g., training and feedback from the users. Adapted to $\boldsymbol{\gamma}$, a scheduler in the AP performs the chunk allocation and power control for downlink transmissions.

III. Adaptive Power Allocation across Subcarriers

Under different power control policies, the optimal chunk allocation strategies differ. Consider first the general case that transmit-powers can be adaptively allocated across individual subcarriers. Let $\alpha_{k,m}(\boldsymbol{\gamma})$ be the chunk allocation decision for transmission to user k at chunk m upon channel realization $\boldsymbol{\gamma}$. An exclusive chunk allocation is typically enforced by the practical systems such that at most one user can be allocated to a single chunk; i.e., $\alpha_{k,m}(\boldsymbol{\gamma}) \in \{0, 1\}$ and $\sum_{k=1}^{K} \alpha_{k,m}(\boldsymbol{\gamma}) \leq 1$, $\forall m$. Let $p_{k,m}^{(j)}(\boldsymbol{\gamma})$ denote the power for transmission to user k at the jth subcarrier of chunk m upon $\boldsymbol{\gamma}$. With $p_{\max}$ being a natural peak-power bound, we have $0 \leq p_{k,m}^{(j)}(\boldsymbol{\gamma}) \leq p_{\max}$.

Let $\mathcal{A}$ denote the set of all schedules satisfying $\alpha_{k,m}(\boldsymbol{\gamma}) \in \{0, 1\}$, $\sum_{k=1}^{K} \alpha_{k,m}(\boldsymbol{\gamma}) \leq 1$, and $0 \leq p_{k,m}^{(j)}(\boldsymbol{\gamma}) \leq p_{\max}$, $\forall k, m, j$. Assuming without loss of generality that the additive white Gaussian noise at the receiver has unit variance and the subcarrier-bandwidth $\Delta = 1$, the scheduler at the AP decides both the chunk allocation $\boldsymbol{\alpha}(\boldsymbol{\gamma}) := \{\alpha_{k,m}(\boldsymbol{\gamma}),\ \forall k, m\}$ and the power allocation vector $\boldsymbol{p}(\boldsymbol{\gamma}) := \{p_{k,m}^{(j)}(\boldsymbol{\gamma}),\ \forall k, m, j\}$ to maximize the utility of average user rate vector $\bar{\boldsymbol{r}} := \{\bar{r}_k,\ \forall k\}$; i.e., it solves:

$$\begin{aligned} &\max_{\bar{\boldsymbol{r}} \geq 0;\ (\boldsymbol{\alpha}, \boldsymbol{p}) \in \mathcal{A}} \sum_{k=1}^{K} U(\bar{r}_k) \\ &\text{s. t.}\ \ \bar{r}_k \leq \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M} \alpha_{k,m}(\boldsymbol{\gamma}) \sum_{j=1}^{J} \log_2\Big(1 + \gamma_{k,m}^{(j)} p_{k,m}^{(j)}(\boldsymbol{\gamma})\Big)\Big] \\ &\qquad \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{k=1}^{K}\sum_{m=1}^{M} \alpha_{k,m}(\boldsymbol{\gamma}) \sum_{j=1}^{J} p_{k,m}^{(j)}(\boldsymbol{\gamma})\Big] \leq \check{P} \end{aligned} \quad (2)$$

where U is a selected *concave* utility function, $\mathbb{E}_{\boldsymbol{\gamma}}[\cdot]$ denotes expectation over fading realization $\boldsymbol{\gamma}$, and $\check{P}$ is the average sum-power budget at the AP for all downlink transmissions. Here we assume that the AP can support continuous rate adaptation up to Shannon's limit per subcarrier.[1]

The utility function in our formulation is to balance the total throughput and fairness among users. For instance, it has been shown that the so-called α-fairness can be attained by the maximizer of a class of concave utility functions [7], [8]:

$$U_\alpha(x) = \begin{cases} x^{1-\alpha}/(1-\alpha), & \alpha \neq 1, \\ \ln(x), & \alpha = 1. \end{cases} \quad (3)$$

The notion of α-fairness includes max-min fairness (with $\alpha \to \infty$) [18], proportional fairness (with $\alpha = 1$) [19], and throughput maximization (with $\alpha = 0$) as special cases. Larger α means more fairness. These α-fair utility functions will be used to test the proposed algorithms in simulations.

A. Optimal Chunk and Power Allocation

Different from the optimal subcarrier scheduling in [12] where each subcarrier is allowed to be time shared by users at the outset, here we assume *a fortiori* an exclusive chunk allocation, i.e., at most one user allocated to a single chunk, for a low-complexity transceiver design. This results a (non-convex) mixed-integer program in (2). However, we next show that this problem (2) can be still solved using a Lagrange dual approach under conditions. Let $\boldsymbol{\lambda} := \{\lambda_k,\ \forall k\}$ collect the Lagrange multipliers associated with the constraints $\bar{r}_k \leq \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_m \alpha_{k,m}(\boldsymbol{\gamma}) \sum_j \log_2\left(1 + \gamma_{k,m}^{(j)} p_{k,m}^{(j)}(\boldsymbol{\gamma})\right)\right]$, $\forall k$, and let μ denote the Lagrange multiplier for

[1]Generalization to continuous- or discrete-rate adaptation using practical adaptive modulation and coding (AMC) schemes is also possible along the lines of our work [10].

$\mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k,m}\alpha_{k,m}(\boldsymbol{\gamma})\sum_j p_{k,m}^{(j)}(\boldsymbol{\gamma})\right] \le \check{P}$. Using the convenient notations $\boldsymbol{X} := \{\bar{\boldsymbol{r}};\ \boldsymbol{\alpha}(\boldsymbol{\gamma}), \boldsymbol{p}(\boldsymbol{\gamma}), \forall \boldsymbol{\gamma}\}$ and $\boldsymbol{\Lambda} := \{\boldsymbol{\lambda},\ \mu\}$, the Lagrangian function of (2) is:

$$\begin{aligned} L(\boldsymbol{X}, \boldsymbol{\Lambda}) \;=\; & \sum_k U(\bar{r}_k) - \sum_k \lambda_k \Big(\bar{r}_k - \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_m \alpha_{k,m}(\boldsymbol{\gamma}) \\ & \times \sum_j \log_2(1+\gamma_{k,m}^{(j)} p_{k,m}^{(j)}(\boldsymbol{\gamma}))\Big]\Big) - \\ & \mu\Big(\mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{k,m}\alpha_{k,m}(\boldsymbol{\gamma})\sum_j p_{k,m}^{(j)}(\boldsymbol{\gamma})\Big] - \check{P}\Big). \end{aligned} \tag{4}$$

The Lagrange dual function is then given by:

$$D(\boldsymbol{\Lambda}) = \max_{\bar{\boldsymbol{r}}\ge 0;\ (\boldsymbol{\alpha},\boldsymbol{p})\in\mathcal{A}} \ L(\boldsymbol{X}, \boldsymbol{\Lambda}), \tag{5}$$

and the dual problem of (2) is:

$$\min_{\boldsymbol{\Lambda}\ge 0}\ D(\boldsymbol{\Lambda}). \tag{6}$$

To solve the dual problem (6), we need to specify the dual function $D(\boldsymbol{\Lambda})$ in (5). Upon defining

$$\varphi_{k,m}^{(j)}(p_{k,m}^{(j)}(\boldsymbol{\gamma})) := \lambda_k \log_2(1+\gamma_{k,m}^{(j)} p_{k,m}^{(j)}(\boldsymbol{\gamma})) - \mu p_{k,m}^{(j)}(\boldsymbol{\gamma}), \tag{7}$$

we can rewrite (4) as:

$$\begin{aligned} L(\boldsymbol{X}, \boldsymbol{\Lambda}) = & \mu\check{P} + \sum_k [U(\bar{r}_k) - \lambda_k \bar{r}_k] \\ & + \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_k\sum_m \alpha_{k,m}(\boldsymbol{\gamma})\sum_j \varphi_{k,m}^{(j)}(p_{k,m}^{(j)}(\boldsymbol{\gamma}))\Big]. \end{aligned} \tag{8}$$

From (8), the optimal $\bar{r}_k^*(\boldsymbol{\Lambda})$ maximizing $L(\boldsymbol{X}, \boldsymbol{\Lambda})$ in (5) should solve: $\max_{\bar{r}_k\ge 0}\ [U(\bar{r}_k) - \lambda_k \bar{r}_k]$, $\forall k$. Provided that the selected concave function U, e.g., the one in (3), is differentiable and its first derivative U' has a well-defined inverse U'^{-1}, then we clearly have:

$$\bar{r}_k^*(\boldsymbol{\Lambda}) = U'^{-1}(\lambda_k), \quad \forall k. \tag{9}$$

On the other hand, the optimal chunk and power allocation solves:

$$\max_{(\boldsymbol{\alpha},\boldsymbol{p})\in\mathcal{A}} \ \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_k\sum_m \alpha_{k,m}(\boldsymbol{\gamma})\sum_j \varphi_{k,m}^{(j)}(p_{k,m}^{(j)}(\boldsymbol{\gamma}))\Big]. \tag{10}$$

Regardless of $\alpha_{k,m}(\boldsymbol{\gamma}) \ge 0$, the optimally allocated power should maximize $\varphi_{k,m}^{(j)}(p_{k,m}^{(j)}(\boldsymbol{\gamma}))$, $\forall k, m, j, \forall \boldsymbol{\gamma}$. Since $\varphi_{k,m}^{(j)}(p_{k,m}^{(j)}(\boldsymbol{\gamma}))$ in (7) is a concave function of $p_{k,m}^{(j)}(\boldsymbol{\gamma})$, its maximizer is:

$$\tilde{p}_{k,m}^{(j)*}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) = \Big[\frac{\lambda_k}{\mu \ln 2} - \frac{1}{\gamma_{k,m}^{(j)}}\Big]_0^{p_{\max}} \tag{11}$$

where $[\cdot]_0^{p_{\max}}$ denotes the projection into the interval $[0, p_{\max}]$. Based on $\tilde{p}_{k,m}^{(j)*}(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$, define subsequently

$$\varphi_{k,m}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) := \sum_j \varphi_{k,m}^{(j)}(\tilde{p}_{k,m}^{(j)*}(\boldsymbol{\gamma}; \boldsymbol{\Lambda})).$$

Then the chunk allocation solves per $\boldsymbol{\gamma}$:

$$\max \sum_k\sum_m \alpha_{k,m}(\boldsymbol{\gamma})\varphi_{k,m}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}).$$

Under the constraints $\alpha_{k,m}(\boldsymbol{\gamma}) \in \{0,1\}$ and $\sum_k \alpha_{k,m}(\boldsymbol{\gamma}) \le 1$, the optimal chunk allocation should adopt a "winner-takes-all" strategy per chunk; i.e., chunk m is assigned to the user

$$k_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) = \arg\max_k \ \varphi_{k,m}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}), \quad \forall m,\ \forall \boldsymbol{\gamma}. \tag{12}$$

This establishes the following lemma:

Lemma 1: *For a given $\boldsymbol{\Lambda}$, the optimal $\bar{r}_k^*(\boldsymbol{\Lambda})$ for (5) is given by (9), and the optimal chunk and power allocation amounts to: $\forall \boldsymbol{\gamma}$,*

$$\begin{cases} \alpha_{k_m^*,m}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) = 1, \ \ p_{k_m^*,m}^{(j)*}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) = \tilde{p}_{k_m^*,m}^{(j)*}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}), \\ \alpha_{k,m}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) = p_{k,m}^{(j)*}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) = 0, \qquad \forall k \ne k_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}) \end{cases} \tag{13}$$

where the "winner" $k_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ per chunk m is chosen by (12).

Here the optimal scheduling at the AP amounts to a greedy water-filling solution where power allocation and chunk allocation are decoupled. In the first stage, transmit-power is allocated per user k across subcarriers following a water-filling principle, i.e., with higher power assigned to better channel realizations $\gamma_{k,m}^{(j)}$; see (11). In the second stage, $\varphi_{k,m}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ represents the maximum net-reward (rate reward minus power cost) that user k can obtain over all subcarriers of chunk m. Comparing the net-rewards across users, chunk m is then assigned to the user $k_m^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda})$ with the highest net-reward.

With $\boldsymbol{X}^*(\boldsymbol{\Lambda}) := \{\bar{\boldsymbol{r}}^*(\boldsymbol{\Lambda});\ \boldsymbol{\alpha}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}), \boldsymbol{p}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}), \forall \boldsymbol{\gamma}\}$ provided by Lemma 1 for a given $\boldsymbol{\Lambda}$, the dual function $D(\boldsymbol{\Lambda})$ in (5) can be specified. Using the notation $\boldsymbol{X}$ and $\boldsymbol{\Lambda}$, we arrange the constraints in (2) into a compact form: $\boldsymbol{g}(\boldsymbol{X}) \ge \boldsymbol{0}$. Then it can be also shown that $\boldsymbol{g}(\boldsymbol{X}^*(\boldsymbol{\Lambda}))$ is a (sub-)gradient of the dual function $D(\boldsymbol{\Lambda})$ [20]. Therefore, the dual problem (6) can be solved through the following (sub-)gradient descent iteration [5], [12]:

$$\boldsymbol{\Lambda}[n+1] = [\boldsymbol{\Lambda}[n] - \beta \boldsymbol{g}(\boldsymbol{X}^*(\boldsymbol{\Lambda}[n]))]^+ . \tag{14}$$

Specifically, we have:

$$\begin{aligned} \lambda_k[n+1] = & \Big[\lambda_k[n] + \beta\Big(\bar{r}_k^*(\boldsymbol{\Lambda}[n]) - \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_m \alpha_{k,m}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}[n]) \\ & \times \sum_j \varphi_{k,m}^{(j)}(p_{k,m}^{(j)*}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}[n]))\Big]\Big)\Big]^+ \\ \mu[n+1] = & \Big[\mu[n] + \beta\Big(\mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_k\sum_m \alpha_{k,m}^*(\boldsymbol{\gamma}; \boldsymbol{\Lambda}[n]) \\ & \times \sum_j p_{k,m}^{(j)*}(\boldsymbol{\gamma}; \boldsymbol{\Lambda}[n])\Big] - \check{P}\Big)\Big]^+ \end{aligned} \tag{15}$$

where β is a small stepsize, n is the iteration index, and $[x]^+ := \max(0, x)$. Convergence of the gradient descent iteration (15) to the optimal Lagrange multipliers $\boldsymbol{\Lambda}^* := \{\boldsymbol{\lambda}^*,\ \mu^*\}$ for (6) is guaranteed from any initial $\boldsymbol{\Lambda}[0] \ge 0$, and this convergence can be geometrically fast under general conditions [20], [21].

For the non-convex mixed-integer program (2), there may exist non-zero duality gap; hence solving the dual problem (6) via (15) may not yield the optimal solution for (2). Under the condition that the channel coefficient vector $\boldsymbol{\gamma}$ has a continuous cdf, however, we can show the following proposition:

Proposition 1: *For ergodic fading channels with continuous cdf, the problem (2) has a zero duality gap with its dual (6),* *and the almost surely optimal solution for (2) is given by* $\{\bar{\boldsymbol{r}}^*(\boldsymbol{\Lambda}^*);\ \boldsymbol{\alpha}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*), \boldsymbol{p}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*), \forall\boldsymbol{\gamma}\}$ *specified in Lemma 1, where* $\boldsymbol{\Lambda}^*$ *is obtained from (15) with any initial* $\boldsymbol{\Lambda}[0] \geq 0$.

Proof: The "winner-takes-all" chunk allocation policy per chunk m at channel realization $\boldsymbol{\gamma}$ in Lemma 1 subsumes three cases: i) If $\varphi^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda}) = 0, \forall k$, and thus $\max_k \varphi^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda}) = 0$, all users' channels indeed experience deep fading over all subcarriers of chunk m such that $\gamma^{(j)}_{k,m} \leq \mu \ln 2/\lambda_k$, $\forall j$, $\forall k$. Upon such a deep fading state, any user k, even if scheduled, will be allocated with transmit-power $p^{(j)*}_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda}) = 0$, $\forall j$ [cf. (11)]. Therefore, the *unique* optimal strategy for AP is to defer its transmission at chunk m, which can be represented by the policy in (13) where the chunk is assigned to an arbitrary "winner" but zero transmit-powers are allocated. ii) If $\max_k \varphi^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda}) > 0$ and it is attained by a single "winner", the optimal allocation given by (13) is clearly unique. iii) If $\max_k \varphi^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda}) > 0$ and it is attained by multiple users, the chunk allocation becomes non-unique. However, it can be shown that having multiple "winners" in case iii) is an event of Lebesgue measure zero, provided that the random channels have a continuous cdf. The non-unique "winner" selection in this case then has a "measure-zero" effect, since our criterion is to maximize the *average* net-reward in (10). Therefore, the optimal chunk and power allocation policy $\{\boldsymbol{\alpha}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}), \boldsymbol{p}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda})\}$ in Lemma 1 is almost surely unique.

This almost sure uniqueness is the key to establish the zero duality gap result for the non-convex mixed-integer program (2). By relaxing the chunk allocation decision $\alpha_{k,m}(\boldsymbol{\gamma})$ to a real number $\in [0,1]$ in (2), we have an optimization over continuous variables. By further taking a viable change $\breve{p}^{(j)}_{k,m}(\boldsymbol{\gamma}) := \alpha_{k,m}(\boldsymbol{\gamma}) p^{(j)}_{k,m}(\boldsymbol{\gamma})$, this optimization problem can be reformulated into:

$$\begin{aligned}\max_{\bar{\boldsymbol{r}}\geq 0;\ (\boldsymbol{\alpha},\breve{\boldsymbol{p}})\in\mathcal{A}'} \quad & \sum_{k=1}^{K} U(\bar{r}_k) \\ \text{s. t.} \quad \bar{r}_k \leq & \ \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M} \alpha_{k,m}(\boldsymbol{\gamma}) \sum_{j=1}^{J} \log_2\Big(1 + \gamma^{(j)}_{k,m}\frac{\breve{p}^{(j)}_{k,m}(\boldsymbol{\gamma})}{\alpha_{k,m}(\boldsymbol{\gamma})}\Big)\Big] \\ & \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{j=1}^{J} \breve{p}^{(j)}_{k,m}(\boldsymbol{\gamma})\Big] \leq \breve{P}\end{aligned} \tag{16}$$

where $\mathcal{A}'$ denotes the set of all schedules satisfying: $0 \leq \alpha_{k,m}(\boldsymbol{\gamma}) \leq 1$, $\sum_{k=1}^{K} \alpha_{k,m}(\boldsymbol{\gamma}) \leq 1$, and $0 \leq \breve{p}^{(j)}_{k,m}(\boldsymbol{\gamma}) \leq \alpha_{k,m}(\boldsymbol{\gamma}) p_{\max}$, $\forall k, m, j$.

Following the similar lines of [12], it can be shown that (16) is a convex program. Using the Lagrange multipliers $\boldsymbol{\lambda}$ and μ, we have the similar Lagrangian function (5) for (16). It is not difficult to see that the almost surely unique "winner-takes-all" policy holds true when we maximize the Lagrangian function $L(\boldsymbol{X}, \boldsymbol{\Lambda})$ in (5) even if $\alpha_{k,m}(\boldsymbol{\gamma})$ is allowed to take continuous value. Therefore, we will have exactly the same dual function $D(\boldsymbol{\Lambda})$ and dual problem (6) for the relaxed problem (16). Since the convex program (16) has zero duality gap with its dual (6), the chunk allocation and power control strategy $\{\boldsymbol{\alpha}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*), \boldsymbol{p}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*)\}$ in Proposition 1 is almost surely optimal for (16) [21]. Clearly, the relaxed problem (16) always has an optimal value P'^* not less than the optimal value P^* of the original problem (2); i.e., $P'^* \geq P^*$. On the other hand, since the "winner-takes-all" policy $\{\boldsymbol{\alpha}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*), \boldsymbol{p}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*)\}$ lies within the feasible set of (2), it is clear that the value P'^* obtained by the latter policy is not greater than P^*; i.e., $P'^* \leq P^*$. Therefore we must have $P'^* = P^*$. Let D^* denote the optimal value for the dual problem (6). Due to zero duality gap between (16) and (6), we have $P'^* = D^*$. This implies that (2) has a zero duality gap with its dual (6), i.e., $P^* = D^*$, and thereby $\{\boldsymbol{\alpha}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*), \boldsymbol{p}^*(\boldsymbol{\gamma};\boldsymbol{\Lambda}^*)\}$ is also almost surely optimal for (2). ■

Proposition 1 shows that the dual-gradient iteration (15) can find the optimal chunk allocation and power control strategy for (2). A similar approach was employed to derive the optimal subcarrier scheduling for OFDMA system in [12], where it was assumed that each individual subcarrier can be assigned to a user. Given that transmit-powers can be adaptively allocated across individual subcarriers, the optimal power allocation follows a "water-filling" principle in (11) with or without predefined subcarrier-chunks. When chunk is the minimum unit for subcarrier allocation, the net-reward $\varphi^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda})$ needs to be calculated for each chunk (instead of for each subcarrier) per user to determine the subcarrier-chunk allocation. Clearly this chunk-based optimal resource allocation specifies to the subcarrier-based one in [12], when the number of subcarriers per chunk $J = 1$. But in general the subcarrier allocation policy is different, and thus the optimal Lagrange multipliers λ^*_k and μ^* in the power control (11) are different for the cases with or without predefined chunks. Therefore, albeit in a similar form, the power allocation values (as well as subcarrier scheduling strategy) are significantly different for the optimal schemes with and without predefined chunks.

B. Stochastic Scheduling

To implement the dual-gradient iteration (14), we need *a-priori* knowledge of fading cdf to evaluate the two expected values in (15). Since mobile applications motivate schedulers that can learn the required cdf on-the-fly, we next develop an on-line scheduling scheme to approach the optimal strategy using a stochastic optimization paradigm [9], [10], [22], [23]. To this end, we drop $\mathbb{E}_{\boldsymbol{\gamma}}$ from (15) and get a stochastic version of (15) based on per slot channel realization $\boldsymbol{\gamma}[n]$ as follows:

$$\begin{aligned}\hat{\lambda}_k[n+1] &= \Big[\hat{\lambda}_k[n] + \beta\Big(\bar{r}^*_k(\hat{\boldsymbol{\Lambda}}[n]) - \sum_m \alpha^*_{k,m}(\boldsymbol{\gamma}[n];\hat{\boldsymbol{\Lambda}}[n]) \\ &\qquad \times \sum_j \varphi^{(j)}_{k,m}(p^{(j)*}_{k,m}(\boldsymbol{\gamma}[n];\hat{\boldsymbol{\Lambda}}[n]))\Big)\Big]^+ \\ \hat{\mu}[n+1] &= \Big[\hat{\mu}[n] + \beta\Big(\sum_k\sum_m \alpha^*_{k,m}(\boldsymbol{\gamma}[n];\hat{\boldsymbol{\Lambda}}[n]) \\ &\qquad \times \sum_j p^{(j)*}_{k,m}(\boldsymbol{\gamma}[n];\hat{\boldsymbol{\Lambda}}[n]) - \breve{P}\Big)\Big]^+\end{aligned} \tag{17}$$

where the hat notation with λ_k and μ is used to stress the fact that these iterations are stochastic estimates of those in (15), based on *instantaneous* (instead of average) power and rates.

In (17), a stochastic dual-gradient is employed for Lagrange multiplier updates. For stationary and ergodic channel processes, convergence of this stochastic gradient based iteration (17) to the optimal $\boldsymbol{\Lambda}^*$ in probability as the stepsize $\beta \to 0$ can be established along the similar lines with those in [9],

[10], [11]. As a result, the companion chunk and power allocation scheme $\{\boldsymbol{\alpha}^*(\boldsymbol{\gamma};\hat{\boldsymbol{\Lambda}}[n]), \boldsymbol{p}^*(\boldsymbol{\gamma};\hat{\boldsymbol{\Lambda}}[n])\}$ also converges (in probability) to the globally optimal one for (2).

Since the stochastic iteration (17) can yield the optimal chunk and power allocation for (2) without knowing the channel cdf a priori, a simple on-line scheduling algorithm can be implemented at the AP as follows:

s1) Starting from arbitrary $\hat{\boldsymbol{\Lambda}}[0] \geq 0$, determine the on-line chunk and power allocation upon channel realization $\boldsymbol{\gamma}[n]$ per slot n using (13);

s2) Update $\hat{\boldsymbol{\Lambda}}[n+1]$ from $\hat{\boldsymbol{\Lambda}}[n]$ using (17); then implement s1) and s2) again for the next slot $n+1$.

In this algorithm, the AP needs only to calculate the net-rewards $\varphi^*_{k,m}(\boldsymbol{\gamma}[n];\hat{\boldsymbol{\Lambda}}[n])$ for all the K users per chunk m, and then adopts the "winner-takes-all" strategy in Lemma 1 to determine the chunk allocation and associated power control per slot n. Calculating all the power values via (11) and the net-reward $\varphi^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda})$ for each chunk per user requires a computational complexity of $\mathcal{O}(KMJ)$, whereas determining the chunk allocation via (13) needs a computational complexity of $\mathcal{O}(M\log K)$. Hence, all required operations per slot have a linear computational complexity $\mathcal{O}(KMJ)$ in the number of users and total subcarriers. Yet, this low-complexity algorithm is capable of learning the channel distribution to approach the optimal scheduling and resource allocation on-line. Moreover, convergence of the algorithm could be geometrically fast under conditions due to the geometrically fast convergence of (15) and the trajectory locking of the stochastic (17) and "ensemble" gradient iterations (15) [10], [11].

IV. Adaptive Power Allocation across Chunks

Allowing channel-adaptive power allocation across individual subcarriers enables derivation of an optimum benchmark for system performance. For chunk-based OFDMA systems, however, it is certainly desirable to make "chunk" the minimum unit for overall resource (instead of only subcarrier) allocation. This requires that the same transmit-power is allocated to all the subcarriers of a chunk; i.e., we enforce $p^{(j)}_{k,m}(\boldsymbol{\gamma}) = p_{k,m}(\boldsymbol{\gamma})$, $\forall j$. Upon defining the new power allocation vector $\boldsymbol{p}(\boldsymbol{\gamma}) := \{p_{k,m}(\boldsymbol{\gamma}),\ \forall k,m\}$, the problem to solve becomes [cf. (2)]:

$$\begin{aligned}&\max_{\bar{\boldsymbol{r}}\geq 0;\,(\boldsymbol{\alpha},\boldsymbol{p})\in\mathcal{A}} \sum_{k=1}^{K} U(\bar{r}_k)\\ &\text{s. t.}\ \ \bar{r}_k \leq \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M}\alpha_{k,m}(\boldsymbol{\gamma})\sum_{j=1}^{J}\log_2\Big(1+\gamma^{(j)}_{k,m}p_{k,m}(\boldsymbol{\gamma})\Big)\Big].\\ &\qquad \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{k=1}^{K}\sum_{m=1}^{M}\alpha_{k,m}(\boldsymbol{\gamma})(Jp_{k,m}(\boldsymbol{\gamma}))\Big] \leq \check{P}\end{aligned} \tag{18}$$

Using a similar Lagrange dual approach, we obtain the Lagrangian function of (18):

$$\begin{aligned}L(\boldsymbol{X},\boldsymbol{\Lambda}) \;=\; &\sum_k U(\bar{r}_k) - \sum_k \lambda_k\Big(\bar{r}_k - \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_m \alpha_{k,m}(\boldsymbol{\gamma})\\ &\times\sum_j \log_2(1+\gamma^{(j)}_{k,m}p_{k,m}(\boldsymbol{\gamma}))\Big]\Big) - \\ &\mu\Big(\mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{k,m}\alpha_{k,m}(\boldsymbol{\gamma})Jp_{k,m}(\boldsymbol{\gamma})\Big] - \check{P}\Big).\end{aligned} \tag{19}$$

To obtain the dual function $D(\boldsymbol{\Lambda})$ as in (5), we need to maximize $L(\boldsymbol{X},\boldsymbol{\Lambda})$ over $\boldsymbol{X}$. For a given $\boldsymbol{\Lambda}$, it is clear that the optimal $\bar{r}^*_k(\boldsymbol{\Lambda})$ is still given by (9).

Upon (re-)defining

$$\varphi_{k,m}(p_{k,m}(\boldsymbol{\gamma})) := \lambda_k\Big[\sum_j \log_2(1+\gamma^{(j)}_{k,m}p_{k,m}(\boldsymbol{\gamma}))\Big] - \mu J p_{k,m}(\boldsymbol{\gamma}),$$

the optimal chunk and power allocation maximizing $L(\boldsymbol{X},\boldsymbol{\Lambda})$ should solve:

$$\max_{(\boldsymbol{\alpha},\boldsymbol{p})\in\mathcal{A}} \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_k\sum_m \alpha_{k,m}(\boldsymbol{\gamma})\varphi_{k,m}(p_{k,m}(\boldsymbol{\gamma}))\Big].$$

It is easy to see that $\varphi_{k,m}(p_{k,m}(\boldsymbol{\gamma}))$ is a concave function of $p_{k,m}(\boldsymbol{\gamma})$. Its first derivative

$$\varphi'_{k,m}(p_{k,m}(\boldsymbol{\gamma})) = \frac{\lambda_k}{\ln 2}\sum_j \frac{\gamma^{(j)}_{k,m}}{1+\gamma^{(j)}_{k,m}p_{k,m}(\boldsymbol{\gamma})} - \mu J$$

is a decreasing function of $p_{k,m}(\boldsymbol{\gamma}) \geq 0$. Let $\varphi'^{-1}_{k,m}(0)$ denote the unique solution to the equation $\varphi'_{k,m}(p_{k,m}(\boldsymbol{\gamma})) = 0$. The optimal power value maximizing $\varphi_{k,m}(p_{k,m}(\boldsymbol{\gamma}))$ is clearly:

$$\tilde{p}^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda}) = \Big[\varphi'^{-1}_{k,m}(0)\Big]_0^{p_{\max}},$$

which can be easily obtained by a bi-sectional search over $[0, p_{\max}]$. Upon defining $\varphi^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda}) := \varphi_{k,m}(\tilde{p}^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda}))$, the optimal chunk allocation is to allocate chunk m to a user $k^*_m(\boldsymbol{\gamma};\boldsymbol{\Lambda})$ with the largest $\varphi^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda})$. Therefore, the optimal chunk and power allocation amounts to: $\forall\boldsymbol{\gamma}$,

$$\begin{cases}\alpha^*_{k^*_m,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda}) = 1, & p^*_{k^*_m,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda}) = \tilde{p}^*_{k^*_m,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda}),\\ \alpha^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda}) = p^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda}) = 0, & \forall k \neq k^*_m(\boldsymbol{\gamma};\boldsymbol{\Lambda})\end{cases} \tag{20}$$

where the "winner" per chunk m is: $k^*_m(\boldsymbol{\gamma};\boldsymbol{\Lambda}) = \arg\max_k \varphi^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda})$.

With $\bar{r}^*_k(\boldsymbol{\Lambda})$ given by (9) and $\{\alpha^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda}), p^*_{k,m}(\boldsymbol{\gamma};\boldsymbol{\Lambda})\}$ specified in (20), the optimal $\boldsymbol{\Lambda}^*$ can be then obtained through the stochastic gradient iteration [cf. (17)]:

$$\begin{aligned}\hat{\lambda}_k[n+1] &= \Big[\hat{\lambda}_k[n] + \beta\Big(\bar{r}^*_k(\hat{\boldsymbol{\Lambda}}[n]) - \sum_m \alpha^*_{k,m}(\boldsymbol{\gamma}[n];\hat{\boldsymbol{\Lambda}}[n])\\ &\qquad\times\varphi_{k,m}(p^*_{k,m}(\boldsymbol{\gamma}[n];\hat{\boldsymbol{\Lambda}}[n]))\Big)\Big]^+\\ \hat{\mu}[n+1] &= \Big[\hat{\mu}[n] + \beta\Big(\sum_k\sum_m \alpha^*_{k,m}(\boldsymbol{\gamma}[n];\hat{\boldsymbol{\Lambda}}[n])\\ &\qquad\times Jp^*_{k,m}(\boldsymbol{\gamma}[n];\hat{\boldsymbol{\Lambda}}[n]) - \check{P}\Big)\Big]^+\end{aligned} \tag{21}$$

It can be shown that the non-convex mixed-integer program (18) has a zero duality gap with its dual problem. Consequently, the on-line chunk and power allocation scheme $\{\boldsymbol{\alpha}^*(\boldsymbol{\gamma};\hat{\boldsymbol{\Lambda}}[n]), \boldsymbol{p}^*(\boldsymbol{\gamma};\hat{\boldsymbol{\Lambda}}[n])\}$ resulting from (21) converges to the optimal one for (18) without a-priori knowledge of channel cdf. It is easy to see that the required operations for the proposed algorithm have a computational complexity $\mathcal{O}(KMJ)$, and convergence of the algorithm could be geometrically fast.

V. Constant Power Allocation

To avoid large peak-to-average-power ratio that is detrimental to mobile terminals, practical OFDMA systems actually often require a constant transmit-power allocated to all subcarriers. It is of interest to consider the chunk assignment under such a constant power allocation. With an equally divided power $p_{k,m}^{(j)}(\boldsymbol{\gamma}) = \check{P}/(MJ)$ spent per subcarrier, the utility maximization problem becomes:

$$\max_{\bar{\boldsymbol{r}}\geq 0;\ \boldsymbol{\alpha}\in\mathcal{A}} \sum_{k=1}^{K} U(\bar{r}_k)$$
$$\text{s. t.}\quad \bar{r}_k \leq \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M}\alpha_{k,m}(\boldsymbol{\gamma})\sum_{j=1}^{J}\log_2\Big(1+\gamma_{k,m}^{(j)}\frac{\check{P}}{MJ}\Big)\Big]. \tag{22}$$

Here the power constraint is naturally met; hence, it is absent. With the optimization variable vector $\boldsymbol{X} := \{\bar{\boldsymbol{r}},\ \boldsymbol{\alpha}(\boldsymbol{\gamma}), \forall \boldsymbol{\gamma}\}$ and Lagrange multiplier vector $\boldsymbol{\lambda}$, the Lagrangian function of (22) is:

$$L(\boldsymbol{X},\boldsymbol{\lambda}) = \sum_k U(\bar{r}_k) - \sum_k \lambda_k\Big(\bar{r}_k - \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_m \alpha_{k,m}(\boldsymbol{\gamma}) \times \sum_j \log_2\Big(1+\gamma_{k,m}^{(j)}\check{P}/(MJ)\Big)\Big]\Big)$$

To maximize $L(\boldsymbol{X},\boldsymbol{\lambda})$, the optimal $\bar{r}_k^*(\boldsymbol{\lambda})$ is again given by (9), and the optimal chunk allocation should solve:

$$\max_{\boldsymbol{\alpha}\in\mathcal{A}} \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_k\sum_m \alpha_{k,m}(\boldsymbol{\gamma})\sum_j \log_2\Big(1+\gamma_{k,m}^{(j)}\frac{\check{P}}{MJ}\Big)\Big].$$

It is clear that this amounts to a greedy assignment: $\forall\boldsymbol{\gamma}$,

$$\begin{cases}\alpha_{k_m^*,m}^*(\boldsymbol{\gamma};\boldsymbol{\lambda}) = 1,\\ \alpha_{k,m}^*(\boldsymbol{\gamma};\boldsymbol{\lambda}) = 0, \qquad \forall k \neq k_m^*(\boldsymbol{\gamma};\boldsymbol{\lambda})\end{cases} \tag{23}$$

where the "winner" per chunk m is: $k_m^*(\boldsymbol{\gamma};\boldsymbol{\lambda}) = \arg\max_k \sum_j \log_2\Big(1+\gamma_{k,m}^{(j)}\frac{\check{P}}{MJ}\Big)$.

The optimal $\boldsymbol{\lambda}^*$ can be obtained through the stochastic gradient iteration:

$$\hat{\lambda}_k[n+1] = \Big[\hat{\lambda}_k[n] + \beta\Big(\bar{r}_k^*(\hat{\boldsymbol{\lambda}}[n]) - \sum_m \alpha_{k,m}^*(\boldsymbol{\gamma}[n];\hat{\boldsymbol{\lambda}}[n]) \times \sum_j \log_2\Big(1+\gamma_{k,m}^{(j)}\check{P}/(MJ)\Big)\Big)\Big]^+ \tag{24}$$

Again the chunk allocation $\boldsymbol{\alpha}^*(\boldsymbol{\gamma};\hat{\boldsymbol{\lambda}}[n])$ resulting from (24) converges to the optimal one for (22) without a-priori knowledge of channel cdf. The on-line algorithm is fast convergent and has a computational complexity $\mathcal{O}(KMJ)$ per slot.

VI. Numerical Results

In this section, we provide numerical examples to test the algorithms developed in the previous sections. We consider $K = 4$ user OFDMA downlink transmissions over frequency-selective wireless channels. The total bandwidth is $B = 20$ MHz, and there are 512 subcarriers, each with sub-bandwidth $\Delta = 40$ KHz. The subcarriers are grouped into $M = 64$ chunks, each consisting of $J = 8$ subcarriers. These parameters are the typical setup for European next generation wireless systems in urban scenario [13]. For each user's wireless link, a profile of 20 μs exponentially and independently decaying tap gains is assumed changing independently across slots of 500 μs. The average signal-to-noise ratios (SNRs) for users $k = 1, \ldots, 4$ are 10 dB, 8 dB, 6 dB, and 4 dB, respectively.

For the simulated chunk-based OFDMA downlink, four schemes are tested: i) the optimal scheme with adaptive power control across subcarriers (denoted as APS) in Section III; ii) the optimal scheme with adaptive power control across chunks (denoted as APC) in Section IV; iii) the optimal chunk allocation with constant power allocation (denoted as OCP) in Section V; and iv) a baseline fixed-access scheme (denoted as FCP) where each user is allocated to a fixed set of 16 chunks in order and constant power allocation is adopted. For the first three schemes, the utility function $\sum_k U(\bar{r}_k) := \sum_k \ln(\bar{r}_k)$ is used as the objective, which corresponds to a proportional fair scheduling strategy that is widely adopted in wireless standards. Fig. 1 shows the resulting average sum-rates (i.e., network throughput) from the four schemes under different average sum-power budget $\check{P}$. For comparison, Fig. 1 also includes the average sum-rates achieved by the optimal subcarrier and power allocation scheme (denoted as subcarrier-based APS) when the chuck size is only one subcarrier (i.e., $M = 512$ and $J = 1$); in this case each user can be assigned an arbitrary number of subcarriers. It is clear that both the chunk-based APS and APC schemes substantially outperform the OCP scheme. This shows that the adaptive power control can collect large throughput gain. The subcarrier-based APS scheme yields the highest network throughput; however, it only has an almost negligible performance gain over the chunk-based APS scheme (the two lines are almost identical such that the one for subcarrier-based APS is almost invisible in the figure). Compared to the chunk-based APC scheme, the gain from the APS schemes is also marginal. This evidently indicates that using subcarrier-chunks as minimum unit for resource allocation only incurs small throughput loss while decreasing considerably signaling overhead for practical OFDMA systems. For the baseline FCP scheme, the fairness among the users is not considered. Despite this, all the proposed schemes with optimally adaptive chunk allocation, including the OCP scheme, significantly outperform the baseline FCP scheme. This is because adaptive chunk allocation can attain the multi-user diversity, thereby bringing large gain in throughput.

The channel cdf was assumed unknown in all simulations, and the proposed stochastic schemes were supposed to learn this knowledge on-line to approach the optimal strategies. To verify this, Fig. 2 depicts the evolution of Lagrange multipliers $\hat{\lambda}_k$, $k = 1, \ldots, 4$, and $\hat{\mu}$ in (17) for the APS scheme when the power budget $\check{P} = 1$ Watt and the stepsize $\beta = 0.001$. It is well-known that there exists a tradeoff between convergence speed and optimality for stochastic optimization in the adaptive signals and systems literature [9], [10], [11], [22], [23]. As with any stochastic approximation scheme, Lagrange multipliers in the stochastic gradient iteration (17) can only

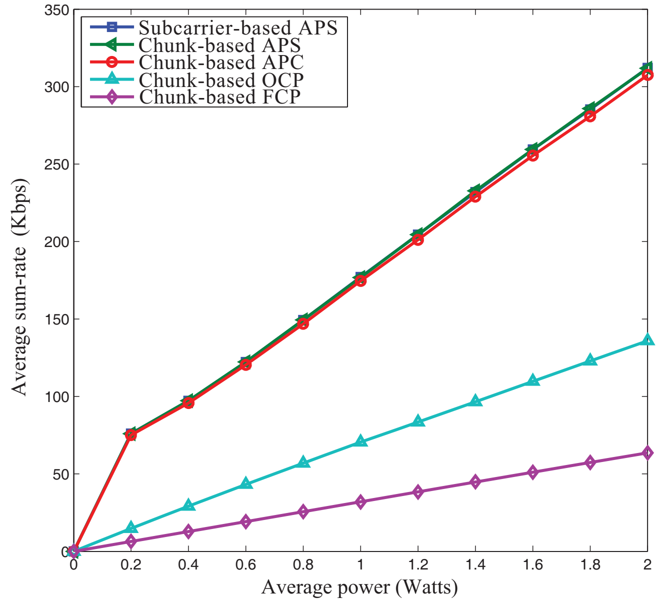


Fig. 1. Comparison of average sum-rates.

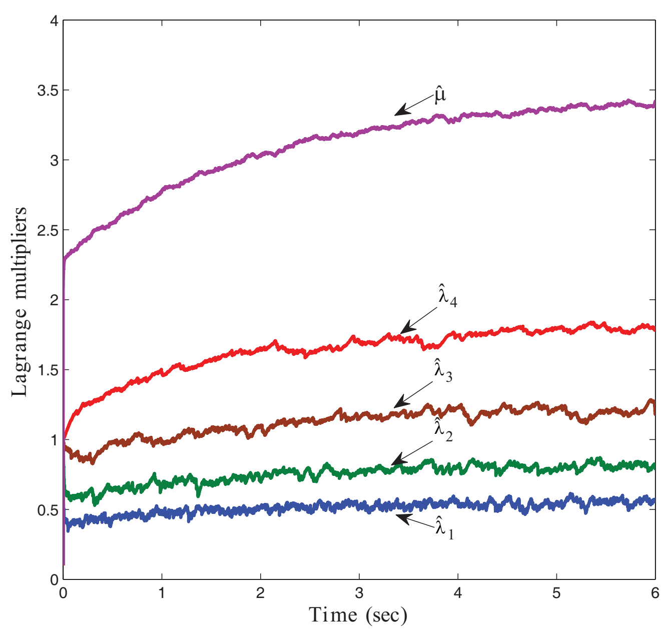


Fig. 2. Evolution of Lagrange multipliers.

converge to (or hover within) a small neighborhood with a size proportional to stepsize β around the optimal values; hence, one needs a small β to come "closer" to optimality, but the smaller β is chosen, the slower convergence speed is experienced. With the given stepsize, such a stochastic convergence is clearly observed, since the Lagrange multipliers quickly converge to the neighborhood of the optimal values.

It is seen from Fig. 1 that both the APS and APC schemes can attain noticeable throughput gains over the OCP scheme. These gains brought by adaptive power control (across subcarriers or chunks) are accompanied by large variation in transmit-powers, thereby large peak-to-average-power ratios. To see it, Fig. 3 shows the ratios of current power to average power at subcarriers 29 and 30 for the APS scheme and

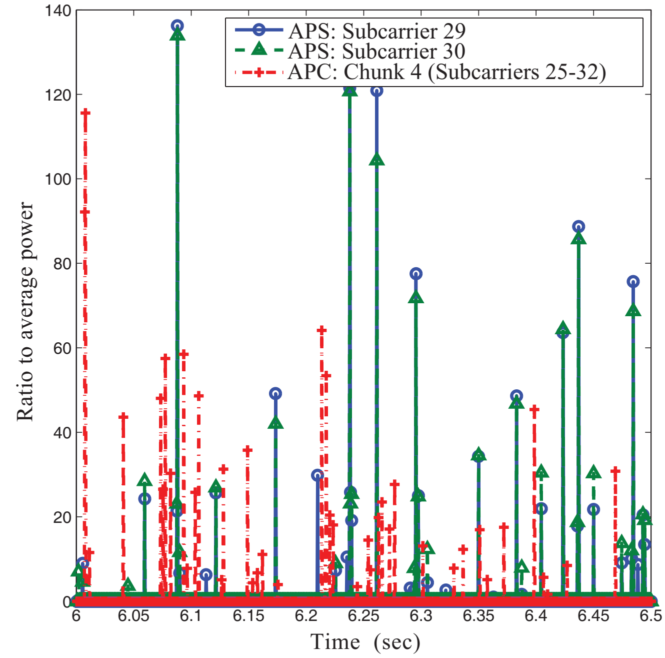


Fig. 3. Ratios of the current power to average power.

that at chunk 4 (subcarriers 25–32) for the APC scheme during the time window 6.0–6.5 second, resulted from the stochastic iterations (17) and (21), respectively, with the power budget $\check{P} = 1$ Watt and the stepsize $\beta = 0.001$. It is clear that implementing adaptive power control results large peak-to-average power ratio (PAPR), whereas the constant power allocation with the OCP scheme maintains a minimum PAPR. To achieve the throughput gain from the APS scheme, coordination of individual subcarriers and very large PAPR need to be dealt with, leading to a high-complexity transceiver design. Compared with the OCP scheme, the throughput gain from the APC scheme is also accompanied by the large PAPR. Therefore, the constant power allocation could be still favored for practical systems in some scenarios from an implementation viewpoint.

To see the effect of utility function selection, the APC scheme based on (21) is employed to solve (18) when different α-fair utility functions defined in (3) are adopted. Fig. 4 (top) shows the average sum-rate of four users, while Fig. 4 (bottom) depicts the resulting average rates for individual users, when the power budget $\check{P} = 1$ Watt and the stepsize $\beta = 0.001$. Since the user-links experience different average SNRs, the resultant average rates differ significantly for small α. It is observed that the fairness improves at the cost of decreasing total throughput as α increases. For instance, when $\alpha = 16$, all users have almost the same average rates, but about 14% total network throughput is lost when compared to the $\alpha = 1$ case. This demonstrates that different α-fair utility functions can trade off network throughput for fairness.

VII. Conclusions

We formulated and solved the optimal chunk allocation for OFDMA downlink transmission with different power control policies. Relying on optimization tools, we proved that the optimal schemes adopt a similar greedy structure and they

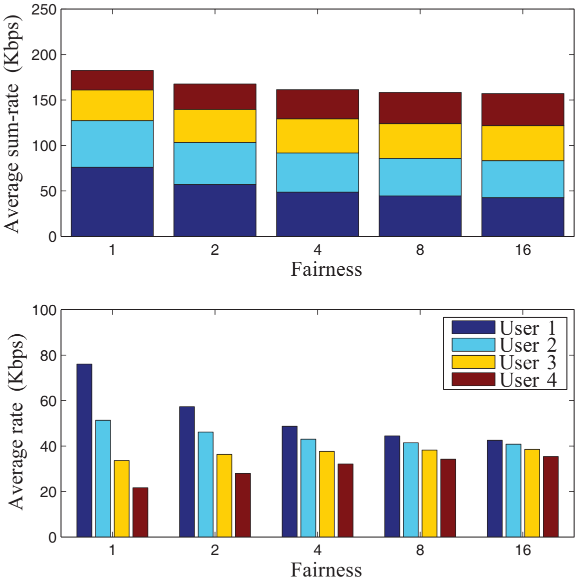


Fig. 4. Average sum-rates of all users (top) and the average rates for individual users (bottom).

can be obtained through dual-gradient iterations with fast convergence and linear computational complexity per iteration. Stochastic optimization methods were also employed to develop on-line algorithms capable of dynamically learning the channel statistics and converging to the optimal benchmarks.

In development of the optimal chunk scheduling, we assumed that the number of chunks and the number of subcarriers in each chunk are predefined by the OFDMA system. It will be interesting to investigate how to perform the subcarrier pairing, i.e., how to determine number of subcarriers per chunk and how to select the (consecutive or interleaved) sets of subcarriers to form the chunks, in the system design, and incorporate it with the optimal chunk scheduling for further performance enhancement. In performance evaluation, we provided the PAPR as an independent performance indicator. Realistically, the PAPR could cause power saturation and this affects the rate and power performance assessment, especially when the nonlinearity of power amplifiers of the transceivers is taken into account. The physical layer effects of the PAPR will be also properly accounted for in future work.

References

[1] J. Chuang and N. Sollenberger, "Beyond 3G: wideband wireless data acces based on OFDM and dynamic packet assignment," *IEEE Commun. Mag.*, vol. 38, no. 7, pp. 78–87, July 2000.

[2] C. Wong, R. Cheng, K. Lataief, and R. Murch, "Multiuser OFDMA with adaptive subcarrier, bit, and power allocation," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 10, pp. 1747–1758, Oct. 1999.

[3] L. Hoo, B. Halder, J. Tellado, and J. Cioffi, "Multiuser transmit optimization for multicarrier broadcast channels: asymptotic FDMA capacity region and algorithms," *IEEE Trans. Commun.*, vol. 52, no. 6, pp. 922–930, June 2004.

[4] G. Song and Y. Li, "Cross-layer optimization for OFDMA wireless networks–part I and part II," *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 614–634, Feb. 2005.

[5] I. Wong and B. Evans, "Optimal downlink OFDMA resource allocation with linear complexity to maximize ergodic rates," *IEEE Trans. Wireless Commun.*, vol. 7, no. 3, pp. 962–971, Mar. 2008.

[6] P. Bender, P. Black, M. Grob, R. Padovani, N. Sindhushyana, and A. Viterbi, "CDMA/HDR: a bandwidth efficient high speed wireless data service for nomadic users," *IEEE Commun. Mag.*, vol. 38, no. 7, pp. 70–77, July 2000.

[7] J. Mo and J. Walrand, "Fair end-to-end window-based congestion control," *IEEE/ACM Trans. Netw.*, vol. 8, no. 5, pp. 556–567, Oct. 2000.

[8] M. Chiang, S. Low, A. Calderbank, and J. Doyle, "Layering as optimization decomposition," *Proc. IEEE*, vol. 95, pp. 255–312, Jan. 2007.

[9] A. Stolyar, "Maximizing queueing network utility subject to stability: greedy primal-dual algorithm," *Queu. Sys.*, vol. 50, pp. 401–457, 2005.

[10] X. Wang, G. B. Giannakis, and A. G. Marques, "A unified approach to QoS-guaranteed scheduling for channel-adaptive wireless networks," *Proc. IEEE*, vol. 95, no. 12, pp. 2410–2431, Dec. 2007.

[11] X. Wang and N. Gao, "Stochastic resource allocation in fading multiple access and broadcast channels," *IEEE Trans. Inf. Theory*, vol. 56, no. 5, pp. pp. 2382–2391, May 2010.

[12] X. Wang, G. Giannakis, and Y. Yu, "Channel-adaptive optimal OFDMA scheduling," *Proc. 41st Conf. on Inf. Sci. and Sys.*, Mar. 2007.

[13] IST-2003-507581 WINNER, "Final report on identified RI key technologies, system concept, and their assessment," Sep. 2005.

[14] 3rd Generation Partnership Project, Technical Specification Group Radio Access Network; Phyiscal layer aspects for evolved Universal Terrestrial Radio Access (UTRA), 3GPP Std. TR 25.814 v. 7.0.0, 2006.

[15] H. Zhu and J. Wang, "Chunk-based resource allocation in OFDMA systems–part I: chunk allocation," *IEEE Trans. Commun.*, vol. 57, no. 9, pp. 2734–2744, Sep. 2009.

[16] H. Myung, J. Lim, and D. Goodman, "Single carrier FDMA for uplink wireless transmission," *IEEE Veh. Technol. Mag.*, vol. 1, no. 3, pp. 20–38, 2006.

[17] H. Lim, T. Yune, and G. Im, "QoS-constrained opportunistic scheduling for SC-FDMA with iterative multiuser detection," *IEEE Commun. Lett.*, vol. 13, no. 1, pp. 4–6, Jan. 2009.

[18] D. Bertsekas and R. G. Gallager, *Data Networks*. Prentice Hall, 1991.

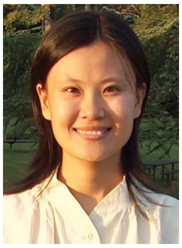
[19] F. Kelly, A. Maulloo, and D. Tan, "Rate control in communication networks: shadow prices, proportional fairness and stability," *J. Operational Research Society*, vol. 49, no. 3, pp. 237–252, Mar. 1998.

[20] D. Bertsekas, *Nonlinear Programming*. Athena Scientific, 1999.

[21] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[22] A. Eryilmaz and R. Srikant, "Fair resource allocation in wireless networks using queue-length-based scheduling and congestion control," *Proc. INFOCOM*, vol. 3, pp. 1794–1803, Mar. 2005.

[23] M. Neely, E. Modiano, and C. Li, "Fairness and optimal stochastic control for heterogenous networks," *IEEE/ACM Trans. Netw.*, vol. 16, no. 2, pp. 396–409, Apr. 2008.



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